An exploratory study of Grade 8 Albanian students' solution processes in mathematics word problems

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Preface

The one year practice that I experienced as a teacher, while enrolled on the Mathematics and Informatics Master Program at “Luigj Gurakuqi” University in Albania, revealed difficulties that students encounter during word problem solution process. Furthermore, it appeared that many students were discouraged to work with word problems. They assumed that, to be solved, word problems require higher mathematical competences than they possessed. This practice experience motivated me to choose the topic for the present dissertation. This study focuses on students’ difficulties and especially on two factors: first, the context (real world or abstract) wherein the problem statement is framed; and second, the provision of the ready-made diagram within the problem statement in the solution process.

It is pleasure to thank those who made this master study possible.

I am grateful to the teacher and the students who participated in the study. I honestly appreciate their willingness to become part of the study and to aid the implementation of the field work.

I owe my deepest gratitude to my supervisor, Professor Simon Goodchild. His guidance, encouragement and support helped me to conduct the study. It is an honor for me to have had him as a supervisor. I am heartily thankful to you!!

I would like to thank all professors of Agder University who during these two years of master program contributed in my development as a researcher and as a teacher.

I offer my regards to all my Albanian and Norwegian friends who supported me in any respect during the completion of this master program.

This study is dedicated to my parents and my fiancé who have always support me and dealt with my absence from family occasions with a smile. I wish you to be proud of me!

Kristiansand, June 2012

Brikena Djepaxhija

Sammendrag

Denne masteroppgaven er en kasusstudie med hensikt å undersøke albanske åttendeklassingers vanskeligheter med å løse tekstoppgaver. Studien undersøker spesielt hvilken rolle to bestemte faktorer spiller. Disse faktorene er problemets kontekst og tilgangen til ferdiglagde diagrammer som representerer problemets bestanddeler og relasjonen mellom dem.

For å identifisere elevers vanskeligheter med tekstoppgaver har jeg brukt Duvals (2006) kognitive tilnærming. I Duvals (2006) teori betraktes omgjøring mellom ulike registre og behandling innenfor et register som to typer kognitive prosesser (matematiske transformasjoner) som er uavhengige kilder til en manglende evne til å forstå matematikk.

Studiens forskningsspørsmål er:

1. Hvilke indikasjoner på omgjøring og/eller behandling avdekkes i albanske åttendeklassingers løsninger av matematiske tekstoppgaver?
   1.1 Hva slags innvirkning har en tekstoppgaves kontekst (abstrakt eller virkelighetsnær) på elevenes prestasjoner?
   1.2 Hva slags innvirkning har inkludering (eller utelatelse) av et diagram i problemformuleringen på elevenes prestasjoner?
2. Hva er elevenes meninger om testen og deres prestasjoner?

Fire ulike metoder, inkludert deltakende observasjon, en penn og papir test med tekstoppgaver, et spørreskjema om testen og om elevenes prestasjoner, og oppgavebaserte intervjuer er kombinert for å samle datamateriale fra elevers arbeid med tekstoppgaver.

Resultatene fra studien viser at elevene har vanskeligheter med å løse tekstoppgaver. Størstedelen av elevenes feil oppstår i omgjøringsprosessene, samtidig som en betydelig del av dem oppstår i behandlingsprosessene. Dette viser at begge de kognitive prosessene (omgjøring og behandling) er kilder til vanskeligheter i problemløsningsprosessen. Videre, i samsvar med Duvals (2004) forklaring, ser de kognitive prosessene bak omgjøring ut til å være de mest komplekse og vanskelige for elevene. Elevenes respons på virkelighetsnære og abstrakte kontekster avdekket at konteksten som en problemformulering ble presentert i hadde lite å si for deres prestasjoner. Deres respons på tekstoppgaver med og uten ferdiglagde diagrammer avdekket derimot noen forskjeller i deres prestasjoner som kan ha oppstått i forbindelse med tilgangen til et diagram. Flere mulige forklaringer på de rapporterte resultatene diskuteres. Studien viser at elever vil ha nytte av å bli instruert i hvordan de kan bruke informasjon gitt i tekstoppgaver.

Summary

This dissertation is a case study, which set out to explore Grade 8 Albanian students' difficulties in mathematics word problems' solution processes. In particular the present study explores the role of two factors, these being the context in which the problem statement is framed, and the provision of ready-made diagram that represent the components of the problem statement and the relation between them.

To identify students' difficulties on word problems, I used Duval's (2006) cognitive approach. In Duval's (2006) theory, conversion between different registers and treatment within the same register are considered as two types of cognitive processes (mathematical transformations) that are independent sources of incomprehension in mathematics learning.

The research questions addressed are:

1. What evidence of Grade 8 Albanian students' conversions and / or treatments is exposed in their solution of mathematical word problems?
   1.1 What is the evidence of influence of the inclusion of problem statements in abstract or real world context on students' solutions?
   1.2 What is the evidence of influence of the inclusion (or omission) of a diagram in problem statements on students' solutions?
2. What are the students' opinions about the test and their performance?

Four different methods including participant observation, a pencil and paper test with word problems, a questionnaire about the test and students' performance, and task based interviews are combined to collect data from students' engagement with word problems.

The results of the study confirm that students have difficulties to solve word problems. The majority of students' errors arise in the conversion processes, nevertheless a considerable part of students' errors also arise in the treatment processes. Thus, showing that both cognitive processes (conversion and treatment) are sources of difficulty in mathematics word problem' solution processes. Furthermore, consistent with Duval's (2004, 2006) explanation, the cognitive processes of conversion appear to be the more complex and difficult processes for students. The students' responses to both real world and abstract context word problems exposed that problem statement context appeared to have little difference on their performance. Whereas, the students' responses to both word problems with and without ready-made diagrams exposed some difference in their performance that might arise from the provision of a diagram. Several possible explanations for the reported results are discussed. The study suggests that students will benefit from instruction in the use of information provided in word problem statements.

Contents

1 Introduction

In teaching and learning mathematics, students are known to experience difficulties with word problems. There are many factors that may affect students' attempts to solve word problems and there is a large body of research conducted to address this issue; this study is intended to contribute to this growing body of research evidence.

This dissertation explores the students 'difficulties in mathematics word problem' solution processes. In particular, the research study reported here explores the effect of two factors on students' performance in word problems these being: the context of the problem, and the provision of 'ready-made' diagrams within the problem statements.

1.1 Researcher's background

An interpretivist approach is taken for the study and I start, therefore, by outlining some details of my own professional background, which will inevitably affect both the methodological decisions and interpretations I make.

I have completed the Mathematics, Bachelor Program of three years (2006-2009) at "Luigj Gurakuqi" University in Shkoder, Albania. During the Bachelor Program, I followed many pure mathematics courses such as Mathematical Analysis, Numerical Analysis, Hilbert Analysis, Complex Analysis, Group Theory Algebra, Linear Algebra, Geometry, Differential Equations, Fourier Series, Probability and Statistics, etc. In other words the Mathematics Bachelor Program equipped me with general mathematics knowledge. To finish the Mathematics Bachelor Program I wrote a paper, which concerns to the application of mathematics in economy.

I then completed the Mathematics and Informatics Education, Master Program of one year (2009-2010) at "Luigj Gurakuqi" University. During this Master Program, I followed many courses such as Mathematical Didactics, Informatics Didactics, Educational Psychology, Teaching Methodology, Sociology and Philosophy of Education, etc. Also during this master program I engaged in six months of passive practice (in which I observed lessons) and then a further six months of active practice (in which I taught lessons) in different grades (from 6th grade until 9th grade) at one elementary school in Shkoder, Albania. To finish this master program I wrote a paper about the applications of Geometer's Sketchpad (software) in the teaching and learning process.

In 2010, I was given the opportunity to follow Mathematics Education, two year Master Program at the University of Agder in Kristiansand, Norway. During this master program I followed courses such as Problem Solving, Teaching and Learning Mathematics, Research Methodology, etc. During this master program, I have understood better what research is about (the concept, the components, etc.) and how it can be conducted; this dissertation is part of this Master Program.

1.2 Word problems in the Albanian school mathematics curriculum

Word problems take an important place in the Albanian mathematical curriculum and students start to solve these kinds of problems from the first grade. My interest in this research area extends two years back in time, when I experienced my practice as a teacher. I recognized, during that time, that students often meet difficulties during the word problem' solution process. Furthermore the students appeared to be separated into two distinct groups, there are those students who try to solve word problems and there are those students who, it appears, do not even attempt to solve the problems. The second group of students justifies their behavior by assuming that word problems are difficult and therefore they are accessible (only) by students with high mathematical competence. This is the reason why I chose to conduct this study focusing on the solutions of word problems from the students' perspective. My purpose in the study is to explore and expose students' difficulties and especially the effect of two factors within the problem statement in the solution process; these being: the context used by the problem, and the provision of ready-made diagrams.

I hope that this study will provide evidence that will help me (as a future teacher), and my (Albanian) colleagues, to understand better students' difficulties (with word problems). Moreover, the evidence will help Albanian teachers to select and to use appropriate word problems (in terms of contexts and diagrams) during lessons.

This research is a case study and its purpose is to explore:

1. What evidence of Grade 8 Albanian students' conversions and / or treatments[1] is exposed in their solution of mathematical word problems?
   1.1 What is the evidence of influence of the inclusion of problem statements in abstract or real world context on students' solutions?
   1.2 What is the evidence of influence of the inclusion (or omission) of a diagram in problem statements on students' solutions?
2. What are the students' opinions about the test and their performance?

1.3 Outline of the study

The study uses data collected from one Grade 8 class of an elementary school in a small city in the north of Albania. Forty-one students (13-14 years old) participated voluntarily in the study. Boys and girls were almost equally represented in the class. The mathematics teacher who supported my data collection is an experienced teacher.

To address the issue it was necessary to collect data from students' engagement with word problems. To achieve this goal several methods were used, including:

- Participant observation: I engaged in participant observation for one week with four Grade 8 classes. The main goals of that week were: the familiarization of students with my presence in the classroom, the identification of the classrooms milieu and the choice of one Grade 8 class to be part of the rest of the study.
- A pencil and paper test with word problems: the purpose of this test was to clarify the characteristics of the students' performance and to pick eight students for the task-based interviews.
- A questionnaire: The questionnaire was used to gather information about the students' opinions about the content of the test and their performance.
- Task based interviews: The purpose of the task based interviews was to expose the students' thinking processes during their performance on word problem.

[1] Conversion and treatment are terms used by Duval (2006) to describe moments in the problem solving process. The terms are explained in Chapter 2.

1.4 Outline of this dissertation

This dissertation consists of six chapters: Introduction, Theoretical Perspective and Prior Research, Methods, Data Analysis, Discussion and Conclusion.

Theoretical Perspective and Prior Research chapter contains four sections.

The first section provides an overview about word problems. The meaning of ‘word problem’ is exposed through a definition and some examples. Furthermore the historical existence of word problems in mathematics followed by a rationale for their privileged position in the school mathematics curriculum, which appears to be related with the word problem functions, is discussed. Then the four types of mathematics tasks (word problems) depending from the starting and goal situations are described, followed by their place within the Albania curriculum. At the end a four stage process of solving words problem is developed.

The second section summarizes briefly the meaning of representations and their main role in mathematics learning and problem solving. Since individual representations cannot be understood in isolation (they exist within a wider system), the meaning of a system of representation is considered through its constituent components. Furthermore the types of representations (internal and external) and the relation within and between them are discussed. Then, Bruner’s (1964) three stage process of human cognitive growth and Goldin’s (2002, 2003, 2008) three stage process of the development of the representational structures are presented and discussed drawing attention to the similarities and the differences between these ‘three stage’ models. At the end of this section the role of representations in mathematics learning and problem solving is presented through some previous studies. In addition, the main role of representations in the process of solving word problem is highlighted in relation to the four stage process developed in the previous section.

The third section contains a general overview of Duval’s (2006) cognitive approach, which is used as the theoretical framework for this study. The important role of semiotic representations in the development of the mathematical thought and their essential function (processing) are exposed. In addition four types of semiotic systems (registers) and two types of transformations of semiotic representations (within and between semiotic systems) are discussed. At the end, Duval’s (2004) three-stage process of solving word problems is presented and compared with the four-stage process (of solving word problem) developed in the earlier section. The fourth section provides an overview of the results of some earlier studies conducted to evaluate the effect of contexts and diagrams on students’ performance in word problems.

The Methods chapter contains nine sections.

The first section describes the choice of methods and the reasons (the methods limitations and advantages) for combining four methods: participant observation, a pencil and paper test with word problems, a questionnaire about the test and students’ performance, and task based interviews. The second section provides an overview about context wherein this study took place. The school context and the participants’ features (students and mathematics teacher) are described there. In addition, the selection method of the (mathematics) textbooks in Albanian schools is exposed.

The third section provides an overview about the design of word problems used in the tests and interviews. The section begins by outlining the goals that led the design of the word problems used in this study. The problems produced were required to be equivalent in terms of mathematical content but varying in respect of context (abstract and real world) and in the provision of ready-made diagrams. Following this is a description of how the students ‘mathematical competences and the curriculum are taken into account in the design of the word problems. Finally, the foundations to design each word problem and ready-made diagram are described. The fourth section describes how some statements (relevant for my study) from a previous questionnaire are adopted, organized and used in this research study. The fifth section describes the implementation of the methods during data collection processes. The first contact with the school’s Principal and students is presented. Further, for each method the date of implementation and the way of being implemented are described.

In the sixth section my conjectures about student performance in relation to the contexts and the ready-made diagrams are set out. Consequently, a conjecture about the best version of the test in terms of the students' improved performance in word problems is raised. The seventh section describes how the analysis for each type of data is carried out. Further, the errors classification and the way the students’ performance is represented in graphs are presented. The eighth section describes the ethical and methodological challenges of the study such as: the anonymity of the school, the teacher and the students, the translation challenge and methods’ limitations. The ninth section discusses issues such as subjectivity, general validity, trustworthiness, etc.

The Data Analysis chapter comprises three sections.

The first section begins by providing a general overview of the students’ performance in the test (in relation with each item and each type of item). Then, in order to show more practically the way these general conclusions are achieved detailed accounts of four individual students’ performance in relation to each item to which they responded in the individual tests are presented. The second section, the questionnaire analysis, sets out findings about the students’ opinions in relation to the test and their performance. The third section sets out by providing a general overview about the students’ performance in the interviews. Then, in order to show more practically the way these general conclusions are achieved, detailed observations of one individual student’ performance in relation with each item he had in his individual test is presented.

The Discussion chapter contains four sections.

The first section provides a discussion about students’ conversion and treatment reported in the data analysis and Duval's (2004, 2006) assertions in terms of these cognitive processes. The second section describes how the findings in this study regarding the effect of the problem statement context (real word and abstract) on students’ performance in word problems confirmed some pervious research (set out in Chapter Two). The third section, based in the design of the study and previous research (set out in Chapter Two and Chapter Three), provides a discussion around the reported results about the interaction of the context and diagram. In the fourth section reported results, about the effect of the ready-made diagram on students’ performance in word problems, are discussed based in the design of the study, the theory and previous research (set out in Chapter Two and Chapter Three).

The Conclusion chapter contains five sections.

The first section provides a brief summary of the dissertation, wherein are described the aims of the research, the methods used to collect data, the study context and the attained results. The second section describes the elements that can affect the trustworthiness of the study such as: the word problems which students faced during the test and the task based interviews, the methods used to collect data and the theoretical framework (Duval’s (2006) cognitive approach) used to analyze the concerned data. The third section describes how close the implementation of the study was to the intended methodology (methods and design). The fourth section provides an overview about the implication of findings in terms of theory (Duval’s (2006) cognitive approach), future research, teachers' perspective and curriculum planners’ and policy makers’ perspective. The fifth section describes what I (as a researcher and as a teacher) have learned during the conduction of the present research.

2 Theoretical Perspective and Prior Research

The expression "problem solving" is used rather loosely in mathematics but it is useful to present problem solving from two points of view: From a cognitive point of view a problem exist for a person when he/she does not have a ready-made strategy to solve the presented mathematical task; From a mathematical point of view tasks can be classified considering two aspects: in which domain of mathematics they are set (algebra, arithmetic, geometry, etc.); and whether the task is a pure or an applied task (Verschaffel, Greer & De Corte, 2000).

In my study I define the word problems as applied tasks which can be from different branches of mathematics.

This chapter is organized into four parts:

- Word problems

In this part I provide an overview about the meaning, the functions, the type and the solving process of a word problem and make reference to the work of van den Heuvel-Panhuizen (2005); Torkildsen (2010); and Verschaffel, et al., (2000).

- Representation

This part summarizes briefly: the meaning of a representation and of a system of representations; the types of systems of representations and the relationship between them; the parallelism between the stages of the cognitive growth of a human and the development of the representational structures; and the main role of representations in mathematics learning and problem solving. Reference is made to the works of Bruner (1964); Duval (2000, 2006); Goldin (2002, 2003, 2008); Goldin and Shteingold (2001); and Preston and Garner (2003); etc.

- Theoretical framework

In this part I explain why I have chosen Duval's (2006) cognitive approach as the theoretical framework for the study; the predominate role that the transformations of semiotic representation plays in any mathematical activity; the four kinds of semiotic systems used for these transformations; and the two kinds of transformations (conversions and treatments) that often are the (independent) sources of incomprehension in mathematical learning.

- Research on context and diagram of word problems

In this part I provide an overview about the results of some earlier studies conducted to evaluate the contexts' and the diagrams' effect on students' performance on word problems. In particular reference is made to the research published by Boaler (1994); Booth and Thomas (2000); De Bock, Verschaffel, Janssens, Dooren, and Claes (2003); Diezman and English (2001); Hart (1996); Movshovitz-Hadar and Shriki (2009); Pantziara, Gagatsis, and Elia (2009); and Pantziara, Gagatsis, and Pitta-Pantazi (2004); etc.

2.1 Word problems

Word problems do not have an agreed definition. One would use words such as text problems, verbal problem, story problem, etc. to construct a definition for word problems. Verschaffel, et al., (2000) offer the following formulation:

> *Word problems can be defined as verbal descriptions of problem situation wherein one or more questions are raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem*

> *statements. In their most typical form, word problems take the form of brief text describing the essentials of some situation wherein some quantities are explicitly given and others are not, and wherein the solver – typically a student who is confronted with the problem in the context of a mathematics lesson or mathematics test – is required to give a numerical answer to a specific question by making explicit and exclusive use of the quantities given in the text and mathematical relationships between those quantities inferred from the text. (p. ix)*

A variety of problems within mathematics can be illustrated with the help of three examples.

Example 1

Evaluate 12+15=?

Example 2

If we have 12 and we add 15. What is the result?

Example 3

Ben has twelve candies. His sister gave him another fifteen candies. How many candies does Ben have now?

The first and second examples are not word problems because they do not describe any existent or imaginable meaningful context. They are only mathematical calculations about operations on mathematical entities where whole numbers and the operations are given. In contrast the third example has a meaningful problem statement that is presented through a story. Students have to discover by themselves which are the quantities to be used and which mathematical operation is to be used to relate these quantities in order to achieve an answer for the problem.

Word problems in mathematics have existed for millennia, for example word problems are found in Egyptian papyri 4000 from years ago; in ancient Chinese and Indian manuscripts; and the Treviso (Italy) arithmetic textbook from 1478 (Colebrooke, 1967; Libbrecht, 1973; & Swetz, 1987; as cited in Verschaffel, et al., 2000). Moreover word problems are also present in the modern mathematics curriculum, but this history raises questions about the rationale for the privileged position of word problems.

Verschaffel, et al., (2000) propose that word problems serve a number of functions, they explain:

1) *"Application function"* – the application of knowledge required to solve a word problem that is placed into a certain situation.
2) *"Motivation function"* – since word problems are presented in a certain context they engage students in their solution because the context convinces the students that mathematics is necessary for the life out of school.
3) *"Selection Function"* – to evaluate the new generation's mathematical competencies that in the future will guide and serve society.
4) *"Thought provoking function"* - to develop students' creative thinking and problem-solving competencies.
5) *"Concept formation function"* - word problems with their advantage of having context can help students to construct new mathematical concepts. (pp. xi-xii)

It is evident from the functions that word problem are assumed to accomplish that the context they carry has an important role. Suitable contexts can be formed using real world situations or the formal world of mathematics if they are “real” in the students’ minds (van den Heuvel-Panhuizen, 2005). I will discuss this in more detail in section number 2.4.1.

Mathematical tasks have both a starting and a goal situation. Depending on whether their situations are exactly described (explained) or not they can be characterized into four types as in Table 1 (Torkildsen, 2010).

Task type	Starting situation	Goal situation
A	Closed	Closed
B	Closed	Open
C	Open	Closed
D	Open	Open

Table 1. Different types of tasks; From Torkildsen (2010) table 12.1

Example of a type A task (closed starting point, and closed goal): Find out the number of calories which your friend Sara consumes during one day (24 hours) if she eats and drinks the ingredients presented in the Table 2:

Morning 200 gr - Tine Ekstra Lett Melk (Extra Light Milk)	100gr/162kj
Lunch 250 gr – Barilla Penne Rigate Integrale 75gr – Barilla Saus (Sauce) 100 gr – Tine Kremost (Cream Cheese) 20gr - Pulver Kaffe (Coffee)	100gr/1487kj 100gr/460kj 100gr/1152kj 100gr/1499kj
Dinner 300 gr – Tine Naturlig Yoghurt (Natural Yoghurt)	100gr/310kj

Table 2. Ingredients and Calories

Example of a type B task (closed start, open goal): Find out the approximate number of calories that you consume during one day (24 hours).

Example of a type C task (open start, closed goal): Find out the number of calories which your friend Sara consumes during one day (24 hours) if she eats and drinks the ingredients presented in the Table 3:

Morning 200 gr - Tine Ekstra Lett Melk (Extra Light Milk)
Lunch 250 gr – Barilla Penne Rigate Integrale 75gr – Barilla Saus (Sauce) 100 gr – Tine Kremost (Cream Cheese) 20gr - Pulver Kaffe (Coffee)
Dinner 300 gr – Tine Naturlig Yoghurt (Natural Yoghurt)

Table 3. Ingredients

Example of a type D task (open start, open goal): Find out the approximate number of calories which a human consumes during one day (24 hours).

Van den Heuvel-Panhuizen and Van Rooijen noted two features of word problems in relation to their starting and goal situations. The operations in "*most cases*" can be taken directly from the text and "*nearly always*" the word problems have just one answer (1997, as cited in Torkildsen, 2010, p. 186). The above sentence leaves open the possibilities that word problems can be of all four types (closed-closed; closed-open; open-closed; open-open). Often in recent years, open-end problems (closed-open, open-open) are referred to as reform oriented and their role esteemed. Unfortunately these kinds of problems are unknown for Albanian students since they are not yet included in the Albanian mathematics curriculum. Therefore I will not consider these types of problems in this case study.

In this study I define the process of solving word problems in mathematics to consist of four general steps as in the following scheme (Figure 1).

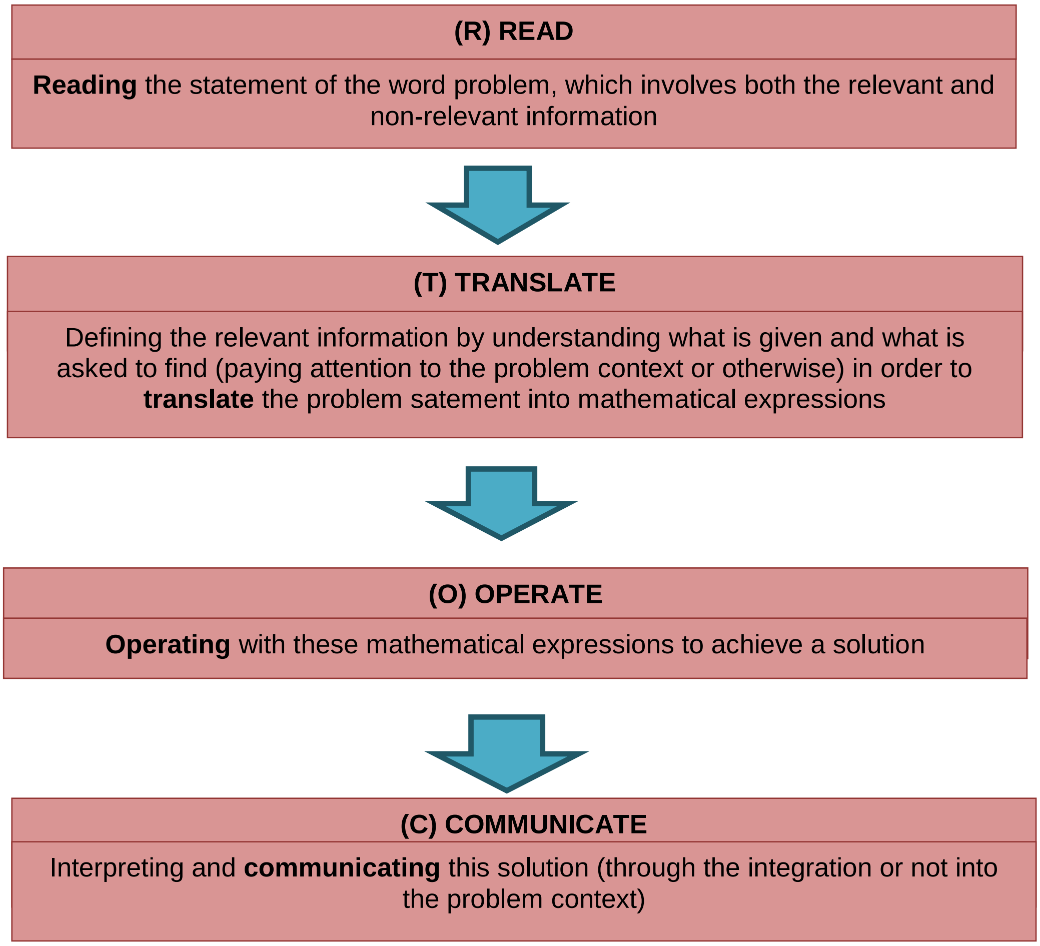


Figure 1. The four stage process of solving word problems.

2.2 Representation

According to Duval (2006, p. 103) "*A representation is something that stands for something else*". This definition leaves open the representing relationship (stand) and the types of configurations (something). In a more rigorous definition, produced by Goldin (2003), the representing relationship is determined.

> *A representation is a configuration of signs, characters, icons, or objects that can somehow stand for, or "represent", something else. According to the nature of the representing relationship, the term represent can be interpreted in the many ways including the following (the list is not exhaustive): correspond to, denote, depict, embody, encode, evoke, label, mean, produce, refer to, suggest, or symbolize. (p. 276)*

It is essential to compare external and internal representations because in this way we see the interaction between them and their characteristics. Some examples of external representations are: written words, numerals, graphs, algebraic equations, etc. While in internal representations include: natural language, visual and spatial imagery, problem solving heuristics, etc. Unlike external representations, internal representations cannot be observed directly. The external and internal representations have a bidirectional relation in terms of representing each other (Goldin 2002, 2003, 2008). Internal representations represent external ones, for example when the teacher is explaining something, the student visualizes internally (mentally) the information. Conversely, external representations represent internal ones, for example the student uses graphs to express his\her idea.

Individual representations cannot be understood in isolation, they exist within a wider system. A typical example in this case is the Cartesian graph which belongs to a system of conventions for associating pairs of numbers with points in the plane by means of orthogonal coordinate axes (Goldin 2002, 2003, 2008).Therefore, in the context of the study, it is important to clarify the meaning of a representational system. According to Goldin (2002, 2003, 2008) the meaning of a representational system can be achieved by considering:

- Primitive components

They form a class of elementary characters and signs but the relationship between them does not exist in this step. These characters and signs may be part of well-defined sets (for example, the letters of an alphabet, arithmetic symbols, etc.) or partially-defined sets (for example, the real-life objects, etc.)

- Configurations

Here the rules to combine elementary characters and signs take place. These rules may be well-defined (for example, the rule of combining letters and operation signs to form the formula for the area of a circle) or partially-defined (for example, the rule of combining numerals, letters and operation signs to form a mathematical equation).

- Structures within representational systems

At this stage complex structures are involved such as configurations of configurations, mathematical operations, etc. Here the rules may be well-defined (for example, rules for moving from one configuration [from an equation] to another [to graph]) or partially-defined (for example, algebraic rules to transform and solve an equation).

Goldin has drawn attention to five types of internal cognitive systems of representations.

> *1. Verbal/syntactic systems which include natural language capabilities, lexicographic competences, verbal association, as well as grammar and syntax.*
>
> *2. Imagistic systems, including visual/spatial, tactile/kinesthetic, and auditory/rhythmic encoding.*
>
> *3. Formal notational systems, including the internal configurations corresponding to learned, conventional symbol systems of mathematics and how to manipulate them.*
>
> *4. A system of planning, monitoring and executive control that guides problem solving, including the strategic thinking, heuristics, and much of what are often referred to as meta-cognitive capabilities.*
>
> *5. An affective system that includes not only the "global" affect associated with relatively stable beliefs and attitudes, but also the changing states of feeling as these occur during mathematical learning and problem solving. (2002, pp. 211-212; 2003, p. 278; 2008, p. 182)*

An individual's internal representation of a^2 may include: its pronunciation "a squared"; a verbal phrase such as "the area of the square side a"; or a visual/spatial image such as that of a square with side a. This example demonstrates that internal representations can represent each other in many different ways.

Duval (2000, 2006) classifies semiotic systems in four types: Natural language (orally explanations, written theorem, written proofs, etc.); Symbolic (written computation, etc.); Non-iconic (different geometrical figures) and Iconic Images (drawings, sketches, etc.); and Diagrams and Graphs. He stresses the transformations importance of semiotic representations within each other, "*unlike the other areas of scientific knowledge, signs and semiotic representation transformation are at the heart of the mathematical activity*" (Duval, 2006, p. 107). (For more detail see the theoretical framework, section 2.3)

A person is not borne provided with knowledge and systems of representations constructed, they develop over time during the person's life. It is an ontogenetic process rather than a phylogenetic process. Bruner (1964) identifies the cognitive growth of a human with a skilled activity, where to achieve the maturation the human needs to combine the simple components (by which the skilled activity is composed) into an integrated segment.

Bruner (1964) proposed a three stage process of the cognitive growth of a human based on fundamental form of representation:

1. *"Enactive representation"*- the past events are represented through appropriate (effective) "*motor response*". In other words, during this stage the thinking process of the child is based on physical action, he/she learns by doing. For example, tying knots, riding a bicycle, swimming, etc.
2. *"Iconic representation"* (from 18 months) - the events are summarized by choosing the precepts and images, "*by the spatial temporal and qualitative structures of the perceptual field and their transformed images".* In other words, during this stage the thinking process of the child is based on mental images. For example, reading a map.
3. *"Symbolic representation"* (from 6-7 years) - features which include "*remoteness"* and "*arbitrariness"* are designed to represent things. For example, explaining by words the school location. (p. 2)

One then might inquire into the sequence of the development of learned representational competence.

Goldin has proposed a three stage process of the development of the representational structures:

1. The first one is "*inventive\semiotic"* stage – based on of previous representations new internal configurations are constructed.
2. The second stage is a period *of "structural development"* - the earlier system is used as a template to construct the structure of the new system.
3. The third and last one is an "*autonomous"* stage - the new system detaches from the earlier one because it already has its own meaning. (2002, p. 212; 2003, p. 279; 2008, p. 183)

Even though, Bruner (1964) and Goldin (2002, 2003, 2008) have each developed three stages processes for different aims (respectively, Bruner for the cognitive growth of a human and Goldin for the development of the representational structures) these processes have some points in common. Their entrance during the cognitive growth of a human or the development of the representational structures, for each of the above three stage processes, follows the same order. Each subsequent stage is dependent upon the pervious stage for its development.

Moreover there appears to be a parallelism between each stage of each above development processes. However the authors have used different terms to describe each stage of the development process, terms that are appropriate for their theories.

In section number 2.3, I will explain Duval's (2006) cognitive approach, which is the theoretical framework of this study. In Duval's (2006) theory terms such as iconic and symbolic representations are also used, but they are used for different purposes than that of Bruner (1964). Bruner (1964) has used iconic and symbolic representation terms to describe the second and third stages of cognitive development through which the human passes to achieve his\her cognitive growth. While Duval (2006) has used these terms for other purposes that will be discussed in section number 2.3.

2.2.1 The main role of representations in mathematics learning and problem solving

> *Representations can work as vehicles to understand and interpret mathematical ideas and tools for developing individual strategies in problem solving, as they may well operate as multiple instances of a concept. A certain concept or mathematical structure can be subsumed under different representations and the discovery of common properties in such representations contributes to the assimilation of the concept or structure. (Amado, Carreira, Nobre, & Ponte, 2010, pp. 138-139)*

Representations have an essential role when introducing mathematical concepts. According to Goldin, (2002, 2003, 2008), in order to learn a new mathematical concept it is necessary to develop strong and flexible internal systems of representation. However to develop a new internal system of representation the previously developed systems have to be used as a template. For example, Goldin and Shteingold (2001) in their study introduced the concept of negative numbers, for first and second grade children, using previously developed representations such as a number line representation in one case and signed cardinality representation in another case.

The role of representation in problem solving (on the reasoning process, strategy selection and solution communication) is emphasized by Preston and Garner (2003). In their study, middle grades students worked on "the class party problem". In which three types of celebration were presented, students had to select the best one focused mainly on the price per person to be present in these parties. For more detail consider the class party problem below:

> *The Class Party Problem*
> *Ms. Simpson formed a committee of students to investigate sites for a class party. The committee does not know how many of the students will attend and has not yet decided whether each class member can invite a guest. However, they have brought back the following details on parties:*

- *Water World: Swimming, hot dog, chips and drink. Cost: $100 to reserve the pool and $5 per person.*
- *Pizza Pi and MoviePlex: Pizza, drink, movie. Cost: $10 per person.*
- *Skate' Til Late: Skating and ice cream (eat before you come). Cost: $200 to rent the skating rink and $2 per person for skate rental.*

> *If price is the primary focus of your decision, which party option is the best?*
> *How would you convince other class members of this?*
> *How would you advise other groups planning a party, so that they could make decision, no matter how many guests they had? (Preston & Garner, 2003, p. 40)*

The results showed that students used different kinds of representations (such as tables, graphs, equations and word rules, etc.) to evaluate each version of the party, to conclude which party option is the best and to communicate it to other class members. They also tried to convince each other of the obtained results through different representations. Almost the same conclusion about the role of representations in mathematics learning and problem solving is achieved by Greeno and Hall (1997). They conclude that students' performance in problem solving depends on their skills to use different forms of mathematical representations to construct and communicate their understanding.

In the preceding section (2.1 word problems) the steps through which the solutions of a word problem proceed have been outlined. The representations are present from the first step to the last. In the first step the student reads the problem statement that is presented through external representations; in the second the student uses internal (mental) representations to understand and to organize the information of the problem and then translates these internal representations into external (semiotic) ones to build mathematical expressions; in the third and the last step also the translations among external and internal representations are used to achieve a solution and to communicate it. It is for this reason that I have looked into the literature to explain representations in some detail because through them every word problem and its solution process can be analyzed.

2.3Theoretical framework

I chose the Duval's (2006) cognitive approach as theoretical framework for my study because it is interesting and relevant for my theme. Duval's (2006) cognitive approach can be used as a tool to analyze any mathematical activity and to search for the root of difficulties that many students have with understanding mathematics. Therefore, it also can be used to analyze a specific mathematical activity such as solving word problems and to find the root of difficulties that students meet during their solution process.

The important role of semiotic representations is emphasis by Duval (2006), he consider them as the main "*condition for the development of mathematical thought*" (p. 106). Semiotic representations are present in any mathematical activity because, the only way to have access to mathematical objects is through semiotic representations. The way that the semiotic representations are used and the cognitive requirements involved have to be considered in any mathematical activity. Semiotic representation systems can be used to design, to communicate and to work on mathematical objects but their essential function (property) is processing, which consists in the transformation of one representation into another (Duval 2000, 2004).

Any mathematical activity requires using different semiotic representation systems. Therefore any mathematical activity requires developing internal co-ordination between each pair of semiotic representation systems used. In this way, semiotic representations must not be confused with mathematical objects themselves, because the same mathematical object can be represented by two or more semiotic representations that are produced by different semiotic systems. Precisely here lies the source of possible cognitive conflict, this concerns how mathematical objects can be distinguished by the semiotic representations used when the only way to have access to them is through these semiotic representations (Duval, 2004, 2006). The relation between semiotic representations and conceptual understanding is illustrated in Figure 2. An example of the relation between semiotic representations and conceptual understanding is presented in Figure 3. The Content A of representation in this case is a function which is generated by the mono-functional and non-discursive symbolic register. The Content B of representation is a function which is generated by the mono-functional and non-discursive graph register. In both cases one can identify the same mathematical objects, such as one independent variable x and one dependent variable y and the relation between them (the dependent variable y is two time the independent variable x).

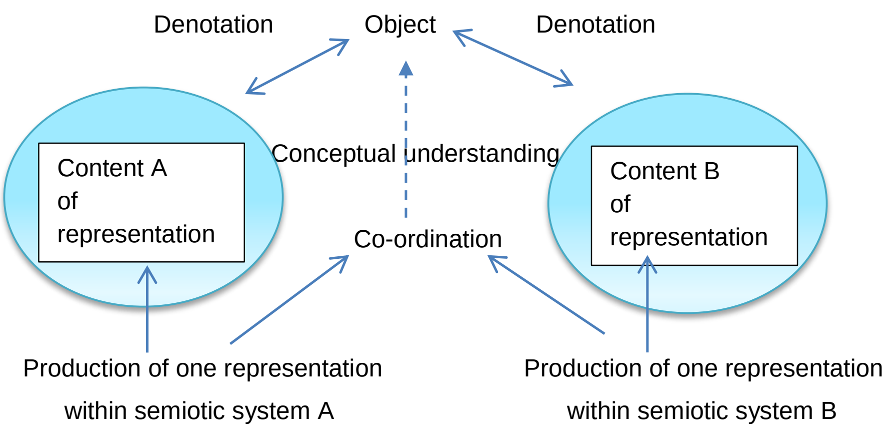


Figure 2. Cognitive conditions of mathematics understanding; From Duval (2000, p. 1-65) figure 6

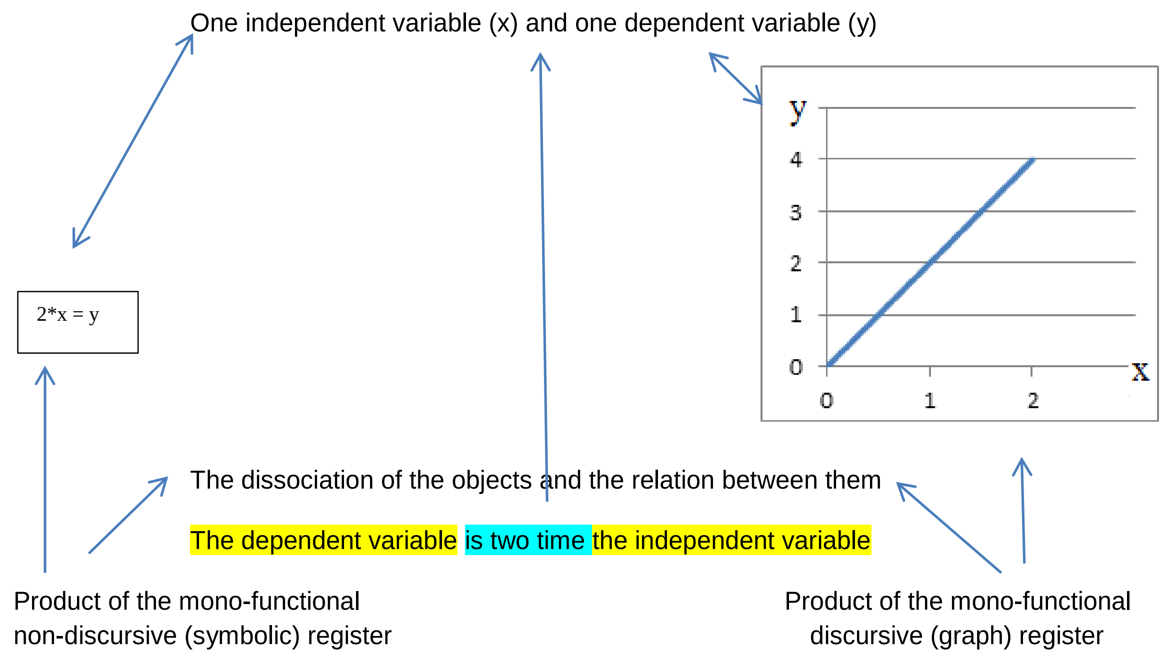


Figure 3. An example of cognitive conditions of mathematics understanding

The meaning and capacity of each semiotic representation depends on the semiotic systems wherein they are produced. To determine the types of semiotic systems Duval (2000, 2006) considers the process through two steps:

1) Opposing the discursive (natural language; symbolic language) with non-discursive (iconic images [drawings, sketches, etc.]; non-iconic images [geometrical figures]; and graphs and diagrams).

2) Dividing the semiotic systems on the basis of their cognitive functions in mono-functional (here mathematical process takes the form of an algorithm, for example, solving a system of equations, see Figure 5) and multi-functional (here mathematical process does not have the form of an algorithm).

According to Duval (2006) four types of semiotic systems are formed (see Figure 4):

- The first semiotic system is the multi-functional (communication, information processing, etc.) system of natural language, here is included oral and written language.
- The second semiotic system is the mono-functional (mathematical processing) symbolic system.
- The third semiotic system is the multi-functional (awareness, imagination, etc.) system of iconic and non-iconic figures.
- The fourth semiotic system is the mono-functional (mathematical processing) diagrams and graphs system.

	REPRESENTATIONS resulting from one the three kinds of DISCURSIVE OPERATIONS: 1. *Denotation of objects (names, marks…)* 2. *Statement of relations or properties* 3. *Inference (deduction, computation…)*	NON-DISCURSIVE REPRESENTATION (Shape configurations 1D/2D, 2D/2D, 3D/2D)
MULTI-FUNCTIONAL REGISTERS: Processes CANNOT BE made into algorithms	IN NATURAL LANGUAGE: two nonequivalent modalities for expressing - ORALLY *explanations*, ?? - WRITTEN (visual): *theorem proofs…*	ICONIC: drawing, sketch, pattern NON- ICONIC: geometrical figures which can be constructed with tools
	Transitional AUXILIARY Representations *No rules of combination (free support)*	
MONO-FUNCTIONAL REGISTERS: Most processes are algorithmic	IN SYMBOLIC SYSTEMS Only written : *impossible to tell orally otherwise than by spelling* *Computation, proof*	D2 COMBINATION OF D1 AND D0[2] SHAPES, oriented (arrows) or not. *Diagrams, graphs*

Figure 4. Classification of semiotic systems; From Duval (2006, p.110) figure 1

These four semiotic systems are called *"representation registers"* (Duval, 1995, p. 21, as cited in Duval, 2006) since they permit the transformation of representations.

[2] It is not clear why Duval uses symbols D2, D1 and D0.

Two types of transformation of semiotic representations are identified Treatment (curved arrows in Figure 4) and Conversion (straight arrows in Figure 4). Treatment type of transformation occurs when the transformations of representations happen within the same register. Conversion type of transformation occurs when the transformations of representations consist of changing the register and keeping the denoted object unchanged (Duval 2000, 2004, 2006). I developed two examples (see Figure 5 and Figure 6) to explain practically these two types of transformations.

The sum of three given numbers is seventy-two. The half of the first number is equal to the third of the second number and the fourth of the third number. Which are these three numbers? (Gazidede, 2007, p. 145)	This example is a word problem that is given through a multi-functional register such as natural language.
$\begin{cases} x+y+z=72 \\ \frac{1}{2}*x=\frac{1}{3}*y \\ \frac{1}{2}*x=\frac{1}{4}*z \end{cases}$	This transition is the translation of problem statement into mathematical expressions. It is a **conversion** from source register into target register that in this case is the symbolic mono-functional register. This **conversion** is **a congruent one** (that looks like a simple coding or like one to one mapping between all constituents of source register and the target one) because the sentences in natural language are converted into algebraic expressions keeping the order of signs, by translating each word to its similar algebraic symbol.
$\begin{cases} x+y+z=72 \\ \frac{3}{2}*x=y \\ 2*x=z \end{cases}$	The transitions that take place in these steps are operations with mathematical expressions to achieve a solution They are called **treatments** within the symbolic register.
....	
$\begin{cases} x=16 \\ y=24 \\ z=32 \end{cases}$	The last transition is done to communicate the solution. It is another **congruent conversion** from symbolic register into natural language register.
The first number is sixteen, the second twenty-four, and third thirty-two.	

Figure 5. Example A, Concrete examples of two types of transformations of semiotic representations

<table>
<tr><td>A pizzeria serves two round pizzas of the same thickness in different sizes. The smaller one has a diameter of thirty centimeters and costs thirty zeds. The larger one has a diameter of forty centimeters and costs forty zeds. Which pizza is better value for money? Why? (OECD, 2006, p. 25)</td><td>This example is a word problem that is given through a multi-functional register such as natural language.</td></tr>
<tr><td>$A = \pi r^2 = \frac{1}{4}\pi d^2$;
$A1 = \frac{1}{4}\pi d1^2 = \frac{1}{4}\pi 30^2 = 225\pi \text{ cm}^2$; $A2 = \frac{1}{4}\pi d2^2 = \frac{1}{4}\pi 40^2 = 400\pi \text{ cm}^2$</td><td>This transition is the translation of problem statement into mathematical expressions. It is a conversion from source register into target register that in this case is the symbolic mono-functional register. In this case the conversion is non-congruent because it is not possible to get the same semantic segment of the problem data within the linguistic statement and within symbolic statement.</td></tr>
<tr><td>$\begin{cases} 30 \text{ zeds} \leftrightarrow 225\pi \text{ cm}^2 \\ 1 \text{ zed} \leftrightarrow x \end{cases} \rightarrow x1 = ?\ \text{cm}^2$
$\begin{cases} 40 \text{ zeds} \leftrightarrow 400\pi \text{ cm}^2 \\ 1 \text{ zed} \leftrightarrow x \end{cases} \rightarrow x2 = ?\ \text{cm}^2$</td><td rowspan="3">During these transitions many treatments are done within symbolic register to achieve the solution.</td></tr>
<tr><td>…………………………………………………………</td></tr>
<tr><td>$x1 = 8.5\,\pi \text{ cm}^2$
$x2 = 10\,\pi \text{ cm}^2$</td></tr>
<tr><td>The larger pizza is the better value for money because it offers bigger area for one zed than the smaller one.</td><td>To communicate the solution another non-congruent conversion is used, from symbolic register into natural language register.</td></tr>
</table>

Figure 6. Example B, Concrete examples of two types of transformations of semiotic representations

From a mathematical point of view treatments and conversions are mixed together in the problem solution process. Usually treatments constrain the choice of target register wherein the conversion has to take place, in order that the target registers to be relevant to carry out treatments powerfully. While from a cognitive point of view the two types of transformation correspond to different cognitive processes that often are the sources of incomprehension in mathematical learning. Conversion appears to be a more cognitively complex process because going from one semiotic system into another taking unchanged the denoted object is a difficult cognitive transition that students often cannot overcome (Duval, 2004, 2006).

According to Duval (2004) the process of solving word problems consists of three general steps and various semiotic representations needed for posing and solving the task. In the first step one faces the scenario and the question of the word problem (both the relevant and non-relevant information are involved), which are evoked through semiotic representations produced by the natural language register. This step is analogous to the first step (R) through which the solutions of a word problem proceeds which I set out in section 2.1. In the second step a conversion into a target register takes place which is about the identification of the relevant information in order to choose the appropriate arithmetic operations or the unknown quantity which would be denoted with a letter, etc. Another parallelism between this step and the second (T) step, through which the solutions of word problem proceeds, as I outlined in section 2.1, is evident here.

In the third step a range of treatments (within the register where the conversion take place) can be employed to achieve the solution of the problem. One more parallelism between this step and the third step (O) through which the solutions of a word problem proceeds as outlined in section 2.1, is again evident at this point. It is apparent that Duval's (2004) three steps process of solving word problems does not consider the last step (C) through which I have proposed that the solutions progress as I outlined in section 2.1. In Duval's words this step can be explained as another conversion from the source register into the target register (usually, natural language register) to generate an appropriate answer for the problem.

2.4 Research on context and diagram of word problem

Students may experience difficulties in each step through which the solutions of a word problem proceeds (these steps have been outlined in section 2.1). In the first step (R) the misreading of the word problem may be apparent. In the second step (T) a lack of comprehension of the problem statement or difficulties in translating the problem statement into mathematical expressions may be apparent. In the third step (O) the absence of an appropriate strategy or difficulties in translating the mathematical expressions from one form to another may be apparent. In the fourth step (C) the difficulties in translating the mathematical solution into an appropriate answer to the problem may be apparent. Moreover numerous factors may affect students' attempts to overcome the above difficulties. The effect of two factors such as the context of the problem and a provided 'ready-made' diagram with the problem statement will be explored (evaluated) in the present study. In the following sections, I present the results of some earlier studies, which explored the effect of context and diagram.

2.4.1 Context

In this study I will categorize the context of word problems into two broad groups: abstract and real world. The term "abstract context" will be used to define word problems in which the text involves mathematical world situations (or uses mathematical objects) such as: *"One circle has a diameter of 12 centimeters. Another circle has a radius of 7 centimeters. Which circle has a greater area? Explain your reasoning."* (Larson, Boswell, Kanold, & Stiff, 2008, p. 529). While the term "real world context" will be used for word problems in which the text involves a real world situation (or uses real world objects) such as: "*Heather is working out/calculating her income tax for the year. She earned $5,367. The tax-table shows that for incomes between $5000 and $6000, the tax is $200 plus 15% of any amount over $5000. How much tax will Heather owe?"* (Public Schools of North Carolina, n.d, p. 34).

Several studies have been initiated with the hypothesis that students' performance will be improved by introducing word problems in realistic context such as Boaler (1994), De Bock, et al. (2003), Movshovitz-Hadar and Shriki (2009), etc.

In the study by De Bock, et al. (2003), the performance of 13-14 years old and 15-16 years old students on non-proportional geometric word problems (about the relationship between lengths, areas and volumes) have been tested. The same word problems were formulated in a 'realistic' context using the fantastic world of Lilliputians (in the world of the Lilliputian, all lengths are 12 times as small as in our world, the world of Gulliver), for example, *"In Gulliver's world, a cheese cube has a volume of 172 800 mm^3. What is the volume of a Lilliputian cheese cube?"*(p. 448) and in abstract context using the mathematical world, for example, *"The side of a cube I is 13 times as large as the side of a cube J. If the volume of cube I is 2197 cm^3, what is the volume of cube J?"* (p. 448). The results showed that students who were set realistic context items performed worse than others. 41% of students gave correct responses on non-proportional abstract items and only 25% on realistic items.

Almost the same result was achieved by Boaler (1994), where students and especially girls performed better in problems presented in an abstract context than in their counterpart problems presented in real world context such as a “fashion problem” (where the jobs of five persons who work in a fashion workshop have to be organized). In addition, Movshovitz-Hadar and Shriki (2009) conducted a study where the role of real context in which the introduction of the logic of mathematics was examined. In this case the realistic context was given through children’s literature such as Alice’s adventures in wonderland. Fourteen of the eighteen students that participated in this study expressed their satisfaction to learn the logic of mathematics in this context. Teachers were convinced of the efficacy of context because through it they can enter in the children’s world and speak their language during the mathematical lessons. However after some lessons using the Alice book students were tested (using realistic context problems) and the final grades of eighteen students compared with previous year’s groups of students (who learned the logic of mathematic in abstract context) produced no significant differences. De Bock, et al. (2003) explained the results by conjecturing that students are used to work with abstract word problems since these types of problem are usually present in their mathematical lessons at school. Another conjecture they suggested was that when students experience realistic word problem, they engage more in the context of the problem, which directed students away from its deeper mathematical structure.

Smith (2003) used data from a previous study he had conducted (Smith, 1999, as cited in Smith, 2003) to examine the construction and the use of representations from children’s perspectives. The fresh analysis exposed that when students are engaged within real contexts they see little need to move from their idiosyncratic to general (mathematical) representations. In other words, they design representations to solve particular problems, not as tools for general use. On the other hand students who build general representations tend to strip the real context away thus losing the benefits that realistic context is expected to provide. However to understand and communicate mathematics a bridge between these two kinds of representations is necessary.

Some other studies such as Duval (2004), Graumann (1989), and Hart (1996) (in contrast with the above studies) highlight the importance of the real world context providing evidence for its positive effect on students’ performance.

From an educational point of view the problems of daily life are positively evaluated. To be solved, these problems require students to call on their everyday experience and mental representations. Therefore, students can comprehend the mathematical concepts and then could make sense of the semiotic representations used (Duval, 2004). Graumann (1989) emphasized the positive role of geometric problems in real world contexts and at the same time gave some real situations wherein the real world context may be established. For example, handicraft and planning: building a house, extending a loft, painting the room, etc.

Hart (1996) conducted a study wherein the effect of the personalized context (the real world context which students might find enjoyable, interesting and humorous) on students’ attitudes and abilities toward solving word problems was examined. Hart was a sixth-grade mathematics teacher and she carried out the study with her class. Over eight weeks of the study she transformed the standard text book word problems into their counterparts into personalized contexts and used both the standard textbook form and personalized forms in her teaching. In a subsequent test the students performed better on the personalized form of the problems than on the standard textbook form. From the achieved results Hart (1996) concluded that personalized contexts activate students' schemata, aiding them to build a bridge to connect old and new knowledge. Therefore students have higher achievement on these types of problems. Additional data such as a student questionnaire at the end of the study revealed that students were motivated to solve personalized word problems. However, the generalizability of these results is not to be taken for granted given the localized and subjective nature of the research and strong possibilities of a Hawthorne effect occurring.

At the outset, the examination of the above literature (Boaler, 1994; De Bock, et al., 2003; Movshovitz-Hadar & Shriki, 2009; & Smith, 2003) leads me to conjecture that in my study the real world context would not affect positively the Grade 8 Albanian students' performance. Albanian students are familiar with traditional word problems (which I have classified as abstract). Therefore the real world context may be an obstacle in their performance. It may cause uncertainty among students because they may be confused for the reason that they have not solved such problems before. Another conjecture based on De Bock, et al. (2003) was that the Albanian students will engage more in the context of the problems than in the mathematical content involved there. However, after I read the Duval (2004); Graumann (1989); and Hart (1996) studies, my above conjectures (about the effect of the real world context) were shaken. Based on the examination of these studies, I guess that the real world context would affect positively the Albanian students' performance when it is interesting and familiar for the students. It is factual that Albanian students are used working on traditional word problems but if the real world context would be interesting and familiar for them (as a new experience) it could motivate students to work on real world problems. Therefore they could use their life experience and their old knowledge (mental representations) to pass successfully through each step of the process of solving word problems.

2.4.2 Diagrams

A diagram is a visual representation which presents information in a spatial form. Moreover to use a diagram successfully one needs to understand that a diagram represents the components of the situation and their organization (Diezman & English, 2001).
Often the term diagram is considered equivalent with terms such as picture and drawing. However important differences exist between these terms. As mentioned above, the diagrams represent the structure of the problem, wherein the surface details are not important. While pictures and drawings show more surface details such as the form of the components, the color of the components, etc. (Diezman & English, 2001).

Booth and Thomas (2000) and Dirkes (1991) evaluate the importance of a diagram in the problem solving process as a strategy used to solve word problems (Dirkes, 1991) and as a useful tool in the solution of the problem (Booth & Thomas, 2000). A diagram is not necessary for the solution of a problem but it may support the process of problem solving (Gagatsis & Elia, 2004). Diagrams could be used *"to unpack the structure of a problem"* (Diezman & English, 2001, p. 77) in order to help students to understand better what is given and what is required to be found. Furthermore it could be used to reorganize information in a way that makes sense for students, and to *"lay the foundation for its (the problem's) solution"* (Diezman & English, 2001, p. 77), helping students to find a correct solution strategy. According to Duval (2004) auxiliary representation can accomplish only a specific function in problem solving, a function qualified to imagination (iconic representations), to conversion (non-iconic representation) or to treatment (non-iconic representation). However what matters is to find a variety of relevant representations and to co-ordinate them.

Pantziara, Gagatsis, and Elia (2009) conducted a study to explore the role of diagram in the problem solving process. 194 Cypriot students in grade six (12 years old) were presented with two tests: test A (comprised of six non-routine problems without diagrams) and one week after with test B (comprised of six [isomorphic in relation with the problems of test A] non-routine problems but in this case each problem was accompanied by a diagram). The analysis of the data from this study revealed no statistically significant difference in terms of correct and incorrect responses between two tests. However, the results show that the performance of a significant proportion of students who encountered difficulties to solve the problems in the test A was markedly improved in test B. In other words the diagrams provided a significant aid for these students. On the other side another significant proportion of students who succeeded to solve the problems of test A did not succeed to solve the isomorphic problems that included diagrams in the test B. Similar results were provided earlier by Booth and Thomas (2000). They concluded that the presented diagrams were useful for some students (with average spatial abilities). However, they were not useful for some other students (with low spatial abilities), who cannot identify the structure of the problem in the presented diagrams.

In the study by De Bock, et al. (2003)13-14 years old students and 15-16 years old students were tested to show their performance on non-proportionality (about the relationship among the lengths, areas and volumes) on geometric word problems. Students were divided in two groups. The first and second groups had to solve the same non-proportional word problems, but the non-proportional word problems of the second group were accompanied by diagrams of the objects introduced into the problems and students were asked to complete the diagrams before starting to solve word problems. The study set out the hypothesis that such diagrams will have a positive effect on improving students' performance, but results showed the opposite. The second group of students had a worse performance, in other words a negative effect of these diagrams was evident.

Diezman and English (2001) and Pantziara, Gagatsis, & Pitta-Pantazi (2004) suggest that to improve students' performance on problem solving it has to be accompanied by the development of students' diagrammatic literacy through instructional activities. In this way the diagrams could be used by students as effective tool for thinking. The above assumption is supported by studies such as, Hamaker and Van Essen (1990); Ng and Lee (2009) and Willis and Fuson (1988). In these studies students are taught (instructed) to construct diagrams to represent the information presented in problem statements and to use them to solve the word problems. The results of these studies showed that after students took instructions in terms of how to construct and use diagrams their performance on word problems improved. These results emphasize the helpful role of the diagram in the solution process of a word problem.

In the Albanian curriculum a lot of word problems which are accompanied with ready-made diagrams are involved, therefore the teachers are expected to have instructed the students to use these diagrams effectively. Furthermore, from my experience as student and as student-teacher, I identified many cases in which Albanian teachers orient students to construct a diagram before starting to solve the problem. The customary approach in Albanian lessons (in terms of using diagrams while are solving word problems) and the examination of the above literature leads me to conjecture that in my study the ready-made diagrams would affect positively the Grade 8 Albanian students' performance on word problems. The term ready-made diagram, in this study, is used to name the diagram which is presented within the problem statement of the word problem.

2.5 Summary

This chapter has presented relevant literature to provide an overview in the field of inquiry. At the beginning the notion of a word problem was explained through: a definition, some examples, the functions of a word problem, the types of word problems, and the four stage process of solving a word problem. Then, the meaning of representations (system of representations) and their main role in the process of solving word problems were set out. Further in this chapter detail of Duval's (2006) cognitive approach are given, this is adapted and used to analyze students' responses to word problems. The important role of semiotic representations in the development of mathematical thought, their essential function, four types of semiotic systems and two of representations' transformations (treatment and conversions) are reported. In this theory, conversions between different registers and treatments within the same register are considered as two types of cognitive processes that are independent sources of incomprehension in mathematics learning. In addition, Duval's (2004) three stage process of solving word problem is presented and compared with the four stage process of solving a word problem set out earlier. Finally the chapter has presented the results of some earlier studies conducted to evaluate the context of the problem and the presence of a diagram in the problem statement on students' performance to word problems.

The literature presented in this chapter provides the foundation for the research questions of this inquiry, these being:

1. What evidence of Grade 8 Albanian students' conversions and / or treatments is exposed in their solution of mathematical word problems?
   1.1 What is the evidence of influence of the inclusion of problem statements in abstract or real world context on students' solutions?
   1.2 What is the evidence of influence of the inclusion (or omission) of a diagram in problem statements on students' solutions?
2. What are the students' opinions about the test and their performance?

The issues presented in this chapter constitute the basis for the construction of the study design which is discussed in the next chapter. The next chapter will present and justify: the choice and the implementation of methods, the context wherein the study took place, the design of word problems, the adaptation of a previous questionnaire, the way wherein each type of data is analyzed, my conjecture about students' performance, the ethical challenges and trustworthiness.

3 Methods

This methods chapter is divided into nine parts:

- The choice of the methods

In this part I describe the methods I chose to use (to collect data) and the reasons for combining four methods: participant observation, a pencil and paper test with word problems given to one Grade 8 class, a questionnaire that asks students to reflect on the test, and clinical interviews with a small sample of students.

- The research context

Here I describe the school and class context where this study took place.

- Design of the word problems for the test and the task-based interviews

In this part the approach to designing the word problems used in the tests and interviews is described.

- The adoption of some statements from a previous questionnaire.

In this part I describe how some statements (relevant for my study) from a previous questionnaire are adopted and used in this research study.

- The implementation of the methods

In this part I describe how methods were implemented during the data collection process.

- Hypotheses

In this part I set out my conjectures about student performance in relation to the contexts and the ready-made diagrams.

- Analysis of the data collected.

Here the approach to data analysis is explained. The largest part of data is analyzed on the basis of Duval's (2006) cognitive approach.

- Ethical and other challenges

In this part ethical and methodological challenges presented by the study are described such as: the protection of identities of the school, the teacher and the students, the translation challenge (from Albania into English) and the methods' limitations.

- Trustworthiness

Here issues such as subjectivity, general validity, trustworthiness, etc. are discussed.

3.1 The choice of the methods

Before starting to organize my research I had an idea about what I wanted to study in my research. I wanted to explore the difficulties which students meet in the mathematics word-problem' solution process. Also, I wanted to explore the effect of the context and ready-made diagrams in the students' word problem solution process. I discussed this idea with a mathematics teacher at an elementary school and I asked if she might help by allowing me to collect data in her classes. The teacher expressed enthusiasm about my idea and recommended me to focus on the Grade 8 word problems because she would teach in Grade 8 in the academic year that I would collect data. Thus, the context and sample of students used in this study might be best described as ‘convenient’ rather than representative.

In the middle of November, when my data collection would take place, the students were expected to have worked through at least the four first chapters of their mathematics textbook (Lulja & Babamusta, 2007). Therefore I focused on word problems that to be solved required the mathematical knowledge students were expected to have appropriated in the first and the third chapters of their textbook. I focused on the mathematical content of these two chapters (fractions, percentage, and the length of line segments) because this mathematical content offered enough scope to focus on word problems in different contexts and different types of ready-made diagrams.

My research questions are:

1. What evidence of Grade 8 Albanian students’ conversions and / or treatments is exposed in their solution of mathematical word problems?
   1.1 What is the evidence of influence of the inclusion of problem statements in abstract or real world context on students’ solutions?
   1.2 What is the evidence of influence of the inclusion (or omission) of a diagram in problem statements on students’ solutions?
2. What are the students’ opinions about the test and their performance?

To answer these research questions it is necessary to collect data from students’ engagement with word problems (in different contexts and in the presence or the absence of the ready-made diagrams). To achieve this goal I can use several methods but each of them offers both strengths and its limitations. Therefore, I decided to combine methods to collect data, in order to address the limitations of one method by another method.

At the beginning I thought to use participant observations to collect my data. In this way, at the same time I could observe and interact with students while they are working with different word problems during their mathematics lessons. However, then some limitations became apparent. If I were to use a video camera to record these mathematics lessons I would risk affecting the performance of some students because they might feel the pressure of the video camera. However, if I were not to use a video camera I would risk losing a part of data because of the impossibility of memorizing in detail and being able to report in a trustworthy manner everything that happens in 45 minutes. By using participant observations I also risked gaining access to some students performance because of the time restriction but also due to the possible reluctance of students to express their thinking (in relation to their understanding of the word problem and their solution strategy) in front of their classmates. Therefore to avoid the above constraints I decided to use another method to collect data; that is a pencil and paper test.

In the test, it was intended that pupils would work individually (which is the customary working approach in Albanian classrooms) and if they have questions during this process I would be present to clarify their misunderstandings without, I hope, affecting their thought processes. However, if the test were to take place before participative observations, my presence in the class (during the test) might affect students’ performance because I would be previously unknown to the students. Moreover they might hesitate to ask an unknown person questions they may have in relation to the test. Also, only being present in the class during the test I would not be able to explore the classrooms milieu. This is important because it is necessary to have some knowledge of the context within the research took place. Therefore to avoid the above restrictions I decided to make participant observations one week before the pencil and paper test was to take place. Another advantage that I can gain from participant observations is the possibility to choose between four different Grade 8 classes within the school. I would choose the class where I experienced the greatest diversity of students' in terms of mathematical performance (from low through to high mathematical level).

I decided to use another method, a questionnaire (that has been used in a similar study and developed by De Bock, et al., (2003)) in order to gain access to students' opinion in relation to: test difficulties, if they felt satisfied with the test, if test were similar with the tests they completed before, if they used the diagrams, if the diagrams helped them, etc. The students would complete this questionnaire individually after they finished the test. In this way, each student would have the opportunity to express his/her personal opinion without being affected by their peers. I hoped this would also help to avoid time restrictions placed on my fieldwork activity in the class.

By the pencil and paper test I hoped to obtain an insight to the students' proficiency in solving word problems and an indication of the source of error where they occurred (conversion processes/ treatment processes) in the problem solving process. The information that comes out from this type of data is limited because it cannot expose the students' thinking processes. To avoid these limitations I thought to use task-based interviews but it would lead me into another constraint, this being the restricted time available. Therefore I decided to use both methods. Moreover on the basis of students' performance in the test I would choose which students would be interviewed. The teacher suggested choosing students with average and higher mathematical competencies because this group of students would be better able to express their thinking orally.

3.2 The research context

The data collection was done in one Grade 8 class of an elementary school. It is the largest public school located in a small city in the north of Albania. The school was founded in September 1967. At that time, in this school 407 students were organized in 12 classes (or groups), while at the time of writing 1116 students are organized in 37 classes. In this school 50 teachers are responsible for the different school subjects. Boys and girls are almost equally represented in the school. The language that is used in this elementary school is Albanian.

There are many mathematics textbooks approved by the Albanian Ministry of Education and Sciences for Grade 8 (also for other grades) of elementary school. These books are written by different authors. The mathematics teachers of the school have the responsibility to choose the mathematical textbook, which they think is most appropriate to be used. All Grade 8 students of the school have to work from the same textbook that their teachers choose. In this way, their peers in another school may not work with the same mathematics textbooks. The mathematics teachers of this school have chosen to work with Matematika 8 (Lulja & Babamusta, 2007).

Forty-one Grade 8 students (13-14 years old) participated in the study. Students came from the same class of this elementary school. Boys and girls were almost equally represented in the class which participated in my study. The students' participation in my study was voluntary. The mathematics teacher who supported my data collection is an experienced teacher (with about 24 years as a teacher in the elementary school).

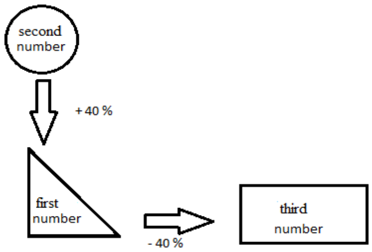
3.3 Design of the word problems for the test and the task-based interviews

When I began to design the word problems (based in the notion of a word problem, outlined in Chapter Two) for the test and task-based interviews my goal was to produce two word problems which in relation with mathematics content would be equivalent but in different contexts (abstract and real world). Another goal, but not less important, was to produce for each word problem an effective ready-made diagram, which represents the components of the situation and their organization (Diezman & English, 2001). In order to achieve my goal effectively I needed to take into account the students' mathematical competence and the curriculum.

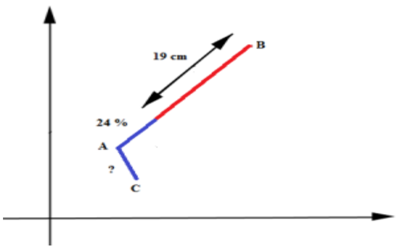
At the beginning I referred to the mathematics textbook (Lulja & Babamusta, 2007) which students use. Moreover I considered that the students were expected to have covered the content of at least the four first chapters of their textbook. Therefore I was looking for the mathematical content involved in these chapters, for the context of the word problems and for the diagrams. I noticed that in the first and third chapters they have more work with word problems. Moreover, the mathematical content involved in these chapters (percentage, fractions, decimal numbers, the point, the segment, the line, the angle, etc.) offered sufficient scope to produce word problems in different contexts and different types of ready-made diagrams. However, I still lacked the mathematical level of Grade 8 students (in the elementary school where I expected to collect data). Therefore I decided to contact the teacher and to ask her about students' mathematical competencies. She helped me by sending some word problems that she prepared for her students taking into account their level of performance. Then based on these problems' level of difficulties I formed an idea what mathematical level of difficulties I should involve in my problems.

The first and the second items (which I produced) are closed-closed word problems (see Chapter Two) and to be solved require knowledge that students were expected to have appropriated from the first chapter (from the lesson 1.3 Perqindja [The Percentage] until the end of this chapter) of their textbook (Lulja & Babamusta, 2007).
The abstract contexts of these two word problems (1Abs and 2Abs) are produced by framing the problem statements into mathematical world situations. Mathematical objects such as numbers, the coordinate plane, points and the distance between two points are used to build these mathematical world situations. The above mathematical objects are common for students because they often encounter them in school mathematics (word problems). The ready-made diagrams of the first and the second abstract word problems are built using the components of each mathematical world situation (numbers, the coordinate plane, points and the distance between two points) and the relation between these components.

1Abs (abstract context) % Bigger & % Smaller Problem
Three numbers are given. The first number is forty percent bigger than the second number. The third number is forty percent smaller than the first number.
Can you say that the second and the third numbers are equal? Why?
Can you find the sum of these three numbers if the second number is one-hundred-fifty? (Please explain briefly how you approached this problem)

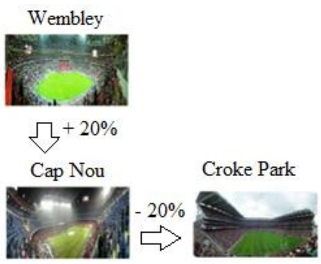


2Abs (abstract context) % Difference Problem
In the coordinate plane are given three points A, B, and C. The distance between point A and point C is twenty-four percent of the distance between point A and point B. Knowing that the difference between these two distances is nineteen centimeters, can you find how far away the point C is from point A? (Please explain briefly how you approached this problem)



The real world context of the first task (1Rel) is produced by framing the problem statement into a real word situation. Real world objects such as the three largest stadiums of Europe measured by the number of the spectators they can accommodate are used to build this real word situation. I think this real world context would be interesting for students because soccer is a popular sport in Albania and students (girls and boys) are fans of different national and international teams. The ready-made diagram for this problem is constructed using the components of the problem statement which in this case are three stadiums, and the relation between them by the number of the spectators they can accommodate.

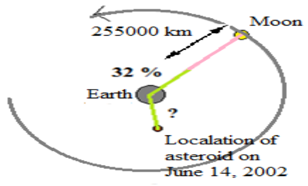
1Rel (real world context) Stadiums Problem
The capacity of stadiums is measured by the number of the spectators they can accommodate. Europe's three largest stadiums are: "Cap Nou" stadium in Barcelona city in Spain, "Wembley" stadium in London city in England and "Croke Park" stadium in Dublin city in Ireland. "Cap Nou" stadium has the capacity twenty percent bigger than "Wembley" stadium. "Croke Park" stadium has the capacity twenty percent smaller than "Cap Nou" stadium.
Can you say that "Wembley" and "Croke Park" stadiums have the same capacity? Why?
Can you find the capacity of three stadiums together if the capacity of "Wembley" is ninety thousand spectators? (Please explain briefly how you approached this problem)



The problem statement of the second problem (2Rel) is framed into a real world situation by using 'real world' objects such as objects in the earth's near space, the earth, the moon and an asteroid. I think this real world context would be interesting for the students because sometimes Albanian' news (for example on television) involves information about the relation of objects in space on certain days. The ready-made diagram for this problem is constructed using these three near space objects, the relation between them and how those objects might be represented in the space.

2Rel (real world context) Asteroid Problem

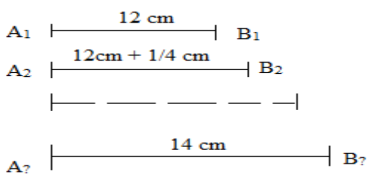
On June 14, 2002 an astronomer measured the distance between the Earth and a traveling asteroid, which was near to the Earth. He found that the distance between the Earth and the asteroid was thirty two percent of the distance between the Earth and the moon. Knowing that the difference between these distances was two hundred fifty five thousand kilometers, how far away from the Earth was the asteroid at that time? (Please explain briefly how you approached this problem)



The third and the fourth items are closed-closed word problems (see Chapter Two) and to be solved require also knowledge that students were expected to have appropriated from the first chapter (from the lesson 1.1 Thyesat dhe Numrat Dhjetore [The Fractions and The Decimal Numbers] until the end of this chapter) of their textbook (Lulja & Babamusta, 2007).
The abstract context of these two word problems (3Abs and 4Abs) is produced by framing the problem statements into mathematical world situations. Mathematical objects which students meet often in their school mathematics such as segments, sequence of segments, rectangles are used to construct these mathematical world situations. The same objects and respectively the relation between them are used to build the ready-made diagrams for the third and the fourth abstract word problems.

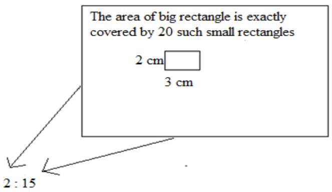
3Abs (abstract context) Sequence Problem

It is given the sequence of segments $[A_1 B_1]$; $[A_2 B_2]$; $[A_3B_3]$; … where each segment is one-quarter centimeters bigger than the previous segment. If the first segment $[A_1 B_1]$ is twelve centimeters, which segment of the sequence has the length fourteen centimeters? (Please explain briefly how you approached this problem)



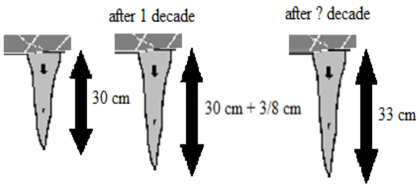
4Abs (abstract context) Rectangles Problem

The area of a big rectangle is exactly covered by twenty small rectangles with dimensions two centimeters and three centimeters. Can you find the perimeter of big rectangle if its dimensions are in the ratio two over fifteen? (Please explain briefly how you approached this problem)



The problem statement of the third word problem (3Rel) is framed into a real world situation by using stalactites as an example of a real world object. I think the real world context would be interesting for students because from the geography courses they followed in the previous years they are expected to have learned about stalactites. In these conditions students will use the knowledge which they took from another school subject to imagine the situation in which this word problem is set. The same objects (exactly the sketches of the stalactites) and the relation between them over decades are used to build the ready-made diagram.

3Rel (real world context) Stalactites Problem
Stalactites are icicle-shaped stone formations found on cave ceilings. They form from minerals deposited by dripping water. Suppose a stalactite is thirty centimeters and is growing at a rate of about three over eight centimeters per decade (ten years). How long it will take for the stalactite to reach a length of thirty three centimeters? (Please explain briefly how you approached this problem)



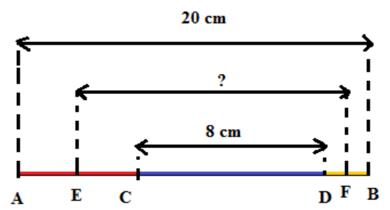
The statement of the fourth problem (4Rel) is framed into real world situation by using the real world object such as “Mother Teresa” piazza (a famous piazza in the Albania’ capital), and the tiles which will be used to fix the floor. I think this context would be interesting for the students because the construction of this piazza is much discussed by Albanian’ television programmers. The components of the problem statement (the same objects as above) and relation between them are used to build the ready-made diagram.

4Rel (real world context) "Mother Teresa" Piazza Problem
A construction company has won the tender to fix the floor of "Mother Teresa" piazza in Tirana city, which has a rectangular form. To fix the floor of this piazza the company will use forty thousand rectangular tiles with dimensions thirty centimeters and one hundred centimeters. Can you find the perimeter of "Mother Teresa" piazza, if its dimensions are in the ratio five over six? (Please explain briefly how you approached this problem)

The fifth and the final word problem to be solved requires knowledge that students were expected to have appropriated from the third chapter (from the lesson 3.4 Matja e Segmenteve

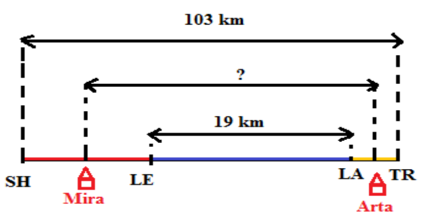
(Gjatesia e Segmenteve) [The Measurement of The Segments (The Length of The Segments)]). The statement of this problem (5Abs) is framed in the mathematical world situation by using common mathematical objects for students, in particular, as line segments. This item (in abstract context) is a closed-closed word problem (see Chapter Two). Also the ready-made diagram in this problem is built using the same mathematical objects and the relation between them.

5Abs (abstract context) Line Segment Problem
One segment with the length twenty centimeters is divided into three not equal segments. The length of the middle segment is eight centimeters. Can you find the length between two middle points of the biased segments? (Please explain briefly how you approached this problem)



The statement of the fifth problem (5Rel) is framed into real world situation by using the real world objects such as Albanian' road segments (Shkoder – Tirane road segment). I think that this real world context would be interesting for the students because from their geography courses they are expected to have learned about this road segment. Therefore they can see the connection between the knowledge obtained from two different school subjects. Also this road segment connects their city with the north and the center of Albania. Therefore they are expected to know this road segment from their everyday life. This item (in real world context without diagram) is an open-closed word problem (see Chapter Two). The ready-made diagram in this problem is built using the sketch (map) of this road segment and the relation between sections of this road segment.

5Rel (real world context) Road Segment Problem
Mira and Arta study in the University of Shkodra. Every Friday after school they take Shkoder – Tirane bus line to go home. Mira lives in a small village situated in the middle of the Shkoder -Lezhe road segment. Arta lives in a village situated in the middle of the Lac – Tirane road segment. The length of the Shkoder - Tirane road segment is one hundred and three kilometers, while the length of the Lac – Lezhe road segment is nineteen kilometers. Can you find how far away from Mira lives Arta? (Please explain briefly how you approached this problem)



As I hope is evident from the above, I aimed to set all problems into the real world contexts which are expected to be familiar to the students and additionally to be interesting contexts for them. After I produced these tasks, before using them for the test and for the interviews I discussed them with the teacher who was collaborating with me. In some problems she recommended me to re-formulate the text of the problems statement in order to be clearer and the teacher worked with me to effect improvements to the problem statements.[3]

3.4 Adoption of some statements from a previous questionnaire

As mentioned in Section 3.1, to gain access to students' opinions about the test and their performance I decided to use a questionnaire. While I was reading an article about the effect of contexts and diagrams (De Bock, et al., 2003) I encountered a questionnaire used in the different experimental conditions that was created and piloted in an earlier study by De Bock, et al., (2003). I took from this questionnaire only those statements that appeared relevant to my study. I organized the questionnaire into nine statements (see Table 4). Six of them are general statements about the content of test and students' performance and three diagrams related statements (in the Table 4 the general statements and diagrams related statements are divided by the red line).

	Strongly agree	Agree	Not sure	Disagree	Strongly disagree
1) I think I will have a good score on this test.					
2) I liked to work on this test.					
3) The problems are similar to the problems we solve in the classroom.					
4) I did my best to work on this test as much as possible.					
5) I considered this as an easy test.					
6) It was an instructive experience to complete this test.					
7) While solving the word problems, I made use of the diagrams above.					
8) The diagrams helped me to find the answer.					
9) I usually make a diagram before solving a word problem.					

Table 4. Questionnaire used in the different experimental conditions

[3] The real world contexts of problems number 2Rel and 3Rel are adapted from Larson, et al., (2008) because they appear familiar, interesting and relevant in the Albanian context. It needs to be highlighted that these two problems are not equivalent to the problems by Larson, et al., (2008, pp. 345 & 247) but, I developed these problems by adapting the context of the Larson, et al., (2008, pp. 345 & 247) problems.

3.5 The implementation of the methods

My data collection started on November, 7 and lasted until November, 21. During this time I engaged in the following:

1- Participant observations in four classes
2- One test for the whole of one class
3- One questionnaire for the whole (tested) class
4- Task-based interviews of eight students (chosen from the tested class)

The first contact

The first day in the school the teacher presented me to the Principal, from whom I sought permission to gather the data. She accepted me with a simple condition that was to present my project for all mathematics teachers of the school. Then the teacher introduced me to the four Grade 8 classes of this elementary school and with a few words I introduced my project in front of students. I explained that participation in my project was voluntary but students did not show any unwillingness to participate in my project. On the contrary they were so enthusiastic and when I said that after a week of observation I will choose only one class to be part of the rest of my research (test, questionnaire, task-based interviews), they started a competition between classes to demonstrate who performs better.

1- Participant observations in four classes

I engaged in participant observation (helping students with exercises and problems, teaching some lessons etc.) for one week with all four classes. During my participant observation I had some main purposes:

1. I wanted students to become accustomed to my presence in the classroom in order that my presence in the rest of the study would not have a negative impact;
2. I wanted to explore the classrooms milieu;
3. I wanted to choose the class with the students' greatest diversity in terms of mathematical level;

I believe that my goals were achieved because, at the end of the week students were accustomed to my presence, I gathered some information about classrooms milieu, and I chose only one class for the rest of the study, the 8^c class, because there I found the greatest diversity in terms of students' mathematical performance (from low through to high mathematical level).

2- One test for the whole class

The test was completed on 15/11/2011. The purpose of this test was to clarify the characteristics of the students' performance (students' conversions and treatments) in each version of problems and to pick eight students for the task-based interviews. This test was planned to last for 45 minutes (real time of a lesson) but we used an additional 15 minutes from another lesson, so the test lasted 60 minutes. All participants (41 students of 8^c class) were provided with a test paper which consisted of 4 + 1 (four obligatory and one voluntary) problems, two about percentage, two about fractions and one about the length of the line segments. The students had not previously encountered any of the problems included in the test. Students were allowed to use their own scientific calculator while doing the test in the hope that this would minimize calculation errors.

Four different versions of the test were administered: problems with real world context (R), problems with abstract context (A), problems with ready-made diagrams (+) and problems without ready-made diagrams (-), making four types of combination (R+, R-, A+, A-). Students were divided randomly into four groups (as shown in Figure 7): the first group were set the A- model of test, the second were set the R- model of test, the third group were set the A+ model of the test and the fourth group were set the R+ model of the test. Students were organized in this way to prevent copying and encourage them to work individually, which is the customary working approach in Albanian classrooms.

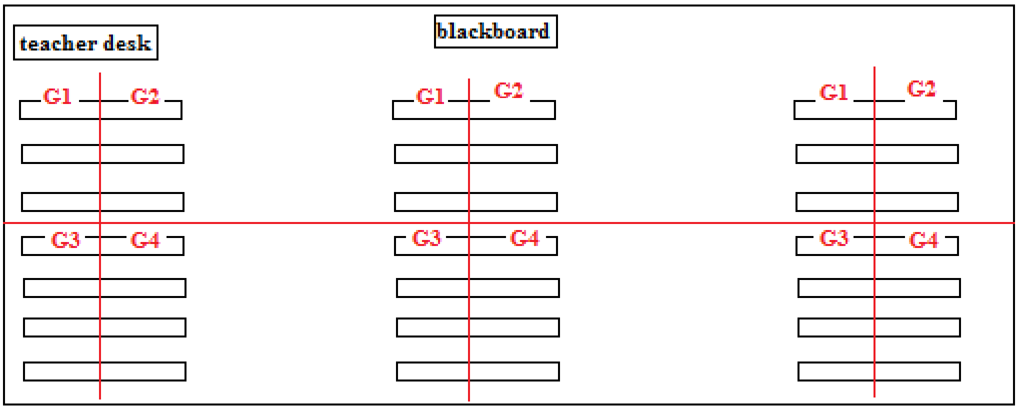


Figure 7. The scheme of the class division into four groups

3- One questionnaire for the whole class

When students finished their test, they were asked to complete the short questionnaire described above, which had to be rated on a five-point scale (from full disagreement to full agreement). As noted in Section 3.4, I organized the questionnaire into nine statements. Six of the statements were general and submitted to all participants, while the other three statements were about the diagrams and were only submitted to the half of the class that had ready-made diagrams in their tests. The questionnaire was used to provide information about the students' opinion (affective feelings) about the test and their performance while they were working on these types of tests.

4- Task-based interviews of eight students

The task-based interviews took place over three days (17, 18, 21 / November/ 2011) and eight students took part in these interviews. The teacher (allowed by the Principal of the school) offered me a room in the school to conduct the interviews. During each interview only I and the interviewed student were present. The interviews took place during school time, therefore each student was absent from one or two class(es) to be part of the interview. Each of the task-based interviews lasted around 50-60 minutes. The interviews were audio and video recorded. The camera was focused on the student's paper work, and not on their face.

In the task based interviews students were asked to solve and to reason aloud for the test questions. Students who were given abstract problems in the pencil and paper test were given real world problems in the interview and vice versa. Therefore, in the interview students met problems that they had not encountered earlier in the written test. In the interview initially no diagram was provided, the diagram was given in a later stage after students had engaged with the pure text version of problem statement. At the end of the interviews they were asked to compare between two tests that they had done (the interview test and the class test).

I chose two students from each group to be interviewed. I chose these students according to the teacher suggestions in relation to their (average and higher) mathematical abilities. I also tried to choose students who made errors that were similar to others in their respective groups. I contacted the teacher and asked her to reflect and comment on my choice because I had to know if these students might be too shy or intimidated by the interviews situation. After the teacher's confirmation, I asked selected students if they would be part of the interviews and I did not meet any refusal from their part. The main questions that I have used during interviews were:

- What did you understand by the problem?
- How do you think to solve it?
- Did you understand the given diagram?
- Can you link it to the problem?
- Did the diagram help you to solve the problem?
- Can you compare between problems of two tests?
- Which test was easy? Why?

3.6 Hypotheses

The first hypothesis is related to real world and abstract contexts of word problems. The reader should recall that the real world contexts are produced by using real life information these being:

- The three largest stadiums of the Europe measured by the number of the spectators they can accommodate;
- The distances from the earth to the moon and from the earth to an asteroid in a certain day;
- The growing rate of stalactites on cave ceilings;
- The area of "Mother Teresa" piazza in Tirana City;
- The length of the road segment Shkoder – Tirane;

My conjecture was that placing students into real and rich contexts would increase their attention. It would be more attractive for them to work on these types of word problems. Students would call on their everyday experience to understand the situation wherein the problem statement is framed and to proceed through each step of the word problem solution. Therefore the students will perform better on these types of word problems than on abstract context word problems which they are usually taught.

The second hypothesis is related to the ready-made diagram, which represents the structure of the problem statement. I predicted that the diagram will facilitate students' attempts to find the relevant information in the problem statement and its organization. Therefore it will affect students' thinking and will help them to find a correct solution strategy. Consequently I conjectured the students will perform better if a ready-made diagram is provided in the problem statement.

Based on the above hypotheses I conjectured the version R+ (real world context with diagram) of test would be associated with the highest level of student's performance on word problems.

3.7 Analysis of the data collected

The following data were collected:

- Notes from the participant observations
- Students' written responses during the test
- Students' written responses from the questionnaire about the test and their performance
- Videos' recordings from eight task-based interviews
- Students' written responses during the task-based interviews

The observation notes were mostly used to describe the research context.

The students' solutions of the mathematics word problems during the test and the task-based interviews are analyzed using Duval's (2006) cognitive approach. In both cases the students' written responses are semiotic representations which are translated within the same register or into various registers mobilized during their solutions process. The errors which students made during the solution process are classified into conversion errors (errors that happen in conversion processes while they were changing the register) and treatment errors (errors that happen in treatment processes while they were making transformations within the same register).

All forty one students' written responses during the test were analyzed individually. In detail I analyzed each student' written response for each item of the test. My intention was to explore if her/his written response about each item were correct (all conversions and treatments are correct), incorrect (errors in conversion, errors in treatment, errors in both conversion and treatment) or if she/he missed it (blank). In addition I intended to explore also if there (in the student' written response) were sufficient evidence to allow the interpretation or possible identification of the source of the error. It needs to be emphasized that these source of error come out from my assertions based on the available evidence. However, the actual source of error is not explicitly evident in students' written responses to the test because in this type of data it is impossible to identify exactly the cause of the (conversions and treatments) error. After I analyzed each test paper (as above), I presented some of the results graphically[4]. For example, Figure 8 shows how many students from group A- (abstract word problem without diagram) solved correct, incorrect, or left blank each item of their test.

[4] The X^2 test is used to explore for any difference between the groups in their performance in the test but, it did not expose statistically significant differences.
Therefore the outcome of this analysis is not presented in the Data Analysis Chapter. An example of the results is reproduced in Appendix 14: X^2 test, Pencil and paper test results.

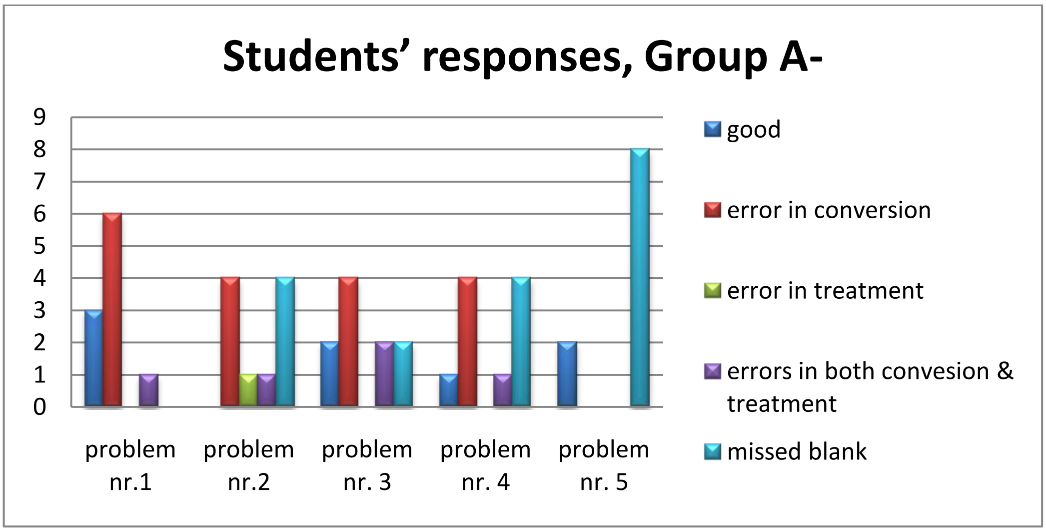


Figure 8. Students' responses, Group A-

The available evidence exposed by the analysis (described above) is used to provide an overview of (groups A-; A+; R-; R+) students' performance on each word problem during the test. I also present the individual analyses of four students (one for each group) in order to clarify (to illustrate) practically how the test analysis is done.

The analysis of the task-based interviews starts during the interview process. Wherein, I followed the student performance (students' written responses and their comments about these responses). Sometimes I posed questions to explore the students' thinking process in different situations when their comments were not clear. Thus, a part of the task-based interviews analysis is made during the time that these interviews are conducted. However, the analysis does not end there. The eight students' written responses during the task-based interviews were analyzed in detail in the same way as the students' written responses during the test (see paragraph above). However, in this case the students' written responses are combined with the video transcriptions[5] of the interviews in order to expose sources of errors. After I analyzed each task-based interview, I presented a part of the achieved results graphically. For example, Figure 9 shows the number of students' correct, incorrect and missed (blank) responses in word problems during the interviews.

[5] All interviews are transcribed. The video transcriptions are not included in the appendices for two reasons:
1) The language in which the video transcriptions are written. They are written in Albanian language.
2) The volume of the video transcriptions, about 100 pages.
However another researcher wishing to reanalyse the data can access the transcripts by application to the author .

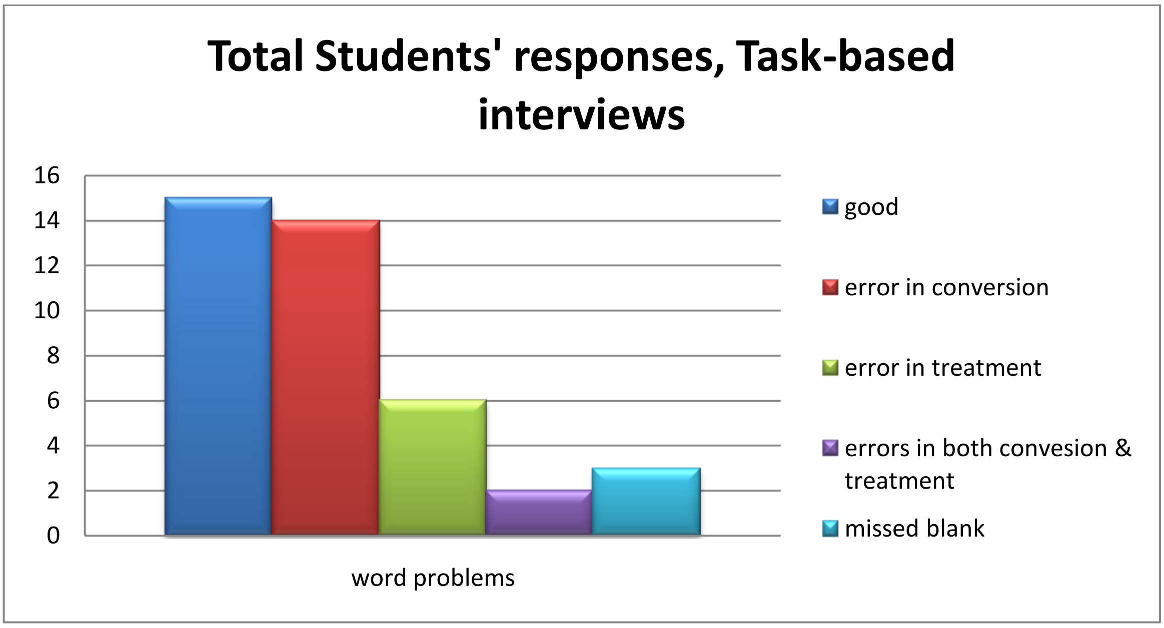


Figure 9. Total Students' responses, Task-based interviews

The available evidence collected through the task-based interviews was used to obtain an overview of eight students' performance on each word problem. I also present the individual analysis of one student in order to clarify (to illustrate) practically how the task-based interviews analysis is done.

The students' written responses from the questionnaire are used to identify their opinions about the test and their performance. Based on the students' answers, I constructed graphs to summarize the results and I used X^2 test to explore for any statistical significant difference between the groups[6]. For example, Figure 10 shows how many students from group A- (abstract word problem without diagram) were in each category: totally agree, agree, not sure, disagree or totally disagree for each statement of the questionnaire.

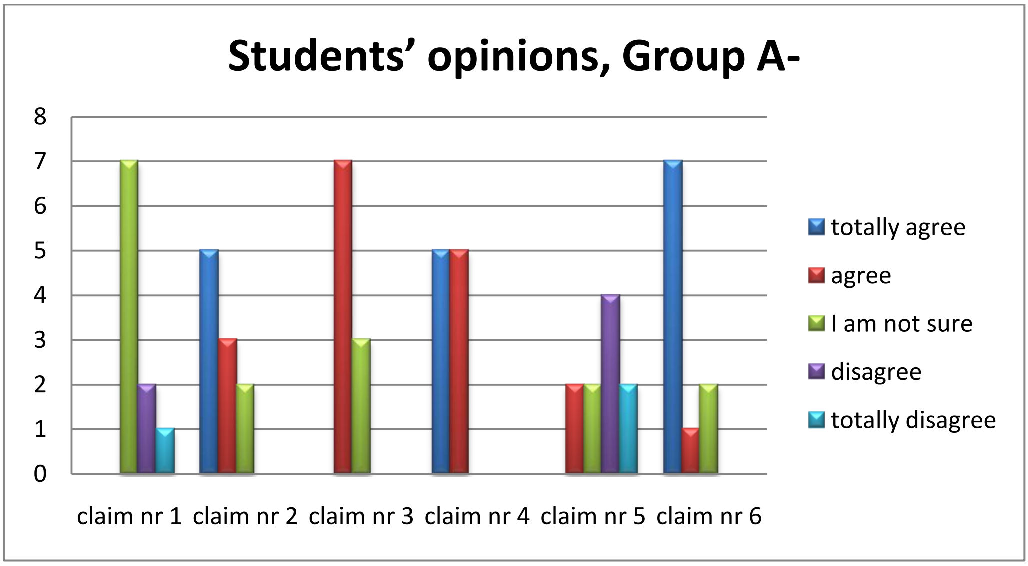


Figure 10. Students' opinions, Group A-

[6] In the Data Analysis Chapter are presented (only) the results of X^2 test which were meaningful. See Appendix15: X^2 test, Questionnaire results.

3.8 Ethical and other challenges

Before starting the data collection I informed the "Norsk Samfunnsvitenskapelig Datatjeneste" (NSD, Norwegian data protection service) about my research. After I provided all the necessary information about my research NSD gave its seal of approval.

The teacher and the school principal were informed about the goals of the research. I promised them that the name of the school and where it is located (the city) will not be used in the dissertation. Also the teacher was assured that she would remain anonymous. The students have had the research goals explained and also I tried to explain the purpose and usefulness of the research. The students wrote their names in the tests' papers but afterwards their names are coded into numerals. During the task-based interviews the video camera was not focused on student's faces but on the paper upon which they were writing. The names of eight interviewed students are coded in letters. Therefore in data analyses the numbers (in the test analysis) and the letters (in the task based interviews analysis) are used to address the students. As I hope to be understood from the foregoing, I attempted as far as I could to avoid the possibility that the school, the teacher and the students that participated in my study might be recognized.

As noted in Section 3.1 a mix of methods was used to collect the data in order to complement the limitations of each method. However, I am aware that the research design does not address or avoid 100% all methods' limitations, for example, the restricted time available for the task-based interviews.

The reader should recall that the students' work was been in the Albanian language. Therefore some data was translated into English language in this report in order to use them as illustration in the data analysis. I have attempted to make a good translation but I am aware that the translated text may not preserve 100% the original meaning of the data. In some parts of the report I have included the Albanian text because the analysis rests very firmly on the words and word order experienced by the students.

3.9 Trustworthiness

As in all interpretative studies, it is a challenge to achieve complete trustworthiness. Therefore I tried to optimize the trustworthiness as far as I can. For example:

- I was present for one week in the mathematical lessons because I wanted students to become accustomed to my presence in the classroom. In order that my presence in the rest of the study would not have a negative affective response.
- I contacted the teacher and asked her about the students that I chose to be interviewed because I had to know if these students were too shy or intimidated by the interviews situation or by the presence of camera.

"*Complete objectivity is impossible*" in a research study (Bryman, 2008, p. 379). Therefore in this research study, the inevitable subjectivity challenge is tried to be minimized by explaining in detail:

- the choice of the methods and their implementation;
- the design of the word problems in the different contexts and of the relevant diagrams;
- the way in which the data is analyzed by taking specific examples to illustrate it;
- the involvement in the data analysis of some of the students' written responses during the test and the task-based interviews;

- the translation from Albanian into English of some of the students' written responses during the test and the task-based interviews;
- the translation and the involvement in the data analysis of some fragments from the video transcriptions;

This is a case study and as with this research approach, case studies *"tend to be oriented to the contextual uniqueness and significance of the aspect of the social world being studied"* (Bryman, 2008, p. 378). Therefore I am aware that it is not reasonable to claim that the results will be of a general validity beyond this Grade 8 class. Also, I think it would be not possible (effective) to use exactly the same tests design and task-based interviews design (with exactly the same items) in different eighth grades in other schools. I achieved the above conclusions based on different reasons as:

- The limited number of participants (the reader will recall that in the test forty one students and in the task-based interviews only eight students participated);
- The specific mathematical textbook that the participants follow and their teacher advice about their mathematical level (the reader will recall that I have based the design of the word problems for the test and the interviews on the mathematics text book and teacher's advice);

3.10 Summary

The data collection has been carried out by combining four types of methods, these being: participant observations, a pencil and paper test with word problems, a questionnaire about the content of the test and students' performance, and task based interviews. The limitations, the strengths, the purposes of each method and how these methods are implemented have been described in this chapter. The design of word problems used in the test and in the task based interviews and how some statements from a previous questionnaire are adopted and used in this study has been presented in advance. Further, this chapter also presents how the analysis of each type of data is conducted. Other issues such as: the context of the research study, my conjectures about students' performance in relation to the contexts and the ready-made diagrams, the ethical challenges and trustworthiness are discussed in the present chapter.

The next chapter will present overviews about students' performance in the test and task based interviews, followed by detailed accounts of individual students' performance in order to make evident the way these general conclusions are achieved. Also an overview of students' opinions about the test and their performance will be presented there.

4 Data Analysis

This chapter presents the data analysis. The chapter is divided into three parts where the first part presents the analysis of the tests, the second uses the questionnaire data and the third and last part is the analysis of the interviews.

The first section, the tests analysis contains two subsections:

- The general analysis of the students' performance in the test

In this subsection, the focus is on students' written solutions to the test. The students' performance in relation with each item (item nr 1; item nr 2; item nr 3; item nr 4; item nr 5) and each type of item (in real world context; in abstract context; with ready-made diagram; without ready-made diagram) is presented here.

- The detailed analysis of four individual students (one student from each group)

In this subsection detail accounts of four individual students' performance in relation with each item they had in their individuals' tests are presented. This subsection shows more practically the way in which the tests are analyzed in order to achieve the general conclusions about students' performance in the test, which are presented in the above subsection.

In the second section, the questionnaire analysis, findings about the students' opinions in relation to the test are presented. This reports students' written responses from the questionnaire about the test and their performance.

The third section the interviews analysis contains two subsections:

- The general analysis of the students' performance in the interviews

In this subsection summary observations from interviews about students' performance in relation with each item (item nr 1; item nr 2; item nr 3; item nr 4; item nr 5) and each type of item (in real world context; in abstract context; with ready-made diagram; without ready-made diagram) are presented.

- The detail analysis of one individual student

In this subsection detailed observations of one individual student's performance in relation with each item he had in his individual test is presented. This subsection shows in greater detail the way in which the interviews are analyzed in order to achieve the general conclusions for students' performance to the interviews, which are presented in the previous subsection.

The reader should recall that the data analysis is conducted in the basis of Duval's cognitive approach (2006). The conversions and the treatments are used to identify where the students' solutions errors occur? However, I am aware that the data can be analyzed and from other theoretical perspectives.

4.1 Test Analysis

4.1.1 General analysis of the students' performance to the test

All responses of the students to the word problems were categorized as correct, incorrect or missed (blank). An answer was considered correct only if all conversions and treatments were correct. All other kinds of answers were considered as incorrect (error in conversion, error in treatment, error in both conversion and treatment) or missed (blank). From the test analysis, based in the above categorization, it appears that about 18% of students' responses are correct and 36 % of them are missed (blank) (see Appendix 5, Total students' responses, Groups A-, A+, R-, and R+).

The majority of errors in the work of students, about 83% of errors, seem to arise through faulty reasoning (see Appendix 6, Types of students' errors in the test). In the case when the faulty reasoning errors are made in conversions, usually they are made in the first conversion. These errors appear to be as consequence of (error in one of the two first steps (R) or (T) of the process of solving word problems, outlined in Chapter Two) misinterpreting language, misunderstanding or misusing data, etc. While in the case when the faulty reasoning errors are made in treatment (which correspond to error in the third step (O) of the process of solving word problems, outlined in Chapter Two), usually they are errors in operations with fractions or in the transformations of equations.

The remainder of the students' errors, oversight errors, appears to occur in the process of recording, either from one stage of a procedure to the next, or perhaps in presenting the answer in the form requested in the problem statement. When the oversight errors are made in conversions, they are usually made in the last conversions (which correspond to error in the fourth step (C) of the process of solving word problems, outlined in Chapter Two) where students forget what is requested by the word problem or forget to consider any significant data in their answer. It is notable that when the oversight errors are made in treatment (which correspond to error in the third step (O) of the process of solving word problems, outlined in Chapter Two), they are operations errors or when students replace a number in place of another.

The greatest numbers of errors, about 72% of errors, are made in conversion and just 28% are made in treatment (see Appendix 5, Total students' responses, Groups A-, A+, R-, and R+).

Word problem number 1Abs) % Bigger & % Smaller Problem / 1Rel) Stadiums Problem

The students of all four groups have made almost the same errors in the word problem number one. In this word problem it seems that students understand the percentage as a fraction but do not relate it as a percentage of a quantity. Moreover, neither placing the word problem in a real world context, nor giving ready-made diagrams appeared to provide any help to the students to overcome this difficulty.

Word problem number 2Abs) % Difference Problem / 2Rel) Asteroid Problem

There appear to be two main types of errors: errors due to a different understanding (that is more related with the lack of understanding of the percentage concept) and errors that appear to have arisen in the decoding of the meaning of the text. A tendency to draw diagrams before starting to solve this word problem can be perceived in the solutions of groups A- and R-. While a tendency to use the ready-made diagram as a springboard to solve this problem can be perceived in the work of group A+ and R+. The students of groups A+ and R+ who used the given data in a wrong way did not consider the ready-made diagrams. The performance of students who have had ready-made diagrams in this word problem, respectively students of group's A+ (27% correct answers) and R+ (40% correct answers), is better than the performance of students who have not had ready-made diagrams in this word problem, respectively students of group's A- (0% correct answers) and R- (10% correct answers) (see Appendices: 1, Students' responses, Group A-; 2, Students' responses, Group R-; 3, Students' responses, Group A+; 4, Students' responses, group R+). A similar relationship between the ready-made diagram and students' performance in this problem will be described later in section number 4.3.

Word problem number 3Abs) Sequence Problem / 3Rel) Stalactites Problem

In this word problem the students of group A- have tried to draw diagrams before starting the solution, while the students of group R- did not. This might arise from the real-life context in which the problem was presented to group R-, because it appears that the students are unfamiliar with the context. This conjecture is supported by responses of group A+ (45% correct answers) in which the ready-made diagram is used effectively by students resulting in them having a noticeably better performance than students of group A- (20% correct answers). While the effective usage of ready-made diagram by students of group R+ is not clearly displayed. Furthermore, it appears that students experience it as unfamiliar. Their performance in this word problem is almost similar with the performance of students of group R- (see Appendices: 2, Students' responses, Group R-; 4, Students' responses, group R+). This final observation is apparently contradicted in the students' interviews, where students' performance was (better) affected when this problem was provided with ready-made diagram. This apparent contradiction will be discussed in Chapter Five.

Word problem number 4Abs) Rectangles Problem / 4Rel) "Mother Teresa" Piazza Problem

It appears that the students of all four groups have made almost the same errors in this word problem. Generally the students attempted to understand it but they cannot find a correct strategy to connect the area of the rectangle (piazza) to the ratio of their sides in order to find the perimeter of the rectangle (piazza). Neither of the factors of interest (the context in which the problem is framed or the ready-made diagram) appear to have had any significant effect in order to aid students to overcome their difficulty in this word problem.

Word problem number 5Abs) Line Segment Problem / 5Rel) Road Segment Problem

This problem was a voluntary task, therefore not so many students chose to attempt it, just about 20%, but all students that tried to do it have drawn a diagram (students of group A- and R-) or have used the ready-made diagram (students of group A+ and R+). The students of group A- have drawn correct diagrams independently of whether they solved the problem in a correct way or not, while the students of group R- have not produced correct diagrams. The real life context in which this word problem is presented appears to have a negative effect on students (of group R-) work because it requires locating the cities in a correct geographical sequence in order to draw correct diagrams. While in the abstract context (in which this word problem is presented for students of group A-) there was no significance about the symbols that could be used to name the segments to draw correct diagrams. The students of group A+ and R+ were supported by ready-made diagrams to solve the problem. However, I cannot compare their performance using statistical analysis respectively with group A- and R- because of the small number of students that participated in each group in this word problem.

The first four items have been produced on the basis of the knowledge that students are expected to have appropriated from the first chapter of their mathematics textbook (Lulja & Babamusta, 2007). In this chapter fractions and percentage (for more detail see Chapter Three) are involved. During the conversations that I had with the teacher, she remarked that students have met a lot of difficulties to understand these mathematical elements. Even after the end of that chapter, they still have uncertainty in relation with fractions and percentage. While item number five is produced on the basis of the knowledge that students expected to have appropriated during the third chapter of their mathematics book [exactly from lesson 3.4 - The Measurement of The Segments (The Length of The Segments)]. In relation to the knowledge that students took from this chapter the teacher expressed optimism for their progress during the lessons.

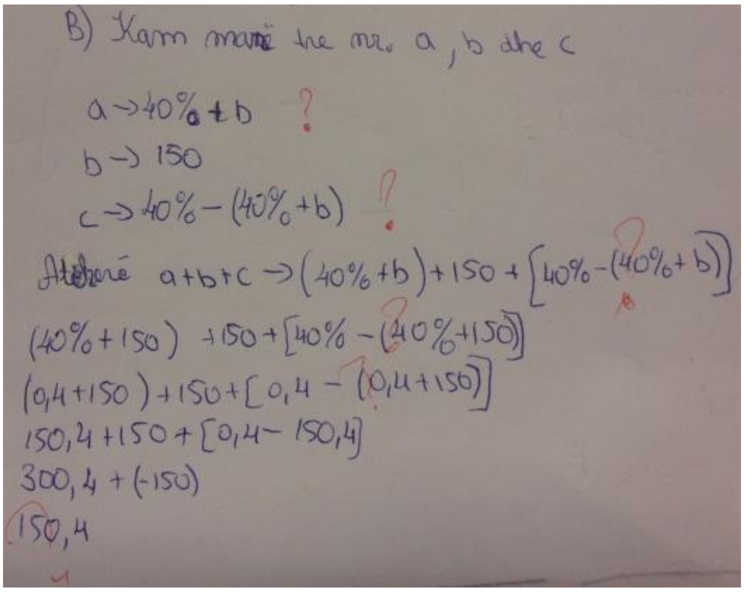
Item number three involves the concept of the sequence (of segments), which has been learned by students in their previous years of school. However, during my participant observations, while I was teaching, I met an exercise that included a sequence of numbers (Lulja & Babamusta, 2007, p. 71). I took the opportunity to explain one more time the meaning of the sequence. I gave that exercise as homework, so the next day we repeated again the meaning of a sequence.

I outlined above the general conclusions that emerge from the tests analysis. However, in the next section I present a more detailed analysis in order to clarify practically the types of errors that I have mentioned above and to demonstrate more concretely the effect of the ready-made diagrams and the contexts. In that section one will find the analysis of the four students' tests (one student from each group). I chose these students because the errors that they made are almost representative of their respective groups, which means that their respective groups' peers have made almost identical errors. I chose to analyze the tests of four girl students because strangely the boy students' performance usually was extremely good (that is almost perfect solutions, leaving little to discuss) or extremely poor (in which case the student has written too little to enable any purposeful analysis).

4.1.2 The detail analysis of four individual students (one student for each group)

Student One from group A-

1Abs) % Bigger & % Smaller Problem
Three numbers are given. The first number is forty percent bigger than the second number. The third number is forty percent smaller than the first number. Can you say that the second and the third numbers are equal? Why?
Can you find the sum of these three numbers if the second number is one-hundred-fifty? (Please explain briefly how you approached this problem)



Student One starts to solve the problem by trying to extract the data provided within the problem statement. While she is making this conversion she appears to be making a one to one mapping between the words of the natural language register (that is the source register) and the symbols of the symbolic register (that is the target register). For example (in the following the top row shows the original Albanian text of the problem statement, with a word to word translation into English):

janë	dhenë	tre	numra	→	kam	marr	tre	numra	a	b	dhe	c
are	given	three	numbers	→	have	took	three	numbers	a	b	and	c

numri	i parë	është	dyzetë	perqind	më i madh	se	numri	i dytë
Number	the first	is	forty	percent	bigger	than	number	the second

a → 40% + b

numri	i tretë	është	dyzetë	perqind	më i vogel	se	numri	i parë
number	the third	is	forty	percent	smaller	than	number	the first

c → 40% - (40% + b)

numri	i dytë	është	njëqind e pesëdhjetë
number	the second	Is	one hundred fifty

b → 150

Student One makes the congruent conversion mathematically incorrect because she misses the meaning of the percentage, since the percentage is not referred to any number. It appears that the reason for the failure of this conversion could be the result of a lack of understanding of the concept of the percentage of a number. In the subsequent step she builds a mathematical model to find the sum of three numbers that is correctly derived from the data that she wrote in the previous step.

At the beginning she makes the sum of three numbers.

a + b + c

Then she replaces these three numbers.

(40% + b) + 150 + [40% - (40% + b)]

(40% + b) + 150 + [40% - (40% + b)]

In this step she replaces the number b with 150.

(40% + 150) + 150 + [40% - (40% + 150)]

(40% + 150) + 150 + [40% - (40% + 150)]

Then she replaces the 40% with 0.4.

(0.4 + 150) + 150 + [0.4 - (0.4 + 150)]

In these steps she makes the calculations inside and outside the brackets

(0.4 + 150) + 150 + [0.4 - (0.4 + 150)]

150.4 + 150 + [0.4 – 150.4]

150.4 + 150 + [0.4 – 150.4]

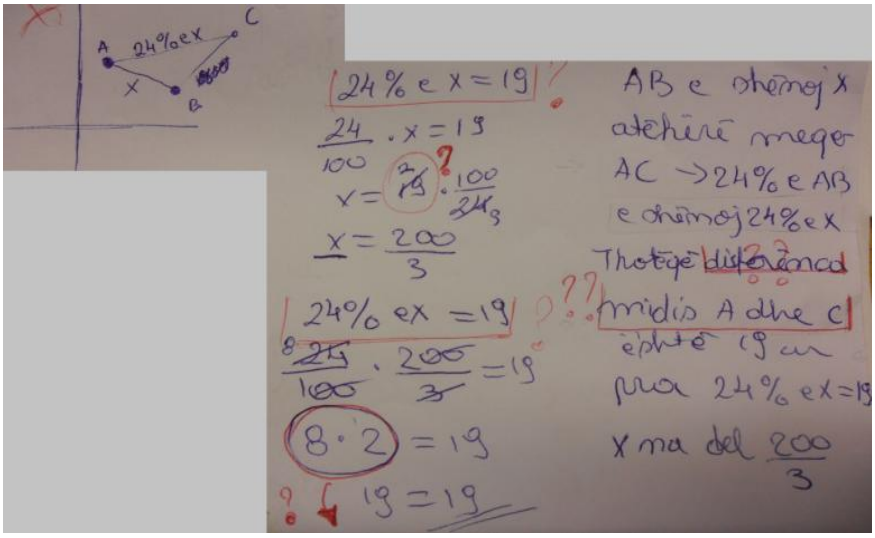
300.4 + (- 150)

300.4 + (- 150)

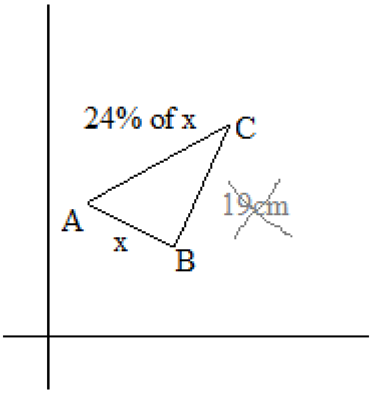
150.4

As it is evident all treatments to achieve the solution are done without further error, but this solution is incorrect in relation with the given word problem. It is interesting to remark here that Student One stops the process of problem solving when she finds a numerical solution for the problem. However she does not integrate this numerical solution into the problem context in order to generate an appropriate answer for the problem.

2Abs) % Difference Problem
In the coordinate plane are given three points A, B, and C. The distance between point A and point C is twenty-four percent of the distance between point A and point B. Knowing that the difference between these two distances is nineteen centimeters, can you find how far away the point C is from point A? (Please explain briefly how you approached this problem)



Student One starts to solve the problem by drawing a diagram in order to present the data that the problem contains. In this diagram she takes three points A, B, C in a coordinate plane. She marks with x the distance AB and with 24% of x the distance AC.



It can be observed that, in the diagram, the distance AC is longer than the distance AB. This might suggest that the student does not understand that 24% of x is smaller than x, which appears to be related to a lack of understanding of the concept of the percentage of a number, which was noted as a possible source of the conversion error in word problem 1, above. It appears that for one moment she joins the points B and C and marks their separation as 19 cm but then for some reason that we cannot know she cancels it. The sentence “the difference between these two distances is nineteen centimeters (diferenca e ketyre dy distancave eshte nëntëmbëdhjetë centimeter)” is not converted into any explicit data in the diagram. As it is visible, the diagram contains just a part of information provided by the problem statement and from the way that she has built the diagram, it is not correct. In other words a conversion error is evident in this point of the solution.

Then, she builds a mathematical model to solve the problem. Where, the distance AC that she has marked before (in the diagram) with 24% of AB is equal to19 cm.

The mathematical model that Student One builds

24% of x = 19

The correct mathematical model

AB - AC = 19

x - 24% of x =19

There are two consecutive conversion errors and at this stage one cannot be sure if she has misinterpreted the information of the problem. However, after her explanation: "*I marked AB with x and since AC → 24% of AB I marked it with 24% of x. It (the problem) says that the distance between A and C is 19 cm, it means that 24% of x = 19, and from here x appears 200/3*", it appears that an error in the reasoning led her to misinterpret the information provided by the problem statement. Because she takes AC = 19 cm while in the problem statement it is the difference between two distances AB and AC that is given as 19 cm and the distance AC is the required answer. She solves this mathematical model in order to find the value of x with which she has marked the distance AB. The mathematical model that she built comprises an equation.

24% of x = 19

She starts to solve this equation by converting the percentage into a fraction.

$\frac{24}{100} * x = 19$

Then she multiplies both sides of equations by $\frac{100}{24}$. $\left\{ \frac{\cancel{24}}{\cancel{100}} * \frac{\cancel{100}}{\cancel{24}} * x = 19 * \frac{100}{24} \right\}$

$x = 19 * \frac{100}{24}$

She writes the numbers 19 and 24 respectively as the product of two numbers.

$x = 2 * 8 * \frac{100}{3*8}$

Then she calculates that 8 / 8 = 1. $\left\{ x = \frac{2*\cancel{8}*100}{3*\cancel{8}} \right\}$

$x = 2 * \frac{100}{3}$

She multiplies 2 by/and 100 and takes the result for the value of x.

$x = \frac{200}{3}$

After which she solves the equation she tries to verify its solution.

24% of x = 19

In this step she transforms the percentage in a fraction and replaces the x with the found value.

$\frac{24}{100} * \frac{200}{3} = 19$

She writes the numbers 24 and 200 respectively as the output of two numbers and then she makes the possible simplifications. $\left\{ \frac{\cancel{3}*8}{\cancel{100}} * \frac{2*\cancel{100}}{\cancel{3}} = 19 \right\}$

8 * 2 = 19

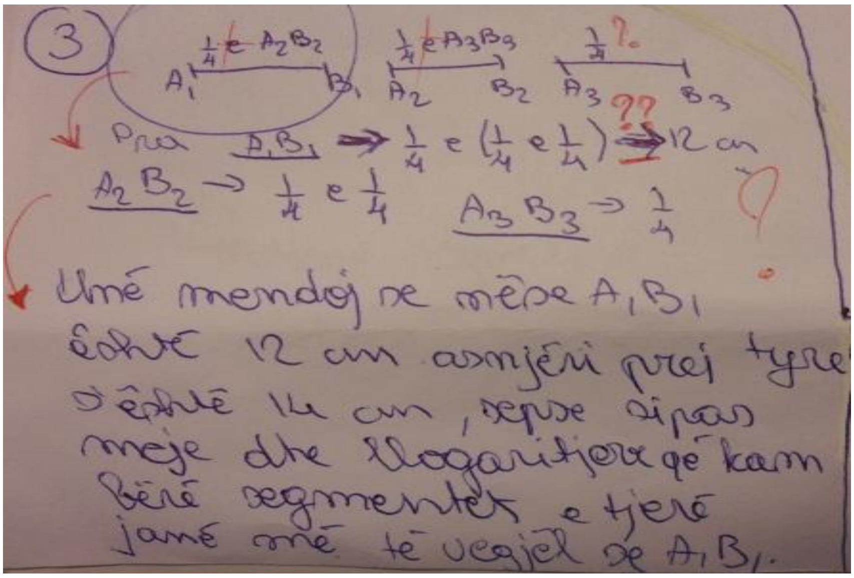
In this step she multiplies 8 with 2.

19 = 19

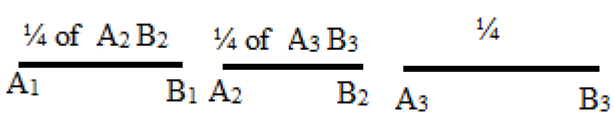
As it can be observed while she is solving this equation she takes 19 = 2 * 8. Then she repeats the error (by taking again 8 * 2 = 19) while she is checking if the answer of the equation is correct. It means that during the treatments (that the Student One has done within the symbolic register) to solve the equation and to confirm its validity she makes an arithmetical error that is related with the multiplication of two single digit numbers. Since the same error is repeated, it appears to be an error by not applying the required knowledge. Also in this problem, Student One does not integrate the achieved numerical solution into the context of the problem.

3Abs) Sequence Problem

It is given the sequence of segments [A_1 B_1]; [A_2 B_2]; [A_3B_3]; … where each segment is one-quarter centimeters bigger than the previous segment. If the first segment [A_1 B_1] is twelve centimeters, which segment of the sequence has the length fourteen centimeters? (Please explain briefly how you approached this problem)



Student One starts to solve this problem by drawing a diagram in order to present the data that the problem contains. In this diagram she draws three segments.



She names the ends of the first segment A_1; B_1; and she marks the length of this segment with "*¼ of A_2 B_2*" which can be read as "the length of segment A_1 B_1 is equal to one-quarter of the length of segment A_2 B_2". She names the ends of the second segment A_2; B_2; and she marks the length of this segment with "*¼ of A_3 B_3*" which can be read as "the length of segment A_2 B_2 is equal to one-quarter of the length of segment A_3 B_3". She names the ends of the third segment A_3; B_3; and she marks the length of this segment with "*¼*" which can be read as "the length of segment A_3 B_3 is equal to one-quarter centimeter". It appears that:

- The sentence "¼ cm më i madhe se segmenti paraardhës" is converted in

  "1/4 cm bigger than the previous segment"

  ¼ e segmentit pasardhës

  ¼ of the subsequent segment

This might suggest that the student has misinterpreted the words of the problem statement.

- She has not considered [A_1 B_1]; [A_2 B_2]; [A_3B_3]; … as a sequence of segments but she has considered just the three first segments ([A_1 B_1]; [A_2 B_2]; [A_3B_3]) because it appears she considers that there is no subsequent segment following [A_3B_3] taking it equal to ¼.

It appears that the student does not understand the meaning of the sequence. I have mentioned in the previous sections that I have explained the concept of the sequence several times during my participant observations. It is evident that the conversion from word problem into data extraction is incorrect, and appears that a wrong reasoning guides her in the misinterpreting and in the misunderstanding of the problem. In Duval's words she changes the denoted objects while changing the register (the problem statement [in natural language register] shows that each segment is based on the length of the previous segment together with a constant length ¼ cm while Student One translate it [into symbolic register] to be that each segment is ¼ of the subsequent segment).

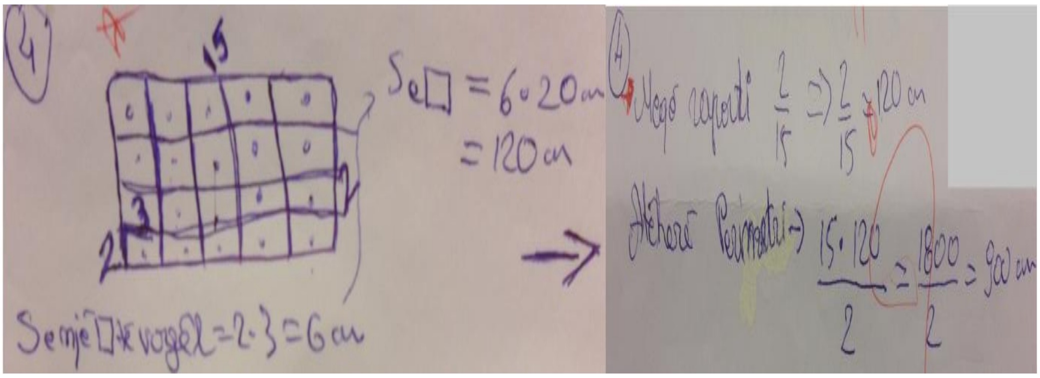
Then to find the solution, she builds a mathematical models, which appears to be derived from the data above by making the replacements (for example: [A_1 B_1] → ¼ of [A_2 B_2] → ¼ of (¼ of [A_3 B_3]) → ¼ of (¼ of 1/4) → 12 cm).

In this mathematical model some suspicions are raised about the meaning of the arrow (→) symbol. At the first glance, it appears to have the meaning of equality. However, the signs of equality is incorrect in the case when [A_1 B_1]→ ¼ of (1/4 of ¼)→ 12 cm because it means that [A_1 B_1] = 1/64 = 12 cm. Another explanation is possible for the meaning of arrow symbol. Student One uses the arrow to mean 'gives' or 'leads to', and that the symbols represent some form of shorthand in which the operand is omitted. Such that " a quarter of something give a quarter of a quarter of something else, gives a quarter of a quarter of a quarter of a third thing, which gives 12.

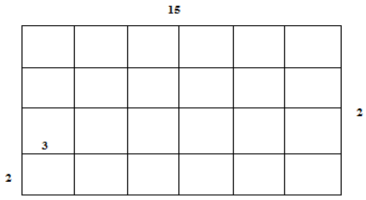
After that Student One offers an answer derived from the mathematical model that she uses *"I think that if A_1 B_1 is 12 cm none is 14 cm, because according to me and the operations that I have done, the other segments are smaller than A_1 B_1".*

Now from this answer it can be confirmed that Student One takes [A_1 B_1]→ ¼ of (1/4 of ¼) from the data above and without making any operation she takes out the segment A_1 B_1→ 12 cm from the given data in the problem statement. While she found the lengths of two other segments by doing calculations that were derived from the mathematical models that she built.

4Abs) Rectangles Problem
The area of a big rectangle is exactly covered by twenty small rectangles with dimensions two centimeters and three centimeters. Can you find the perimeter of big rectangle if its dimensions are in the ratio two over fifteen? (Please explain briefly how you approached this problem)



Student One starts to solve the problem by drawing a (correct) diagram to present the data that the problem contains.



Student One draws a big rectangle and at the edges of this rectangle she puts up the numbers 15 and 2 which show how the dimensions relate to each other. She divides the big rectangle into 20 small rectangles and at the edges of one of them she writes its dimensions 2 cm and 3 cm. Then she builds a correct mathematical model to find the area of a small rectangle. Following this she calculates the product of two numbers and achieves correctly the first solution.

$A_{\text{small rectangle}} = 2 * 3 = 6\ cm^2$

In the current step using some data and the first solution she builds another correct mathematical model to find the area of the big rectangle and makes correct the multiplication of two numbers, achieving the second correct solution.

$A_{\text{big rectangle}} = 6 * 20 = 120\ cm^2$

In this step Student One uses the given data in the problem statement (or in the diagram) and also the second solution to build another mathematics model. However, this time it has no meaning and the conversion was incorrect because she equates the relationship between the sides of the rectangle with the area of the rectangle.

"*Since the ratio is* $\frac{2}{15}$ *then* $\frac{2}{15} = 120\ cm$"

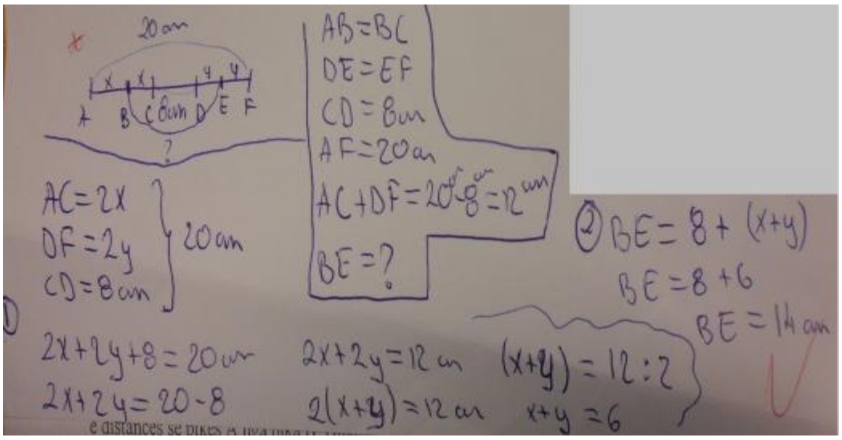
From this incorrect mathematics model another incorrect mathematics model is derived, which is used to find the perimeter, thus Student One makes another error in conversion.

"*Then the perimeter is equal to* $\frac{15 * 120}{2} = \frac{1800}{2} = 900cm$"

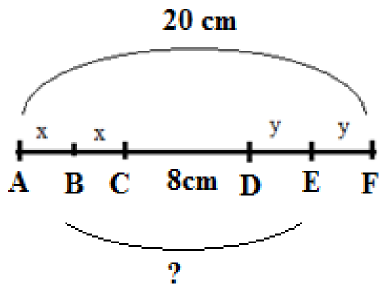
After some operations she manages to find the perimeter of the big rectangle that was correct in relation with the last mathematical model she builds but incorrect in relation to the word problem. It appears that she understood the problem, but that she failed in reasoning to find an appropriate mathematical model for the perimeter which is required.

5Abs) Line Segment Problem

One segment with the length twenty centimeters is divided into three not equal segments. The length of the middle segment is eight centimeters. Can you find the length between two middle points of the biased segments? (Please explain briefly how you approached this problem)



Student One started to solve this problem by drawing a (correct) diagram to present the data that the problem contains and their relation.



It appears that Student One first draws the segment AF and marks it up 20 cm. She divides it in three segments AC, CD, DF and marks up that the segment CD is 8 cm. She marks with B the midpoint of the segment AC and with E the midpoint of the segment DF. Also she marks with x respectively the lengths of segments AB and BC and with y respectively the lengths of DE and EF. The letters that she uses to mark the segments were chosen by her and the letters order has no importance for the validity of the problem solution. Based on the diagram, she starts to organize the data and also builds a mathematical model to find the length of two segments together, exactly AC + DF.

AB = BC; DE = EF; CD = 8 cm; AF = 20 cm; AC + DF = 20 – 8 = 12 cm; BE = ?

$$\left.\begin{aligned} AC &= 2 * x; \\ DF &= 2 * y; \\ CD &= 8\ cm. \end{aligned}\right\} 20\ cm$$

Then she builds another correct mathematical model, which consists on an equation with two variables, to find BC + DE.

$2 * x + 2 * y + 8 = 20$

In this step she is subtracting 8 from both sides of the equation

$2 * x + 2 * y = 20 – 8$

She calculates 20 – 8

$2 * x + 2 * y = 12$

She extracts 2 as a common factor

$2 * (x + y) = 12$

She divides both sides with 2 and achieves a solution.

$(x + y) = 6$

Student One uses the data above and also the last solution to build the third correct mathematical model to find the length between two middle points of the biased segments.

$BE = 8 + (x + y) = 8 + 6 = 14$ cm

After one replacement and some calculations she achieves the solution of the problem.

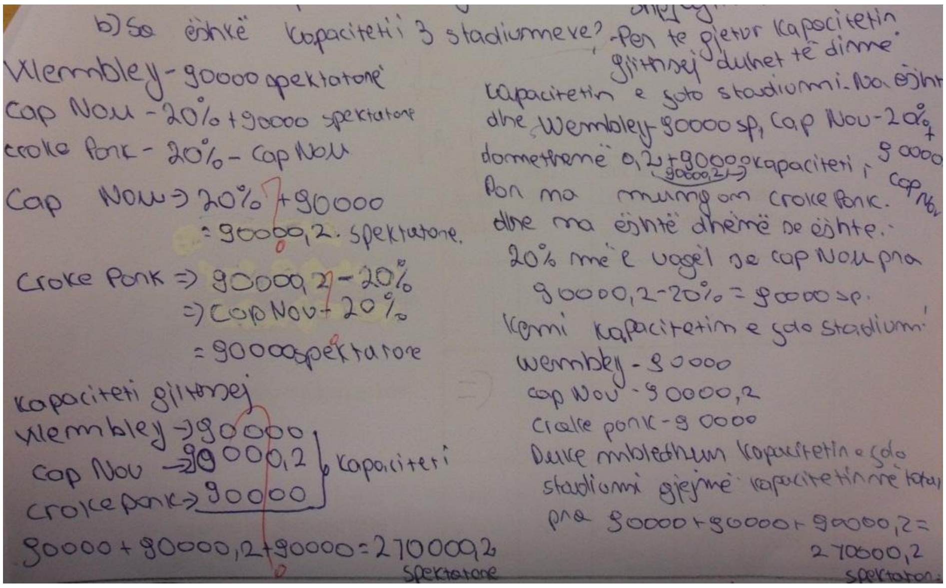
It is evident that the whole process, the data organization and the mathematical models construction, is supported by the diagram.

From above it can be observed that Student One solved correctly just the last word problem. It appears that this evidence supports the teacher's admission that was mentioned in the section number 4.1.1. Students appear to find in dealing with length of segments meaningful. However fractions and percentages appear to create barriers in students' understanding of problem statement and in the choice of the right solution strategy.

Student Two from group R-

1Rel) Stadiums Problem
The capacity of stadiums is measured by the number of the spectators they can accommodate. Europe's three largest stadiums are: "Cap Nou" stadium in Barcelona city in Spain, "Wembley" stadium in London city in England and "Croke Park" stadium in Dublin city in Ireland. "Cap Nou" stadium has the capacity twenty percent bigger than "Wembley" stadium. "Croke Park" stadium has the capacity twenty percent smaller than "Cap Nou" stadium.

a) Can you say that "Wembley" and "Croke Park" stadiums have the same capacity? Why?
b) Can you find the capacity of three stadiums together if the capacity of "Wembley" is ninety thousand spectators?
(Please explain briefly how you approached this problem)



Student Two starts by exploring the data that the word problem contains. It seems that she is doing one to one mapping between the natural language register (in which the word problem is presented) and the symbolic register (in which the data are presented). For example:

kapaciteti	i	Wembley	është	nëntëdhjetë mijë	spektatore
the capacity	of	Wembley	is	ninety thousand	spectators

Wembley - 90000 spectators

stadi-umi	Cap Nou	ka	kapacitet	njëzet	perqind	më te madh	se	stadi-umi	Wembley
Stadi-um	Cap Nou	has	the capacity	twenty	percent	bigger	than	Stadi-um	Wembley

Cap Nou - 20 % + 90000 spectators

stadi-umi	Croke Park	ka	kapacitet	njëzet	perqind	më te vogel	se	stadi-umi	Cap Nou
Stadi-um	Croke Park	has	the capacity	twenty	percent	smaller	than	Stadi-um	Cap Nou

Croke Park - 20% - Cap Nou

She failed to make correct this conversion because she has not referred the percentage to any number. Then she builds two mathematical models to find respectively the capacity of Cap Nou and Croke Park.

Cap Nou → 20 % + 90000

=90000.2 spectators

Croke Park → ~~20% - 90000.2~~

~~= - 90000~~

Croke Park → 90000.2 – 20%

= Cap Nou – 20%

= 90000 spectators

To find the capacity of Cap Nou she rewrites the model that she built to express it in the data above and adds two numbers. She achieves a solution that is 90000.2 spectators, which is correctly derived from the model above but mathematically incorrect in relation with the word problem. The model she built enables her to find 0.2 more than the capacity of Wembley but does not help her to find the capacity of Cap Nou. Also this solution suggests that the student ignores the context of the problem. Because the solution, 90000.2 spectators shows that in the Cap Nou stadium can accommodate 90000 spectators + 1/5 of one spectator, which is not meaningful in real world context. Then to find the capacity of Croke Park it appears that Student Two again rewrites the model that she built to express it in the data above and finds the difference between two numbers. However, since it results a negative number she cancels and exchanges the positions of these two numbers. Moreover she builds a mathematical model that is not derived from the data above. It might suggest that she is attempting to provide a numerical answer for the problem rather than asking herself whether this mathematical model makes sense for the problem or not.

While she is doing the treatments to achieve a solution (in the second step) she replaces the 90000.2 with Cap Nou, perhaps to point out that the number 90000.2 shows the capacity of Cap Nou. Then she subtracts two numbers and achieves a solution that is correctly derived from the model that she built.

Considering the data and the last two solutions she builds another mathematical model to find the capacity of three stadiums together that is correctly derived from above.

Wembley → 90000

Cap Nou → 90000.2

Croke Park → 90000

the capacity

90000 + 90000.2 +90000 = 270000.2 spectators

Student Two then adds three numbers and achieves a solution. This solution is not just mathematically incorrect in relation with the word problem but it has no sense for the context of the problem. It appears that she approaches the word problem in a mechanical way, absent of meaning in the given real world context or the problem statement.

At the end she explains what she has done:

> *To find the capacity of three stadiums together we have to know the capacity of each stadium. It is given Wembley - 90000 spectators. Cap Nou – 20% + 90000 which mean 0.2 + 90000 = 90000.2 spectators is the capacity of Cap Nou. But we miss the capacity of Croke Park and it is given that it is 20% smaller than Cap Nou, which mean 90000.2 – 20% = 90000 spectators. Now we have the capacity of each stadium.*

Wembley → 90000

Cap Nou → 90000.2

Croke Park → 90000

By adding the capacity of each stadium we find the total capacity.

90000 + 90000.2 +90000 = 270000.2 spectators

Her explanation indicates that she converts:

"Croke Park is 20% smaller than Cap Nou"

"*90000.2 – 20%*"

While from the external articulations it can be observed that:

Croke Park is 20% smaller than Cap Nou

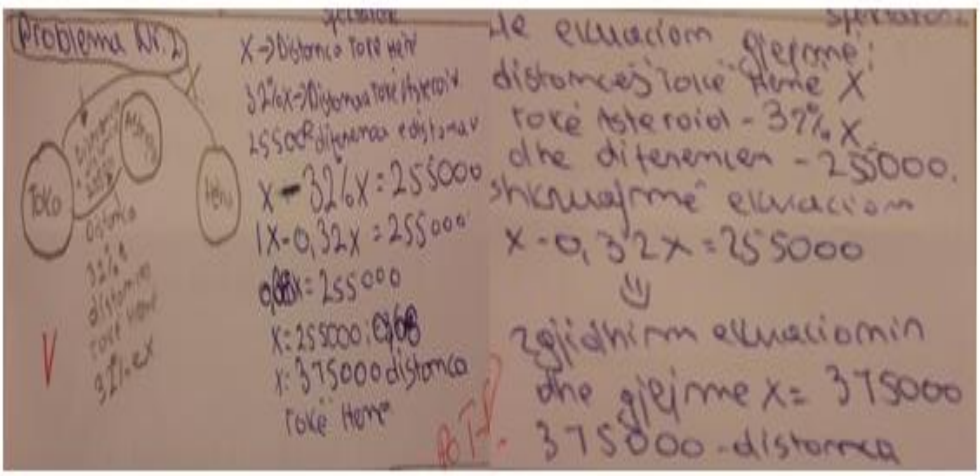
Croke Park – 20% - Cap Nou

Croke Park – Cap Nou – 20%

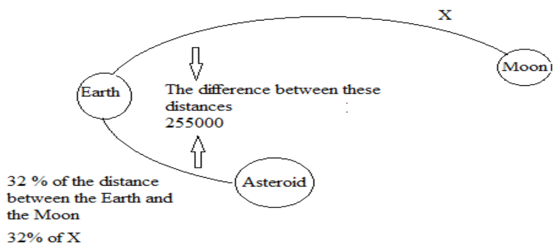
This explicit evidence supports the claim that I mentioned before about the students' effort to provide a numerical answer rather than to make sense of the problem. Because Student Two converted this sentence in that way in order to take a positive number as solution, that we assume, from her point of view, can only be acceptable. It is strange that she appears to 'see' a negative result as unacceptable, in the context, but not 0, 2 persons. On the other hand also this evidence raises another suspicion, which might be: the student does not know or neglects to consider that the difference of two numbers is not commutative. Placing thus the equality sign, 20% - Cap Nou = Cap Nou - 20%. However, since there is no more observable evidence this remains only a suspicion.

2Rel) Asteroid Problem

On June 14, 2002 an astronomer measured the distance between the Earth and a traveling asteroid, which was near to the Earth. He found that the distance between the Earth and the asteroid was thirty two percent of the distance between the Earth and the moon. Knowing that the difference between these distances was two hundred fifty five thousand kilometers, how far away from the Earth was the asteroid at that time? (Please explain briefly how you approached this problem)



Student Two starts to solve the problem by drawing a diagram in order to present the data that the problem contains.



Student Two builds in the diagram the Earth, the Moon and the Asteroid. The distance between Earth and the Moon she marks x. While the distance between Earth and Asteroid she marks 32% of the distance between Earth and the Moon or 32% of x. Also in the diagram she marks that the difference between these two distances (Earth - Moon and Earth – Asteroid) is 255000 km. From this correct conversion (from the text of the problem statement into diagram) she reorganizes the data, again correctly.

x → The distance Earth - Moon

32% of x → The distance Earth - Asteroid

255000 → The difference between distances

Then on the basis of the data above she builds a correct mathematical model to find the distance Earth – Moon, which consists of an equation. Student Two starts to solve this equation by making the necessary treatments as below:

x – 32% of x = 255000

In this step she marks x = 1 * x and transforms the percentage into a decimal number.

1 * x – 0.32 * x = 255000

She makes the subtraction.

0.68 * x = 255000

Student Two divides the both sides of equation with 0.68.

x = 255000 / 0.68

She makes the division and finds the value of x, which is the distance Earth – Moon.

x = 375000

After Student Two solved correct the equation she explains what she has done:

In the equation we can find:

x → The distance Earth - Moon

32% of x → The distance Earth - Asteroid

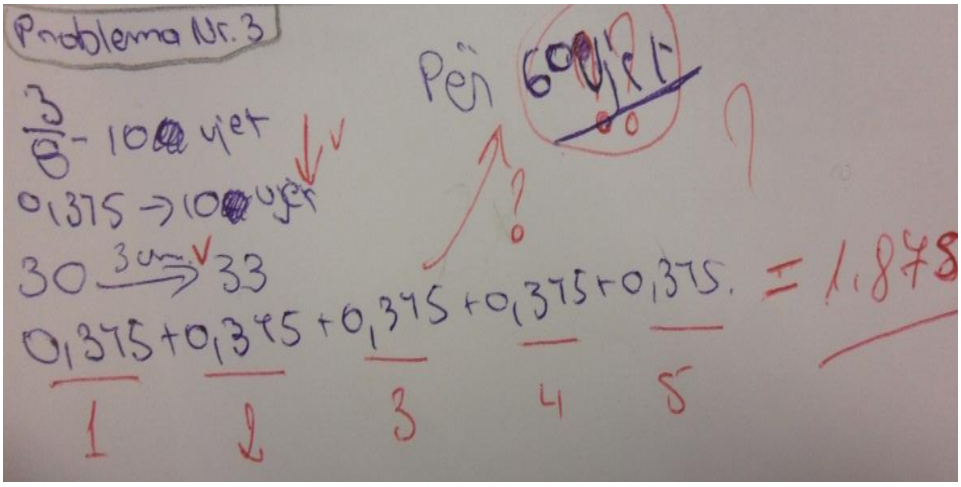
255000 → The difference between distances

We write the equation x – 32% of x = 255000 → we solve the equation and we find x = 375000. 375000 - The distance.

As is evident Student Two explained the components of the equation and also that from the solution of the equation she found the distance (Earth – Moon), considering that the problem is solved. However the problem requires how far away from the Earth the Asteroid was at that time. Therefore, Student Two misses what the question requires to find. This error appears to be made through oversight, because she managed to solve the biggest and the most difficult part of the problem. However, Student Two omits to consider another mathematical model (32% of x) in order to fulfill what is requested by the word problem.

3Rel) Stalactites Problem

Stalactites are icicle-shaped stone formations found on cave ceilings. They form from minerals deposited by dripping water. Suppose a stalactite is thirty centimeters and is growing at a rate of about three over eight centimeters per decade (ten years). How long it will take for the stalactite to reach a length of thirty three centimeters? (Please explain briefly how you approached this problem)



Student Two starts to solve this word problem by exploring the data that the problem statement contains and in the same time by making some treatments to achieve some new data, which she will use to build the mathematical model.

stalaktidi	rritet	mesatarisht	me rreth	tre mbi tetë	centimeter	për	një	dekadë	(dhjetë vite)
the stalactite	grow	at a rate	of about	three over eight	centimeters	per	one	decade	(ten years)

3/8 - 100 years

0.375 → 100 years

ai	do të	arrijë	në gjatësin	tridhjetë e tre	Centi-metra
it	will	reach	the length	thirty three	Centi-meters

stalaktidi	ka	gjatësin	tridhjetë	centimeter
a stalactite	has	the length	thirty	centimeters

3cm

$\underline{30} \rightarrow \underline{33}$

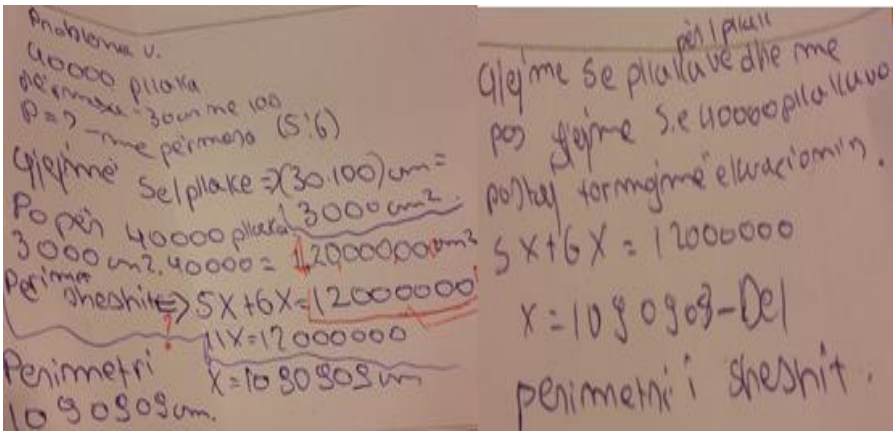
As it can be observed, Student Two converts the first statement into symbols. Then she transforms the fraction into a decimal number, as I mentioned in the previous sections, students had difficulties to operate with fractions. At the beginning, she translates one decade into 100 years because it was incorrectly given so in the problem statement. However, after my explanation about this data error Student Two corrects it. Then, it appears that she converts the two other quotations into symbols reasoning in the same time that to go from the given length into required length the stalactite needs 3 cm. So far Student Two has done all conversions and treatments correctly. Moreover it appears that also the mathematical model to find the solution is built under a correct reasoning. Because it gives the impression that Student Two is adding 0.375 + 0.375… to achieve 3 cm.

The mathematical model that Student Two builds	The correct mathematical model
0.375 + 0.375 + 0.375 + 0.375 + 0.375	0.375 + 0.375 + 0.375 + 0.375 + 0.375 + 0.375 + 0.375 + 0.375 = 3cm
For 60 years	The stalactite will achieve the length 33cm after 8 decades or 80 years

However, from the external evidence it appears that she adds 0.375 just five times which gives 1.875 cm and not 3 cm. Then Student Two answers that the stalactite will achieve the length 33 cm after 60 years, which is incorrectly derived from the mathematical model that she built and also incorrect answer for the problem. It might suggest that subsequent errors in calculations guided her into incorrect treatment and conversion but since there is no more external evidence I cannot be sure for that.

4Rel) "Mother Teresa" Piazza Problem

A construction company has won the tender to fix the floor of "Mother Teresa" piazza in Tirana city, which has a rectangular form. To fix the floor of this piazza the company will use forty thousand rectangular tiles with dimensions thirty centimeters and one hundred centimeters. Can you find the perimeter of "Mother Teresa" piazza, if its dimensions are in the ratio five over six? (Please explain briefly how you approached this problem)



Student Two appears to begin to solve this word problem by discovering and organizing correctly the data that the problem statement contains.

Një kompani ndërimesh ka fitur tenderin për shtrimin e sheshit “Nene Tereza” në Tiranë, i cili ka formë drejtkëndëshe.
A construction company has won the tender to fix the floor of "Mother Teresa" piazza in Tirana city, which has a rectangular form.
Për shtrimin e këtij sheshi kompania do të perdorë dyzetë mijë pllaka drejtkëndëshe me përmasa tridhjetë centimetra dhe njëqind centimetra.
To fix the floor of this piazza the company will use forty thousand rectangular tiles with dimensions thirty centimeters and one hundred centimeters.
Njësoni perimetrin e sheshit “Nene Tereza” nese përmasat e tij rrinë si pesa mbi gjashte.
Can you find the perimeter of "Mother Teresa" piazza, if its dimensions are in ratio five over six?

40000 tiles

With dimensions 30 cm and 100cm

P =? *(It means: the perimeter have to be found)*

With dimensions (5: 6)

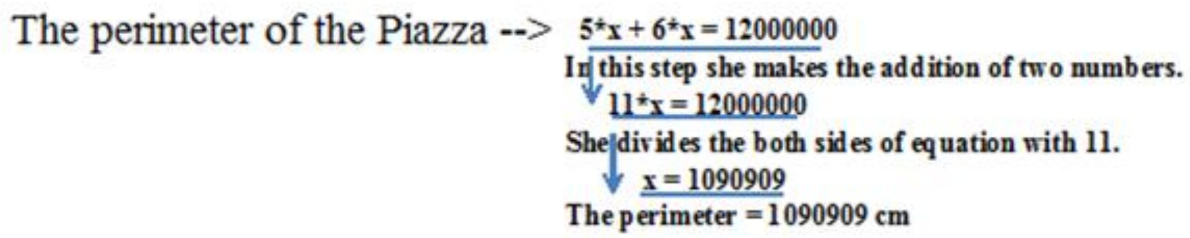
Moreover using the data above, Student Two builds a correct mathematical model to find the area of one tile. Then she works out the product of two numbers achieving the first correct solution.

$$A_{\text{of one title}} = (30 * 100)\ cm^2 = 3000\ cm^2$$

In addition by using both the data and the first solution, she builds another correct mathematical model to find the area of all titles together, which is also the area of the "Mother Teresa" piazza. Student Two makes wrong the multiplication of two numbers but it could be a simple error in writing. It should be remembered that students have had access to their calculators during the test.

$$A_{\text{of 40000 titles}} = 3000\ cm^2 * 40000 \neq 12000000\ cm^2$$
$$= 120{,}000{,}000\ cm^2$$

Furthermore she builds a mathematical model to find the perimeter of the piazza. It consists in an equation where x appears to represent the whole perimeter of the piazza (see her explanation below). However, the construction of this equation is mathematically incorrect and has no meaning. It appears that it is just a combination of the given numbers in the problem used to produce an answer apparently by without awareness of the model for the problem. Even though the equation does not make sense for the problem, Student Two provides the solution of this equation without error in treatment.



At the end Student Two explains her work: *"We find the area of one tile and afterwards we find the area for 40000 tiles. Then we construct the equation 5 * x + 6 * x = 12000000. x = 1090909 – is the perimeter of the piazza."*

It appears that she understood the problem but she did not find a right mathematical model for the perimeter.

5Rel) Road Segment Problem

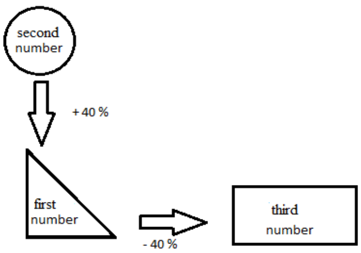
Since this word problem was voluntary item in the test she decided to miss it.

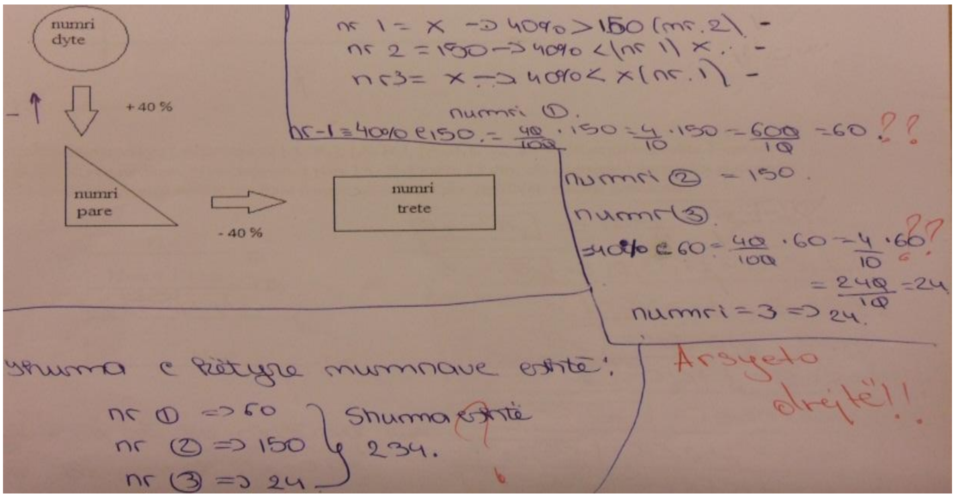
Student Three from group A +

1Abs) % Bigger & % Smaller Problem

Three numbers are given. The first number is forty percent bigger than the second number. The third number is forty percent smaller than the first number.

A) Can you say that the second and the third numbers are equal? Why?
B) Can you find the sum of these three numbers if the second number is one-hundred-fifty? (Please explain briefly how you approached this problem)



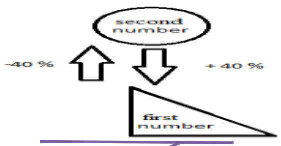


Student Three starts to solve this problem by discovering and organizing the data that the problem statement contains.

numri	i parë	është	dyzetë	përqind	më i madh	se	numri	i dytë
number	the first	is	forty	percent	bigger	than	number	the second

nr.1 = x → 40% > 150 (nr.2)

numri	i dytë	është	njëqind e pesëdhjetë
number	the second	is	one hundred fifty



nr.2 = 150 → 40% < (nr.1) x

numri	i tretë	është	dyzetë	perqind	më i vogel	se	numri	i pare
number	the third	is	Forty	percent	smaller	than	number	the first

nr.3 = x → 40% < x (nr.1)

It is noticeable that she converts the first statement by making one to one mapping from the statement into symbols, and marks the first number with x. To write the second implication Student Three also makes one to one mapping from a text statement into symbols but it appears that she uses also the ready-made diagram to do it. The two first implications indicate that she puts the equality sign between 40% of two different numbers (40% of nr.1 = 40% of nr.2). It suggests that she lacks understanding of the concept of the percentage of a number. The notes in the diagram are not enough to show the student understanding in relation to the ready-made diagram but, they appear to support the conjecture that Student Three misused the diagram. The cause of this misuse appears to be rooted in her understanding of the concept of the percentage of a number. The third implication is also written by one to one mapping from a text quotation into symbols. However, it needs to be emphasized in this implication that Student Three marks the third number with x, whereas earlier she had used x to represent the first number. So the implication takes the form of x is 40% < x. This evidence supports the claim that I mentioned above about the student lack of understanding of the percentage concept.

Then Student Three builds mathematical models to find the first and the third number. These models are incorrectly derived from the data above. Because with the mathematical model that she built to find the first number she can find just 40% of the second number. Moreover with what she built to find the third number she can find just 40% of 40% of the second number which means that she can find 16% of the second number. Again the reason for these wrong treatments appears to be the student’s inadequate understanding of percentage, but other reasons such as a fundamental failure to make sense of the problem statement cannot be excluded. Even though the models are incorrect all treatment processes in achieving numerical solutions are carried out correctly.

Nr.1 = 40% of 150 Nr.3 = 40% of 60

In this step, in both cases Student Three transforms the percentage into a fraction (40% into 40/100)

$= \frac{40}{100} * 150$ $= \frac{40}{100} * 60$

In both cases she makes the possible simplifications

$= \frac{4}{10} * 150$ $= \frac{4}{10} * 60$

Student Three works out the product of two numbers

$= \frac{600}{10}$ $= \frac{240}{10}$

In this last step, in both cases she makes the division of two numbers

$= 60$ $= 24$

Student Three takes the second number directly from the data in the problem statement without using the incorrect model she build (nr.2 = 40% < nr.1).

Nr. 2 = 150

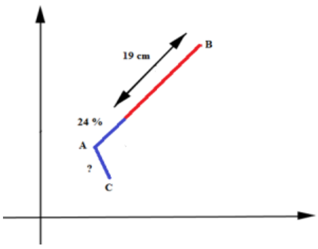
At this point she builds the last mathematical model to find the sum of three numbers together. As can be seen this model is correctly derived from the solutions of three previous mathematical models. Then to achieve the solution she adds three numbers. This solution is correct in relation to the mathematical model from which it is derived but it is incorrect in relation with problem statement.

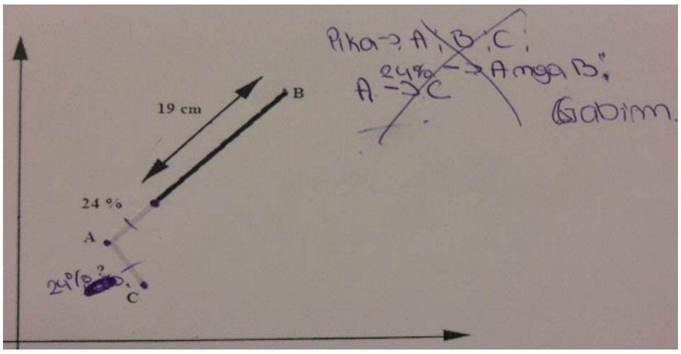
The sum of thre numbers together is :

$$\left.\begin{array}{lcl} \text{nr.1} & \rightarrow & 60 \\ \text{nr.2} & \rightarrow & 150 \\ \text{nr.3} & \rightarrow & 24 \end{array}\right\} 234$$

2Abs) % Difference Problem

In the coordinate plane are given three points A, B, and C. The distance between point A and point C is twenty-four percent of the distance between point A and point B. Knowing that the difference between these two distances is nineteen centimeters, can you find how far away the point C is from point A? (Please explain briefly how you approached this problem)





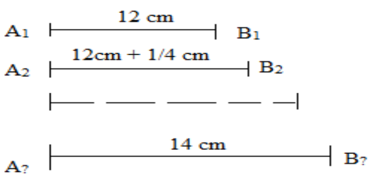
There are some notes in the diagram and some written symbols, but these are cancelled, together they reveal an attempt to solve the problem. Moreover is evident that she understood that are given three points in coordinate plane and that the distance AC is 24% of the distance AB.

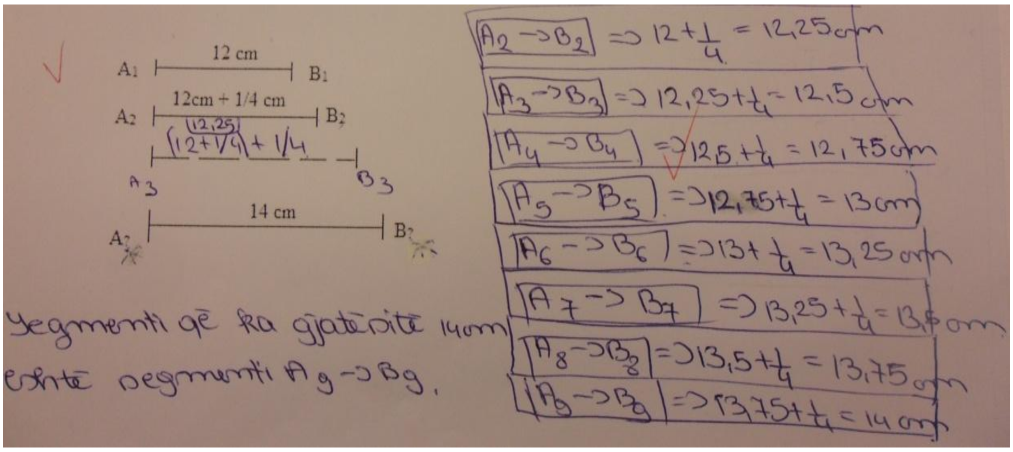
Ne	nje	plan	kordinativ	jane	dhene	tre	pika	A	B
In	one	plane	coordinate	are	given	three	points	A	B
C	Distanca	e	pikes	A	nga	pika	C	eshte	sa
C	The distance	of	point	A	from	point	C	is	as
njëzet e katër	përqind	e	distances	se	pikes	A	nga	pika	B
twenty four	percent	of	the distance	of	point	A	from	point	B

However, Student Three did not go further with the solution of the problem. It appears that the reason she failed is because she did not understand the other part of the problem to get from there the necessary information in order to solve the problem. However, other reasons such as the lack of the motivation to continue working on this item cannot be excluded. In other words she failed to do the first conversion from the problem statement into symbols even though a ready- made diagram was provided.

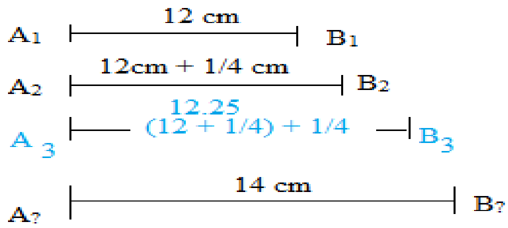
3Abs) Sequence Problem

It is given the sequence of segments [A_1 B_1]; [A_2 B_2]; [A_3B_3]; … where each segment is one-quarter centimeters bigger than the previous segment. If the first segment [A_1 B_1] is twelve centimeters, which segment of the range has the length fourteen centimeters? (Please explain briefly how you approached this problem)





Student Three solved this problem correctly, which means, in other words, that she made all conversions and treatments properly. It is evident that she interacts with the ready-made diagram and uses it as a support to find the right mathematical model to solve the problem.



She builds the mathematical models in this way:

She finds the length of second segment by adding ¼ cm in the length of the first segment

$\boxed{A_2 \rightarrow B_2} \rightarrow 12 + 1/4 = 12.25\text{cm}$

She finds the length of the third segment by adding ¼ cm in the length of the second segment

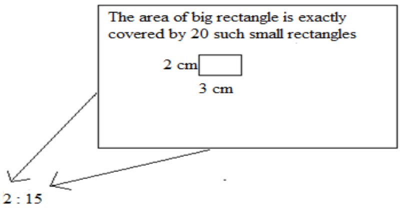
$\boxed{A_3 \rightarrow B_3} \rightarrow 12.25 + 1/4 = 12.5 \text{ cm}$

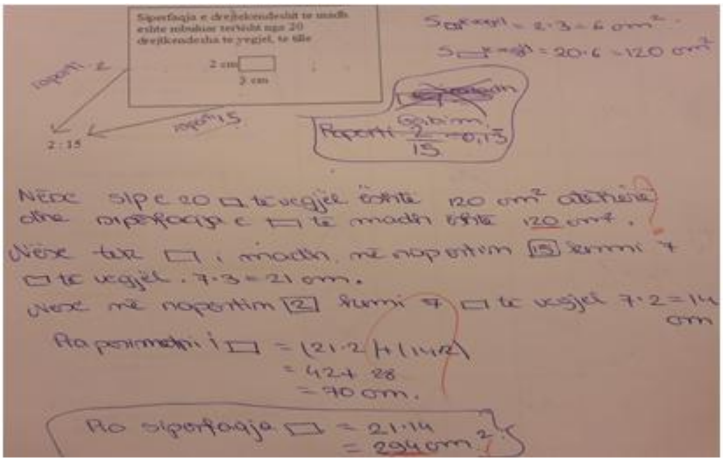
She continues in the same way until she finds the ninth segment by adding ¼ cm in the length of eighth segment.

$\boxed{A_9 \rightarrow B_9} \rightarrow 13.75 + 1/4 = 14\text{cm}$

In this point she answers: "*The segment that has the length 14 cm is the ninth segment A_9B_9*"

4Abs) Rectangles Problem
The area of a big rectangle is exactly covered by twenty small rectangles with dimensions two centimeters and three centimeters. Can you find the perimeter of big rectangle if its dimensions are in the ratio two over fifteen? (Please explain briefly how you approached this problem)





It appears that she starts to solve the problem by interacting with the ready-made diagram and uses it to support her reasoning while she is building the mathematical models.

At the beginning she builds a (correct) mathematical model to find the area of a small rectangle and after she makes the multiplication of two sides of a small rectangle, thus arriving at the first correct solution.

$$A_{\square \text{ small}} = 2 * 3 = 6 \text{ cm}^2$$

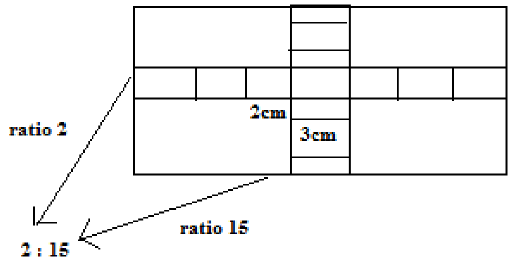
Then using some more data from the problem statement (or from the ready-made diagram) and also the first solution Student Three builds the second (correct) mathematical model to find the area of all small rectangles. After she makes the multiplication of the area of one small rectangle with the numbers of small rectangles (that are used to cover the big one) Student Three finds the second correct solution.

$$A_{\square \text{ all small}} = 20 * 6 = 120 \text{ cm}^2$$

Then she uses the natural language to comment over the derived solution and to explain her reasoning: *"if the area of 20 small rectangles is 120 cm^2 then also the area of big rectangle is 120 cm^2"*.

From the beginning she appears confused about the information that she gets from the problem statement "the dimensions of big rectangle are in the ratio 2:15". In the diagram she marks ratio 2 and ratio 15 that seems difficult to find its meaning. Moreover she writes in symbols the ratio $\frac{2}{15} = 0.13$ then cancels it. Both observations provide evidence that expose her uncertainty about the meaning of this part of the problem statement, which consequently, it appears, guide Student Three in making wrong conversions.

She interacts with the ready-made diagram building inside the big rectangle (respectively horizontally and vertically with the small rectangle that is given) as many rectangles as she can, trying to maintain the same dimensions for each rectangle.



Then using the ready-made diagram she appears to surmise, on the basis of which she builds mathematical models to find the length of each side of big rectangle.

*"If in the big ▭ , in the ratio 15 we have 7 small ▭ s, 7 * 3 = 21 cm. If in the ratio 2 we have 7 small ▭ s, 7 * 2 =14 cm."*

It is evident that she has counted the small rectangles that she built inside the big rectangle horizontally and vertically (in this case 7 horizontally and 7 vertically) and then knowing the dimensions of the small rectangles she managed to construct (incorrect) mathematical models.

She works out the necessary products and it appears that on the basis of her assumption manages to work out the sides of rectangle.

*"Then the perimeter of big ▭ = (21 * 2) + (14 * 2) = 42 + 28 = 70 cm."*

As the evidence above indicates, using the two last solutions she builds a mathematical model to work out the perimeter of the big rectangle, which is correctly derived from solutions above but incorrectly derived from the problem statement. She manages to calculate correctly to achieve a solution as below:

(21 * 2) + (14 * 2)

In this step she works out the products inside the brackets.

=42 + 28

She makes the addition of two numbers achieving the solution that is correctly derived from the mathematical model above.

= 70 cm

However, Student Three has not finished the problem in this point. She builds also a mathematical model to find the area of the big rectangle considering its sides 21cm and 14 cm, "*So the area ▭ = 21 * 14 = 294 cm^2.*" Once she makes the multiplication of the sides, she achieves the solution 294 cm^2. At the first glance, the evidence supposes that she built the last mathematical model to verify if 21 cm and 14 cm can be the sides of the big rectangle (or in other words to verify if this multiplication gives the same area of the big rectangle with that she found above). However, then the evidence shows that she found that the area of the big rectangle is not 120 cm^2 but it is 294 cm^2 (which means that her assumption is incorrect) and she did not reject her assumption. In this case it appears that she has a lack of understanding in relation with problem statement and she made the calculations only mechanically to achieve an answer.

5Abs) Line Segment Problem

Since this word problem was voluntary item in the test she decided to miss it.

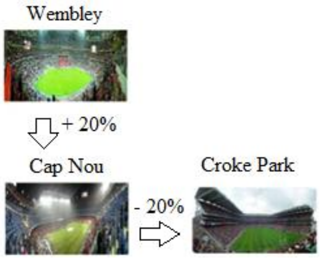
Student number four from group R+

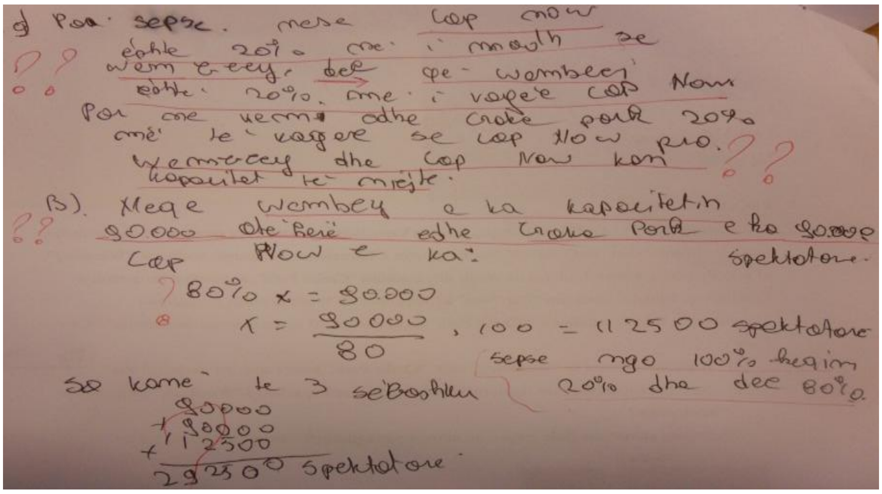
1Rel) Stadiums Problem

The capacity of stadiums is measured by the number of the spectators they can accommodate. Europe's three largest stadiums are: "Cap Nou" stadium in Barcelona city in Spain, "Wembley" stadium in London city in England and "Croke Park" stadium in Dublin city in Ireland. "Cap Nou" stadium has the capacity twenty percent bigger than "Wembley" stadium. "Croke Park" stadium has the capacity twenty percent smaller than "Cap Nou" stadium.

Can you say that "Wembley" and "Croke Park" stadiums have the same capacity? Why?

Can you find the capacity of three stadiums together if the capacity of "Wembley" is ninety thousand spectators? (Please explain briefly how you approached this problem)





Student Four starts to solve the problem by reasoning in order to find an answer for the first question. To explain her reasoning and to answer the first question she chooses the natural language register. It means that the translation of the problem statement is done through the same register or in other words this translation is called treatment.

Stadiumi	Cap Nou	ka	kapacitet	njëzet	për qind	më të madh	se	stadiumi	Wembley
Stadium	Cap Nou	has	capacity	twenty	percent	bigger	than	stadium	Wembley

"I think yes because if Cap Nou is 20% bigger than Wembley then it means that Wembley is 20% smaller than Cap Nou".

Stadi-umi	Croke Park	ka	kapacitet	njëzet	për qind	më të vogël	se	stadi-umi	Cap Nou
Stadi-um	Croke Park	has	capacity	twenty	percent	smaller	than	stadi-um	Cap Nou

"But we have also Croke Park 20% smaller than Cap Nou".

"So Wembley and Croke Park stadiums have the same capacity."

It is evident that Student Four gets correct the information from the problem statement that Cap Nou is 20% bigger than Wembley. It appears that an incomplete understanding of the percentage concept guides her to conclude that Wembley is 20% smaller than Cap Nou, or in the other words to think that 20% of two different numbers is equal. Then using her conclusion and some more information that she retrieves correctly from the problem statement Student Four formulates an (incorrect) answer for the first question. Based on this incorrect answer and on some more data from the problem statement, she starts to find the solution for the second question.

Kapaciteti	i	"Wembley"	është	nëntëdhjetë mijë	spektator
The capacity	of	"Wembley"	is	ninety thousand	spectators

AND

"*Wembley and Croke Park have the same capacity.*"

"Since Wembly has the capacity 90000 spectators then Croke Park has the capacity 90000 spectators"

It is evident that this treatment is correctly derived from the answer and the data above but it is incorrect in relation to the problem statement.

At this point of the problem solution by changing register (from natural language into symbolic register), she builds a mathematical model to find the capacity of Cap Nou, correctly derived from her interpretation of the question. This model consists of an equation with one variable. The equation means that 80% of the variable x (with which she marks the capacity of Cap Nou) is equal to 90000 (with which she marks the capacity of Wembley and Croke Park). This conversion is accompanied by an explanation from her: *"because from 100% we subtract 20% and we get 80%".* It appears that Student Four is attempting to make clear that since the capacity of Wembley (Croke Park) is 20% smaller than Cap Nou, it is equal to 80% of Cap Nou.

Then she carries out the operations (within the same register) as shown below and she achieves a solution that is correctly produced from the model she built.

80% of x = 90000

In this step she multiplies both sides with $\frac{100}{80}$ and makes the necessary simplifications.

$x = \frac{90000}{80} * 100$

She works out the multiplication and the division and achieves a value for the variable x.

x = 112500 spectators

At the end she builds a (correct) mathematical model to find the total capacity of all stadiums together. She constructs the addition of capacities of three stadiums and achieves a solution. It is well produced according to the model she built but it is an incorrect solution from the given problem statement.

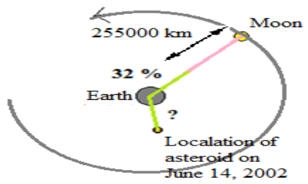
The capacity of three stadiums together

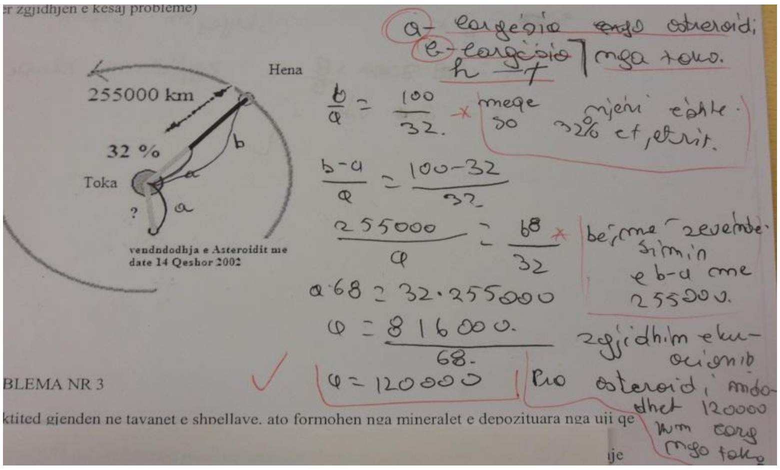
$$\begin{array}{r} 90000 \\ +\ \ 90000 \\ +\ 112500 \\ \hline 292500 \end{array}$$ spectators

In this item it appears that Student Four has not used the ready-made diagram.

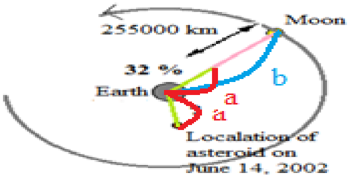
2Rel) Asteroid Problem

On June 14, 2002 an astronomer measured the distance between the Earth and a traveling asteroid, which was near to the Earth. He found that the distance between the Earth and the asteroid was thirty two percent of the distance between the Earth and the moon. Knowing that the difference between these distances was two hundred fifty five thousand kilometers, how far away from the Earth was the asteroid at that time? (Please explain briefly how you approached this problem)





Student Four solved this problem well, making all conversions and treatments without error. She starts to solve the problem by interacting with the given (ready-made) diagram. She marks in the diagram some letters and then she explains with words what these letters mean. In other words Student Four understands the conversion between the problem statement and the ready-made diagram. Moreover she personalizes the diagram by using letters “a” and “b” to identify in it respectively the distances between the Earth and the Asteroid and between the Earth and the Moon.

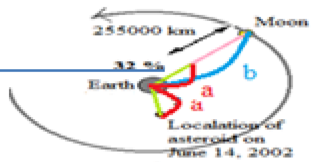


a - the distance from the earth to the asteroid
b- the distance from the earth to the moon

Then Student Four retrieves correctly some data from the problem statement (or the ready-made diagram) and using the letters “a”, “b” (with meaning as above) she converts them as below:

Distanca	mes	tokës	dhe	asteroidit	ishte	sa	tridhjetë e dy
The distance	between	the earth	and	the asteroid	was	as	thirty two
për qind	e	distancës	mes	tokës	dhe	hënës	
percent	of	the distance	between	the earth	and	the moon	

$$\frac{b}{a} = \frac{100}{32}$$ *"since one is as 32% of the other"*



It means that she has considered that the distance “b” has 100 parts in 100 parts while the distance “a” has 32 parts in 100 parts. Then Student Four builds a mathematical model which at the beginning consists in an equation with two unknown variables.

$$\frac{b - a}{a} = \frac{100 - 32}{32}$$

In the current step Student Four gets from the problem statement (or the ready-made diagram) correctly some more data and replaces the difference between distances earth-moon and earth-asteroid with 255000 km. So in this point of the problem solution, the equation is with one unknown variable.

$$\frac{255000}{a} = \frac{68}{32}$$ *"we make the replacement of b-a with 255000"*

Her explanation provides a demonstration of the student' tendency to exclude the real-world context during the solution of the problem.

Then she solves the equation to achieve a value for the variable “a” as below:

$$\frac{255000}{a} = \frac{68}{32}$$

In this step she multiplies the both sides of the equation with (32 * a)

$$a * 68 = 255000 * 32$$

She divides the both sides of the equation with 68 and also makes the multiplication of two numbers.

$$a = \frac{8160000}{68}$$

In this last step Student Four works out the division and she finds the value for the variable “a”.

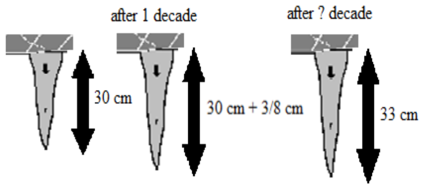
$$a = 120000$$

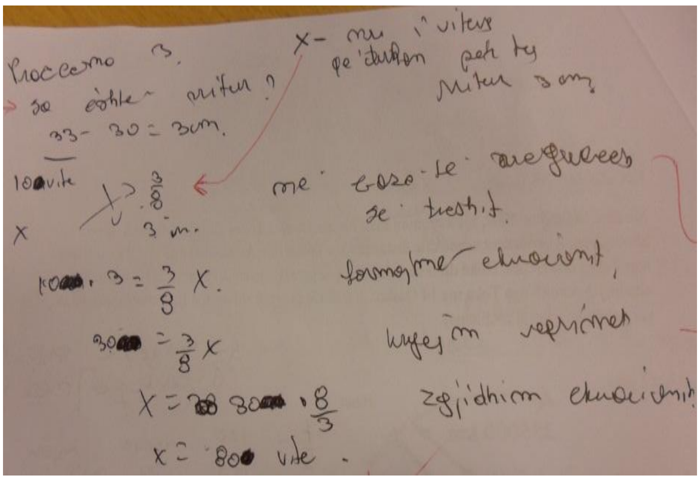
At the end by a conversion (from symbolic register into natural language register) Student Four integrates the numerical solution into the problem context and gives an appropriate answer: *“so the asteroid is 120000 km away from the earth”.*

This evidence suggests that the girl is confronted with word problems in typical school settings therefore she is used to solve the problems in an artificial way without relating them with the real life context.

3Rel) Stalactites Problem

Stalactites are icicle-shaped stone formations found on cave ceilings. They form from minerals deposited by dripping water. Suppose a stalactite is thirty centimeters and is growing at a rate of about three over eight centimeters per decade (ten years). How long it will take for the stalactite to reach a length of thirty three centimeters? (Please explain briefly how you approached this problem)



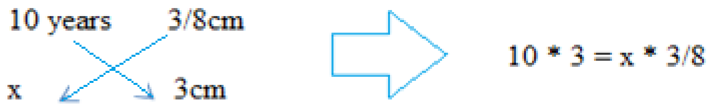


Student Four solves this problem correctly which means that she makes all conversions and treatments without error.

At the beginning by using the length that the stalactite is supposed to be (thirty centimeters) and the length that the stalactite will be after some years (thirty three centimeters) she builds a correct mathematical model to find how many centimeters it will grow. She works out the subtraction and she achieves a correct solution.

33 cm – 30 cm = 3 cm

Then she marks with x the number of years needed for the stalactite to grow 3 cm. She builds a correct mathematical model to find how long it will take for the stalactite to achieve a length of thirty three centimeters. This model consists on an equation that is built on the basis of the rule of three.



$10 * 3 = \frac{3}{8} * x$

In this step she works out the product on the left side of the equation.

$30 = \frac{3}{8} * x$

She multiples the both sides of the equation by 8/3

$x = 30 * \frac{8}{3}$

She makes the multiplication and the division in the right side of the equation, achieving a value for the variable x.

x = 80 years

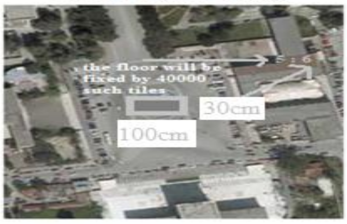
Student Four solves the equation correctly and thus offers a correct answer for the problem. While she was solving this problem she divided the paper in two parts, in the left side she made the calculations and in the right side she explained what she did: *"x – the number of years that needs to be grow up 3 cm. On the base of the triplet rule we construct the equation; we make the operations and solve the equation"*

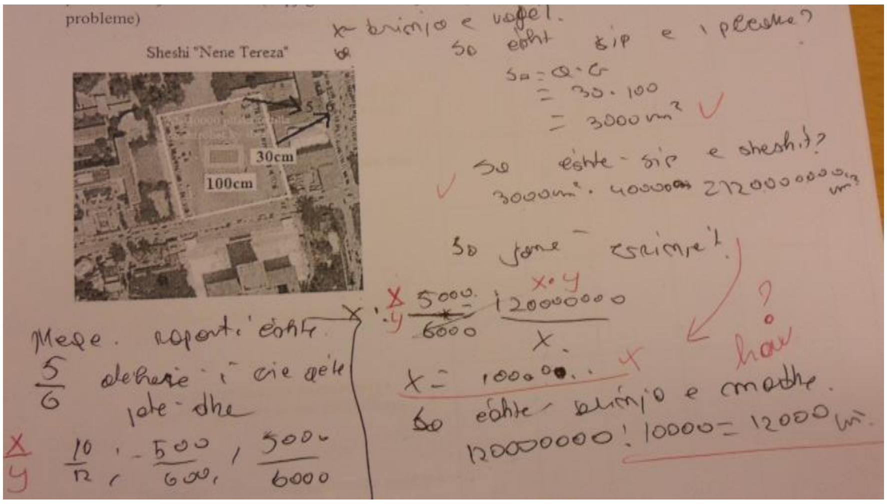
There is no evidence to support or otherwise any claim that the ready-made diagram is useful in her solution.

4Rel) "Mother Teresa" Piazza Problem

A construction company has won the tender to fix the floor of "Mother Teresa" piazza in Tirana city, which has a rectangular form. To fix the floor of this piazza the company will use forty thousand rectangular tiles with dimensions thirty centimeters and one hundred centimeters. Can you find the perimeter of "Mother Teresa" piazza, if its dimensions are in the ratio five over six? (Please explain briefly how you approached this problem)

"Mother Teresa" piazza





Student Four starts to solve the problem by building a (correct) mathematical model to find the area of one tile. She marks with letters a, b the dimensions of a tile and then she replaces them with the dimensions that are given in the problem statement. Moreover Student Four works out the multiplication of two numbers achieving the first (correct) solution.

What is the area of a tile?

$S_{\square} = a * b$
$= 30 * 100$
$= 3000 \text{ cm}^2$

Using some more data from the problem statement and the solution, she builds another (correct) mathematic model to find the area of the "Mother Teresa" piazza. To find it Student Four calculates the product of the area of one tile with the number of tiles (that are used to cover the piazza) thus producing another (correct) solution.

What is the area of the piazza?

$3000 \text{ cm}^2 * 40000 = 120\ 000\ 000 \text{ cm}^2$

At this point of solution she is trying to find the sides of the "Mother Teresa" piazza. Before constructing a specific mathematical model to find those, she gets from the problem statement the data that its dimensions are in the ratio 5 / 6 and finds the expansion of this ratio.

Perhaps she makes this expansion thinking to simplify the next calculations.

$$\overset{*2}{\frown}\quad \overset{*50}{\frown}\quad \overset{*10}{\frown}$$

Since the ratio is $\frac{5}{6}$ than it is also $\frac{10}{12} : \frac{500}{600} : \frac{5000}{6000}$

Then using the last fraction from the expansion, the area of the "Mother Teresa" piazza and the variable x (with which she marks the small side of the piazza, she writes it at the top of the page) she builds an (incorrect) mathematical model to find the small side of the piazza. From the way she built it, she equates $\frac{x}{y} = \frac{x * y}{y}$ (this symbol y is introduced here in this dissertation to represent the other side of piazza in order to aid communication).
Student Four does not show how she makes the calculations to find the value of the variable x but she just gives a value for it. This value is incorrectly derived from the model she built but strangely it is correct for the problem statement. In this case a lot of suspicions are raised, such as the external articulations she wrote can have a different meaning for her from the meaning that these articulations transmit to the reader. Another possible conjecture could be that she has continued the calculation on her calculator … 5000 / 6000 = 10000 / 12000, and 12000 * 10000 = 120000000. However, from this conjecture arises another suspicion, why does she in addition build another mathematical model to find the big side of the piazza?

x-the small side

What are the sides?

x: $\frac{5\ 000}{6\ 000} = \frac{120\ 000\ 000}{x}$

$x = 10000$

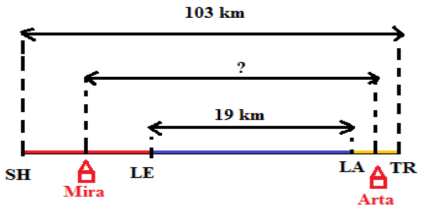
Both errors in conversion and treatments can be detected in the last stage of the problem solution. The error in conversion appears to be made as a consequence of the absence of knowledge to verify if the model she build make sense or not. It appears that she is just attempting to provide a numerical answer. While the consequence of the treatment error is difficult to identify, since the evidences that is available is not enough.
In the next step she builds a (correct) mathematical model to find the length of the other side of the piazza. She divides the area of the piazza with the side which she found above, taking (correctly) the length of the other side.

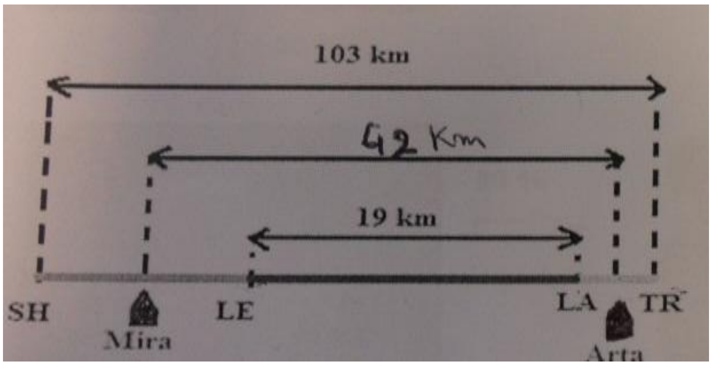
What is the big side?

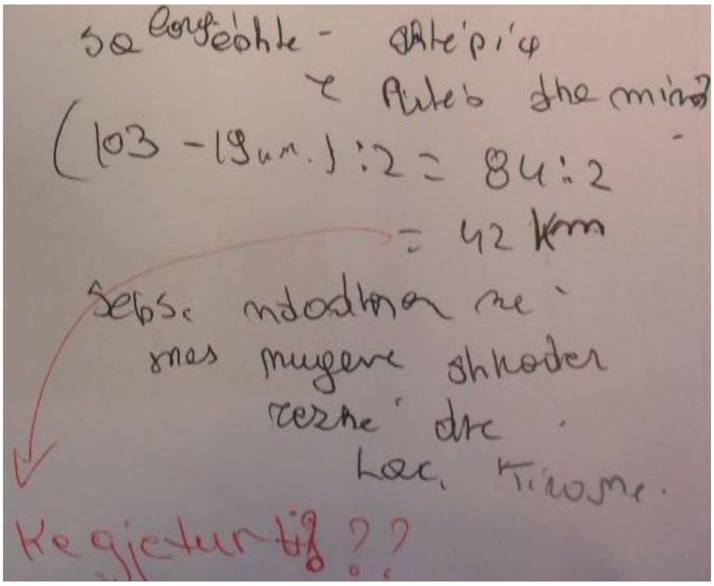
120 000 000 / 10 000 = 12 000 cm

She finishes the solution process of the problems at this point, which means that the student misses what the question requires to find. This error appears to be made oversight because she has found the sides of the piazza that was the most difficult part of the problem but she neglects to consider another mathematical model (the formula for the perimeter of the rectangle that was taken as knowledge since in the 4th year of elementary school) to find the perimeter of the piazza. There is no evidence about whether she has engaged with the ready-given diagram or not.

5Rel) Road Segment Problem
Mira and Arta study in the University of Shkodra. Every Friday after school they take Shkoder – Tirane bus line to go home. Mira lives in a small village situated in the middle of the Shkoder -Lezhe road segment. Arta lives in a village situated in the middle of the Lac – Tirane road segment. The length of the Shkoder - Tirane road segment is one hundred and three kilometers, while the length of the Lac – Lezhe road segment is nineteen kilometers. Can you find how far away from Mira lives Arta? (Please explain briefly how you approached this problem)







Student Four starts to solve this problem by building directly an (incorrect) model to find how far away from Mira lives Arta. To find it, she works out the subtraction between the lengths of the Shkoder - Tirane road segment and the Lac – Lezhe road segment and then divides the product by two.

The calculations of numbers she performs well, which mean that the result is well derived from the model.

How far away are the houses of Arta and Mira?
(103km - 19 km) / 2 = 84 / 2 = 42 km

Then she provides an explanation: *"Because* (the houses) *are situated in the middle of the roads Shkoder- Lezhe and Lac-Tirane".* In the explanation she appears to justify why she has divided by 2 (in the mathematical model). Anyway with the mathematical model that she built she can find just half of road Shkoder- Lezhe plus the half of road Lac-Tirane but not how far away from Mira lives Arta.
In terms of her interaction with the ready-made diagram the only evidence are some notes in the diagram, but these are insufficient to show the effect of the diagram in her solution.

As it is clear, the detailed analysis above support the main part of the general conclusions that have emerged from the analysis of the tests. However, the other part of these general findings remains uncovered as a result of two factors:

1) The lack of evidence. For example, Student Two has missed the word problem number 5. Therefore the trend of the group R- to try to draw a diagram but to fail to draw it correctly cannot be observed in the work of this student.
2) Different performance from the trend of the group. For example, different from the trend of the group A+, Student Three of the group A+ failed to solve the word problem number 2 (in which problem statement was given also a ready-made diagram).

These two factors are unavoidable, because it is impossible to choose a student that has performed exactly as the group' trend.

4.2 The questionnaire analysis

As I explained in the previous section, after the students finished the test they were asked to fill out a questionnaire that was created and piloted in an earlier study by De Bock, et al., (2003) and used in the different experimental conditions. It consisted of several statements, which had to be rated on a five-point scale (from totally disagree to totally agree). Based on the students' answers, I built graphs and used X^2 tests to explore for any statistically significant difference between the groups.

Claim nr. 1: I think I will have a good score on this test.
Most students who filled out the questionnaire, about 73%, were unsure of their performance in the test (see Appendix 11, Total students' opinions, Groups A-, A+, R-, and R+). None of participants expressed that they were totally convinced for his/her good performance.

Claim nr. 2: I liked to work on this test.
The majority of students, about 80%, liked to work on their tests. The students of group's A- (50% totally agree, 30% agree) and A+ (64% totally agree and 9 % agree) have expressed their agreement with higher self-assurance than the students of group's R- (10% totally agree, 80 % agree) and R+ (30% totally agree, 50% agree) $X^2 = 8.52$; df =1; α = 0.05. (See Appendices: 7, Students' opinions, Group A-; 8, Students' opinions, Group R-; 9, Students' opinions, Group A+; 10, Students' opinions, group R+; 15, X^2 test, Questionnaire results).

Claim nr. 3: The problems are similar to the problems we solve in the classroom.
The similarity between the problems they solved in the classroom and the problems they solved in the test is confirmed by the large percentage of students of group's A- (70% agree, 30% not sure) and A+ (91% agree, 9% not sure). While the students of group's R- (40% agree, 40% not sure, 20 % disagree) and R+ (40% agree, 40% not sure, 20 % disagree) were not so convinced for the validity of this statement. Therefore, the students of groups' A- and A+ experienced the presentation and the formulation of word problems as more school-like than those of groups' R- and R+, $X^2 = 8.54$; df = 2; α = 0.05. (See Appendices: 7, Students'

opinions, Group A-; 8, Students' opinions, Group R-; 9, Students' opinions, Group A+; 10, Students' opinions, group R+, 15, X^2 test, Questionnaire results).

Claim nr. 4: I did my best to work on this test as much as possible
93% of the students of all four groups expressed their verdict that they gave their best in the test, 5% were not sure and just 2% disagreed, this outcome appears to support claims about the validity of the collected data. (See Appendix 11, Total students' opinions, Groups A-, A+, R-, and R+)

Claim nr. 5: I considered this as an easy test
Responses to this statement draws attention to the large number of the students of group's A-(60% disagree) and R-(50% disagree) who disagreed that the test was easy versus respectively the number of students of group's A+ (28% disagree) and R+ (20% disagree). (See Appendices: 7, Students' opinions, Group A-; 8, Students' opinions, Group R-; 9, Students' opinions, Group A+; 10, Students' opinions, group R+).

Claim nr. 6: It was an instructive experience to complete this test.
(I learned something while I was completing this test- as I translated in Albanian language)
83 % of the students of all four groups were convinced of the accuracy of this claim and just 2% disagreed. It means that the students believed the test had a positive effect in their learning process. However, it will be left to the analysis of the interviews to expose evidence of what may have been learned. (See Appendix 11, Total students' opinions, Groups A-, A+, R-, and R+)

The following claims were involved only in the questionnaires of group's A+ and R+

Claim nr. 7: While solving the word problems, I made use of the diagrams above.
The majority of both groups, respectively A+ (91%) and R+ (80%) have used the ready-made diagrams while they were solving the word problems. It means that the majority of students of both groups attempted to understand the ready-made diagrams and to use them to find a solution. (See Appendices: 8, Students' opinions, Group R-; and 10, Students' opinions, Group R+).

Claim nr. 8: The diagrams helped me to find the answer.
The large percentage of students was in agreement with this statement, respectively 91% of group A+ and 90% of group R+. This evidence reveals the support that the ready-made diagrams offer from student's point of view. (See Appendices: 8, Students' opinions, Group R-; and 10, Students' opinions, Group R+).

Claim nr. 9: I usually make a diagram before solving a word problem.
About 57% of the students of both groups are expressed that they usually make a diagram before solving a word problem, 24% were not sure and 19 % disagreed (see Appendix 11, total Students' opinions, Groups A-, A+, R-, and R+). It is difficult to identify any rationality guiding students' decision whether or not they should produce a diagram. It appears from the analysis of the test that the production of a diagram might be dependent upon the type of the problem. However, the available data is insufficient to provide any statistical evidence for this issue.

4.3 The task-based-interviews analysis

4.3.1 The general analysis of the students' performance to the interview

As mentioned in the previous section, eight students were interviewed, two from each group (one with average and one with higher mathematical competences). During the task-based interviews students were asked to work on five word problems, which they had not encountered earlier in pencil and paper test. Students who worked with abstract problems in the test were asked to work with real world problems in the interviews and vice versa. In the interviews initially no diagrams were provided. They were given into a later stage after students had engaged with the problem statement without ready-made diagram. I use the students' responses during the interviews to explore and expose their thinking as they engaged to the problem solution. In other words I try to gain access to aspects of students' understanding which are not explicit in their written solutions to the test.

From the analysis of eight interviews it appears that 38% of students' responses are correct and 8% of them are missed (blank) (see Appendix 12, Total students' responses, task- based interviews). The sources of errors in the task based interviews are similar to those exposed by the test, wherein 79% of errors appear to arise through faulty reasoning (see Appendix 13, Types of students' errors in the task- based interviews). During the interviews students made both conversion and treatment errors. Moreover, the greatest numbers of errors, about 68% of errors, arose in the conversion process. Generally, these errors are made by students in the first conversion and appear to occur through faulty reasoning or in last conversion where they appear to occur through oversight. Just 32% of students' errors arise in treatment process, where they occur through faulty reasoning or through oversight (see Appendix 12, Total students' responses, task- based interviews).

The context effect

The interviews are too few to provide enough evidence to achieve a general conclusion about the positive or negative effect of the (real world or abstract) context in students' performance. However, I present below some evidence where the effect of real world context in the problem appears to be visible.

In some real world problems the students' tendency to change the context of the components that are used to formulate those problems into abstract components emerged.

Interview with student A
Brikena: and you will continue in this way until?
Student A: until we get a segment that is 33cm
Brikena: one segment?
Student A: a so stalactite
Brikena: but why you mixed them together?
Student A: because it is more clear, because this (the stalactite) is not an object that we use so we can replace it with an object that we use

Interview with student B
Student B: … … …I am calling rectangle (the piazza)…… …
………………………………………………………………………
Student B:……… and the dimensions of this rectangle………
Brikena: which rectangle?

Student B: the piazza, which means the big rectangle………

The parts of the dialogues above provide evidence of students' tendency to exclude the real-world context during their word problems solution process. The factor responsible for the development of this tendency appears to be the nature of the word problems students encounters usually during their mathematical lessons. However, it is also possible to argue that the students are behaving in a sensible way as they translate from the real world context to the (generalized / abstract) mathematical model.

Another issue arises, apparently, from, the belief the students have about school word problems. More specifically the belief that every piece of the data presented in the problem statement have to be used during problem solutions pushes students to strange questions and answers.

Interview with student C
Student C: Has this date (On June 14, 2002) a lot of importance (in the solution of the problem)?

Interview with student B
Brikena: What do we know from the problem?
Student B: We know that a construction company has won the tender to construct the "Mother Teresa" piazza

Some more evidence of the effect of the context appears to be visible in the third and the fifth real world problems.

3Rel) Stalactites Problem
Stalactites are icicle-shaped stone formations found on cave ceilings. They form from minerals deposited by dripping water. Suppose a stalactite is thirty centimeters and is growing at a rate of about three over eight centimeters per decade (ten years). How long it will take for the stalactite to reach a length of thirty three centimeters? (Please explain briefly how you approached this problem)

The particular context (the stalactite and its growth at a rate) used to produce the third real world problem appears to be unfamiliar for the students. It raises a barrier for them to imagine the situation (in which this item is framed) in order to draw diagrams by themselves.

Interview with student C
Brikena: In this item you never thought to draw a diagram?
Student C: I have not conceived how to draw a diagram

5Rel) Road Segment Problem
Mira and Arta study in the University of Shkodra. Every Friday after school they take Shkoder – Tirane bus line to go home. Mira lives in a small village situated in the middle of the Shkoder -Lezhe road segment. Arta lives in a village situated in the middle of the Lac – Tirane road segment. The length of the Shkoder - Tirane road segment is one hundred and three kilometers, while the length of the Lac – Lezhe road segment is nineteen kilometers. Can you find how far away from Mira lives Arta? (Please explain briefly how you approached this problem)

The real world context used to produce the fifth real word problem is not an unfamiliar context for the students. However, since this problem requires the introduction of a piece of information from real world (locating the cities in a correct geographical sequence) into students' solutions process it appears to be complex for the students. Because, they are not used to introduce some real world data by themselves while are solving mathematical problems.

Interview with student C

Brikena: do you start to draw the diagram just after I show you the location of the cities in a correct geographical sequence?

Student C: yes because I could not imagine how they are

Brikena: did you know the cities order: Shkodra then Lezha then Laci then Tirana?

Student C: we usually do not use these cities but we usually use x or y

In the two problems above it appears that the unfamiliarity with the particular context involved in the problem statement and the lack of necessary real-world knowledge to draw a correct and realistic diagram to present the components of the problem statement and the relation between them result in the real world context having a negative effect on students' performance.

At the end of the interviews all students were asked for their opinion about the difference between the two tests they completed in terms of difficulties. Usually the student's answers are: *"more or less equal"; "tests resemble";* etc. However, when they start two compare the items one to one they start to comment over their personal difficulties with each of them. Moreover according to students' comments, it emerged that the abstract word problems have been easier because the data there were expressed clearer. On the other hand, from another perspective, to work on real world problems was interesting, since it was a new experience, because they did not usually meet those types of word problems in their school mathematics. It is important to note that this last observation comes from students' expressed opinions and is not reflected in their performance.

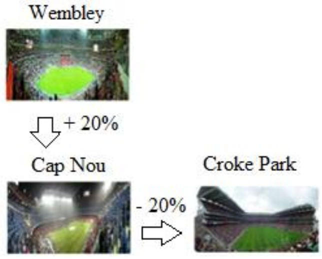
The diagram effect

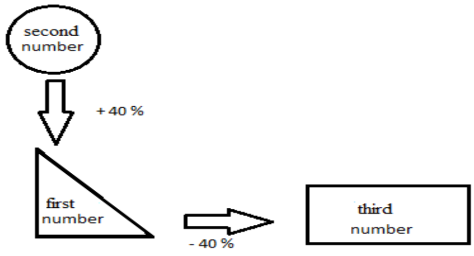
Word problem number 1Abs/ 1Rel

1Abs) % Bigger & % Smaller Problem
Three numbers are given. The first number is forty percent bigger than the second number. The third number is forty percent smaller than the first number.
Can you say that the second and the third numbers are equal? Why?
Can you find the sum of these three numbers if the second number is one-hundred-fifty? (Please explain briefly how you approached this problem)

1Rel) Stadiums Problem
The capacity of stadiums is measured by the number of the spectators they can accommodate. Europe's three largest stadiums are: "Cap Nou" stadium in Barcelona city in Spain, "Wembley" stadium in London city in England and "Croke Park" stadium in Dublin city in Ireland. "Cap Nou" stadium has the capacity twenty percent bigger than "Wembley" stadium. "Croke Park" stadium has the capacity twenty percent smaller than "Cap Nou" stadium.
Can you say that "Wembley" and "Croke Park" stadiums have the same capacity? Why?
Can you find the capacity of three stadiums together if the capacity of "Wembley" is ninety thousand spectators? (Please explain briefly how you approached this problem)

The ready given diagrams in both abstract and real world problems appeared to have no effect, because these diagrams do not affect the students thoughts and therefore the diagrams change nothing in their solutions strategies.



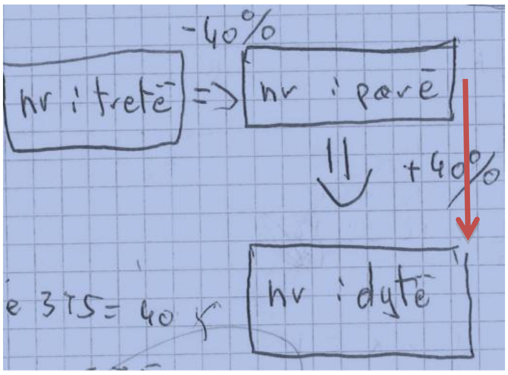


In the case when students solved the word problem before being given the ready-made diagrams they usually commented about these diagrams in this way:

> Interview with student D
> *Brikena: Would a diagram help you to solve the problem?*
> *(After she made correctly the link between the problem statement and ready-made diagram)*
> *Student D: but…more or less it is clear, it is easy so*
> *Brikena: so without a diagram?*
> *Student D: yes, for the first problem yes*

While in the case when they became stuck and could not continue to solve the problem they usually comment in this way:

> Interview with student E
> *Brikena: if I give you this diagram it will help you?*
> *(After he tries to understand it and to express what is given, he concludes)*
> *Student E: it has no link (he failed to understand the conversion from natural language register to graphical register or otherwise)*
> *Brikena: can you draw a diagram that can help you by yourself?*
> *(He constructs a diagram, in the basis of the given diagram)*
> *Student E: I made it*



> *Brikena: Can you use it to solve the problem? Or, Can you explain before and then use it, perhaps it is better so?*
> *Student E: The first number is 40% bigger than the second number and, the third is 40% smaller than the first number.*
> *Brikena: This step here shows that we add 40% of the first number to the first number to take the second number (explaining a step in the diagram, see the red arrow)*
> *Student E: No, it (the first number) is (40%) more than the second number*

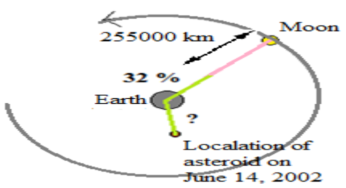
From the dialogue above it appears that he understands the relation between the components of the problem statement. On the other hand, he represents them (in a diagram) in a way that makes sense for him but not for the reader. However he does not use his diagram for the continued process and he failed to solve this problem.

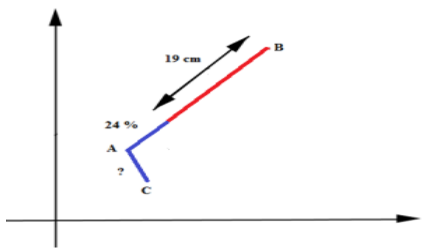
Word problem number 2Abs/ 2Rel

2Abs) % Difference Problem
In the coordinate plane are given three points A, B, and C. The distance between point A and point C is twenty-four percent of the distance between point A and point B. Knowing that the difference between these two distances is nineteen centimeters, can you find how far away the point C is from point A? (Please explain briefly how you approached this problem)

2Rel) Asteroid Problem
On June 14, 2002 an astronomer measured the distance between the Earth and a traveling asteroid, which was near to the Earth. He found that the distance between the Earth and the asteroid was thirty two percent of the distance between the Earth and the moon. Knowing that the difference between these distances was two hundred fifty five thousand kilometers, how far away from the Earth was the asteroid at that time? (Please explain briefly how you approached this problem)

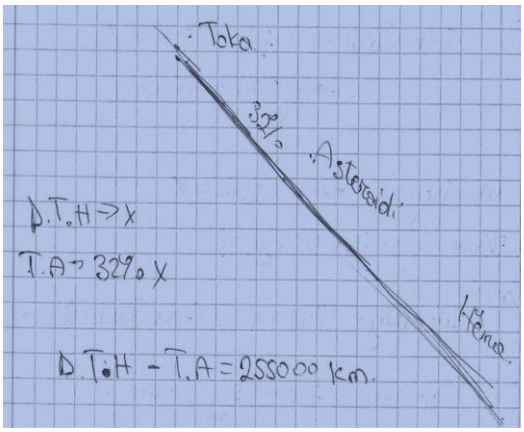
The effect of the ready-made diagrams is clearly evident in both (real world and abstract) word problems.





In the case when students solved the problem before being given the ready-made diagram their comments about these diagrams were usually:

> Interview with student C
> *Student C: if I would have no idea (how to solve the problem) it would helped me a lot*
>
> Interview with student A
> *Student A: it is almost equal with mine (her diagram, see the diagram below) it is no doubt that it will helped…*



Moreover, when students get stuck (meaning they cannot continue to solve the problem), the effect of diagrams is more evident and it takes the form of a positive effect in students' thoughts and therefore in their performance. Below are presented some of students' comments after they took the ready-made diagram and solved the problem.

Interview with student H
Brikena: so in this case what affect did the diagram have in your solution?
Student H: it has a big effect
Brikena: but you cannot solve it without diagram?
Student H: no I would not understand it (the problem)

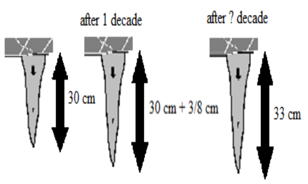
Interview with student E
Student E: can you give me the diagram because I do not know to solve it (the problem) (After he took the ready-made diagram he solved it)

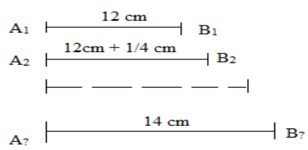
Word problem number 3Abs/ 3Rel

3Abs) Sequence Problem
It is given the sequence of segments [$A_1 B_1$]; [$A_2 B_2$]; [$A_3 B_3$]; … where each segment is one-quarter centimeters bigger than the previous segment. If the first segment [$A_1 B_1$] is twelve centimeters, which segment of the sequence has the length fourteen centimeters? (Please explain briefly how you approached this problem)

3Rel) Stalactites Problem
Stalactites are icicle-shaped stone formations found on cave ceilings. They form from minerals deposited by dripping water. Suppose a stalactite is thirty centimeters and is growing at a rate of about three over eight centimeters per decade (ten years). How long it will take for the stalactite to reach a length of thirty three centimeters? (Please explain briefly how you approached this problem)

The effect of the ready-made diagrams is visible in both (real word and abstract) forms of this problem.





Furthermore, the ready-made diagrams affect positively the students 'performance in two ways:

The diagrams help the students to continue the solution of the problem when they get stuck.

Interview with student E
Brikena: if I will give you this diagram, how do you understand it? Is it related with our word problem?
Student E: it is given the segment A_1B_1 and A_2B_2 is 12cm plus ¼ cm
Brikena: does this diagram present the words of the problem?
Student E: (read again the text of the problem) yes it does
Brikena: have you understand so before? (Before giving the diagram)
Student: no (than he continues to solve the problem correctly)

When students find a correct solution strategy for the problem before being given the ready-made diagrams, the diagram helps students to generate new effective solution strategies to solve the problem.

Interview with student C
Brikena: It means that the diagram affects you to change the solution strategy?
Student C: yes it changes the solution strategy and helps you, but for me both the solution strategies seem easy
Brikena: but if you would have the diagram since at the beginning, you would solve the problem in this way (with an equation) or in that way (with continuous checking until the final result)?
Students C: no, no in that way (with continuous checking until the final result), it is easier because the diagram helps you while in the case when you do not have a diagram you can calculate for x times and it come out.

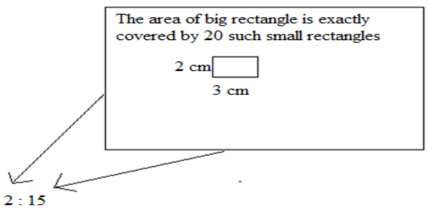
It appears that the presence of the diagram in this word problem prompts students toward an arithmetic solution strategy. While when the diagram is not present close to the problem statement they try to solve it algebraically. Such evidence is also present in the Student G's response (to this problem) which will be presented in the next section.

Word problem number 4Abs/ 4Rel

4Abs) Rectangles Problem
The area of a big rectangle is exactly covered by twenty small rectangles with dimensions two centimeters and three centimeters. Can you find the perimeter of big rectangle if its dimensions are in the ratio two over fifteen? (Please explain briefly how you approached this problem)

4Rel) "Mother Teresa" Piazza Problem
A construction company has won the tender to fix the floor of "Mother Teresa" piazza in Tirana city, which has a rectangular form. To fix the floor of this piazza the company will use forty thousand rectangular tiles with dimensions thirty centimeters and one hundred centimeters. Can you find the perimeter of "Mother Teresa" piazza, if its dimensions are in the ratio five over six? (Please explain briefly how you approached this problem)

The ready-made diagrams of both versions (real world context and abstract context) of this word problem have no effect in students' performance.



For the majority of the students the data of the problem statement are clear thus the presence of a ready-made diagram is not necessary.

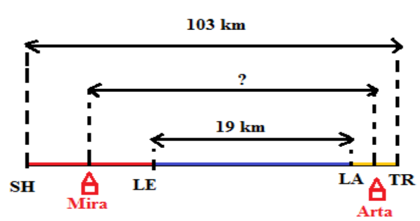
Interview with student H
Brikena: if I will give you this diagram, will it affect somehow?
Student H: no, it is the same

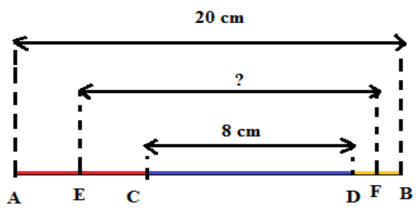
Interview with student B
Brikena: if I will give you this diagram, will it make the things easier or not?
Student B: this is not going to help me but neither it make me confuse

Word problem number 5Abs/ 5Rel

5Abs) Line Segment Problem
One segment with the length twenty centimeters is divided into three not equal segments. The length of the middle segment is eight centimeters. Can you find the length between two middle points of the biased segments? (Please explain briefly how you approached this problem)

5Rel) Road Segment Problem
Mira and Arta study in the University of Shkodra. Every Friday after school they take Shkoder – Tirane bus line to go home. Mira lives in a small village situated in the middle of the Shkoder -Lezhe road segment. Arta lives in a village situated in the middle of the Lac – Tirane road segment. The length of the Shkoder - Tirane road segment is one hundred and three kilometers, while the length of the Lac – Lezhe road segment is nineteen kilometers. Can you find how far away from Mira lives Arta? (Please explain briefly how you approached this problem)





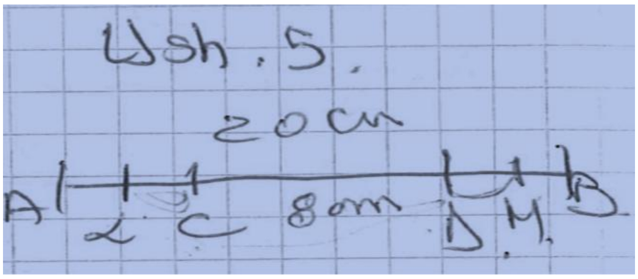
The effect of the ready-made diagram is essential in the students' solution process of this real world problem. They do not manage to solve this problem in the absence of my help (to draw a correct diagram) or the ready-made diagram.

> Interview with student G
> *Brikena: do you score better in the presence or in the absence of the diagram?*
> *Student G: In the presence, because this diagram is a very good guide, it helps me so much.*

> Interview with student C
> *Brikena: Do you start to draw correctly the diagram just after my help (for the cities location)?*
> *Student C: Yes because I cannot imagine how they are.*
> *Brikena: but if you would have the presence of this diagram (I show the ready-made diagram) since at the beginning?*
> *Student C: then it would be very easy; I would solve it immediately, because having this diagram I can base on it....*

Also the effect of the diagram is visible in the students' performance on abstract word problems. They all start to solve the problem by drawing diagrams and when they are asked for the effect of the ready-made diagrams they answer:

> Interview with student F
> *Student F: When it is given it is better because I've already messed it up while here (the ready-made) it is clearer….*

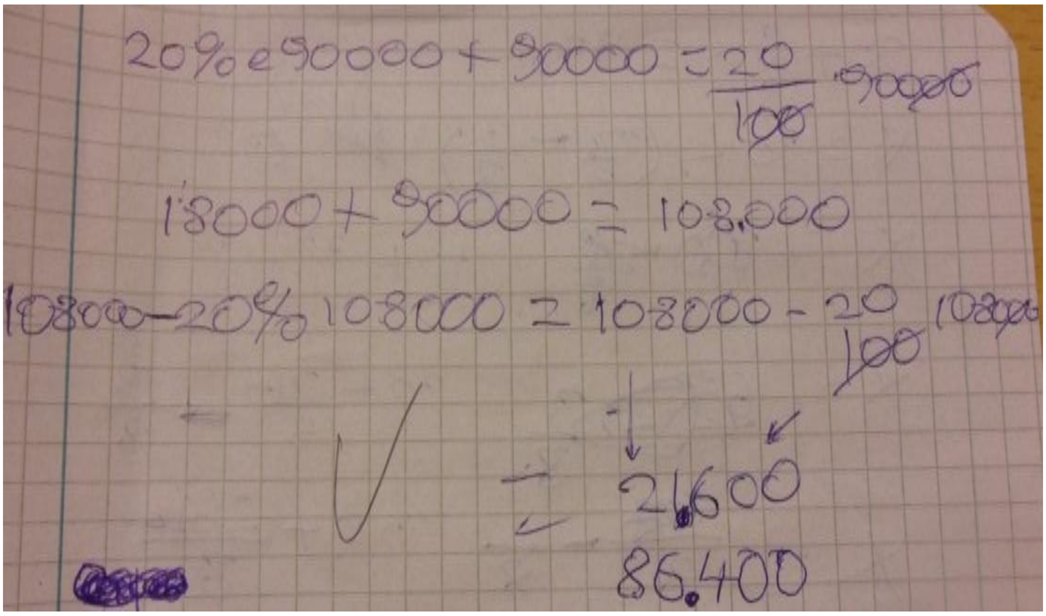


As can be seen from the above in most cases ready-made diagrams have a positive effect on students' performance and otherwise there is no effect, but nowhere is a negative effect visible.

In the next section I will present the detailed analysis of one student interview in order to show practically how the context and the ready-made diagrams have affected his thoughts and therefore and his performance. The selected student is a boy. I chose this boy because his thoughts reactions about the context and the ready-made diagrams appeared to be reasonably representative of the students that were interviewed.

4.3.2 The detailed analysis of one individual student

1Rel) Stadiums Problem
The capacity of stadiums is measured by the number of the spectators they can accommodate. Europe's three largest stadiums are: "Cap Nou" stadium in Barcelona city in Spain, "Wembley" stadium in London city in England and "Croke Park" stadium in Dublin city in Ireland. "Cap Nou" stadium has the capacity twenty percent bigger than "Wembley" stadium. "Croke Park" stadium has the capacity twenty percent smaller than "Cap Nou" stadium.
Can you say that "Wembley" and "Croke Park" stadiums have the same capacity? Why?
Can you find the capacity of three stadiums together if the capacity of "Wembley" is ninety thousand spectators? (Please explain briefly how you approached this problem)



Student G understands the problem and reasons using the given data in the problem statements to give a correct (treatments) answer for the first question.

Student G: I understand that the three biggest stadiums are given; Cap Nou stadium has the capacity twenty percent bigger than Wembley stadium, which is the second in the rank according to capacity. The Croke Park stadium has the capacity twenty percent smaller than Cap Nou stadium. Then, Can you say that "Wembley" and "Croke Park" stadiums have the same capacity? (He reads loudly the question from the problem statement) No I cannot say.
Brikena: why?
Student G: because the twenty percent of Wembley is not the same with twenty percent of Cap Nou

By extracting the data provided within the problem statement the Student G directs his attention into the second question. Student G explains what the problem requires to find and then correctly builds (conversions) the mathematical models and also makes (the treatments) the necessary calculations without error.

Student G: then, can you find the capacity of three stadiums together if the capacity of Wembley is ninety thousand spectators? (He reads the question) Then the capacity of Cap Nou is…twenty percent of ninety thousand…because the capacity of Cap Now is twenty percent bigger than the capacity of Wembley. Then since the capacity of Wembley is ninety thousand, twenty percent of ninety thousand plus ninety thousand is the capacity of…
Brikena: of?
Student G: of Cap Nou stadium. Then Cap Nou capacity is one hundred and eight thousand. Then the capacity of Croke Park is twenty percent smaller than Cap Nou capacity so…so from Cap Nou capacity that is one hundred and eight thousand we subtracts twenty percent of one hundred and eight thousand e… twenty one thousand and six hundred then the capacity of Croke Park is…is eighty six thousand and four hundred. And then just to add all together.

20% of 90000 = $\frac{20}{100}$ * 90000 = 18000

18000 + 90000 = 108000

108000 – 20% of 108000= 108000 - $\frac{20}{100}$ of 108000 = 108000 - 21600 = 86400

Concluding that he understands the problem and he has to make just the addition of three numbers to achieve the solution I interrupt his work to ask if the ready-made diagram would help him to solve the problem.

After he looks the diagram for few minutes he says:

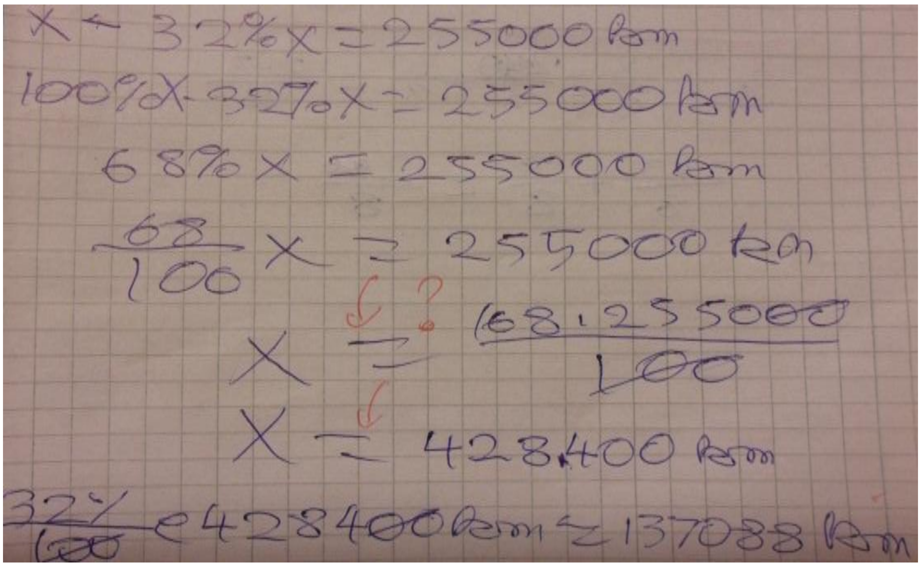
Student G: it is appropriate diagram but for me would be the same with or without it
Brikena: how do you read this diagram?
Student G: the capacity of Wembley is twenty percent bigger than the capacity of Cap Nou and…

Brikena: wait, the capacity of Wembley is twenty percent bigger than the capacity of Cap Nou but here (in the text of the problem statement) writes that the capacity of Cap Nou is twenty percent bigger than the capacity of Wembley?
Student G: e…yes now that I am seeing it well (the diagram) it says the same thing (as the problem statement) but…but it makes me confuse.

From the dialogue above, it appears that the boy after some effort understands the ready-made diagram but he likes to avoid its use. Moreover the reason why this boy likes to avoid this diagram appears to be the unfamiliarity with this type of diagram.

Student G: I like diagrams but as segments, circles but not with stadiums… in other words mathematics figures

2Rel) Asteroid Problem
On June 14, 2002 an astronomer measured the distance between the Earth and a traveling asteroid, which was near to the Earth. He found that the distance between the Earth and the asteroid was thirty two percent of the distance between the Earth and the moon. Knowing that the difference between these distances was two hundred fifty five thousand kilometers, how far away from the Earth was the asteroid at that time? (Please explain briefly how you approached this problem)



After Student G reads the problem he starts to explain the reasoning which leads him during the problem solution process.

Student G: Than the distance between the earth and the asteroid…the difference between these distances, between the earth and the asteroid and…and the earth and the moon is two hundred and fifty five thousand kilometers. Then I mark with x the distance between the earth and the moon and with thirty two percent of x the distance between the earth and the asteroid. x minus thirty two percent of x is two hundred and fifty five thousand kilometers. Then I find x is… four hundred and twenty eight thousand and four hundred kilometers (428400km). I found the distance earth-moon now I have to find the distance earth-asteroid. On June 14, 2002, the distance between the earth and the asteroid is one hundred and thirty seven thousand and eighty eight kilometers (137088)

$x - 32\,\% * x = 255000$ km

In this step Student G replaces 1 with 100/100

100% * x – 32% * x = 255000 km

He works out the subtraction

68% * x = 255000 km

Student G changes the percentage into a fraction

$\frac{68}{100}$ * x = 255000 km

He multiplies the two sides of equations with different numbers making the equality no longer valid (the left side of equation with 100 / 68 whiles the right side with 168 / 100)

$x = \frac{168 * 255000}{100}$

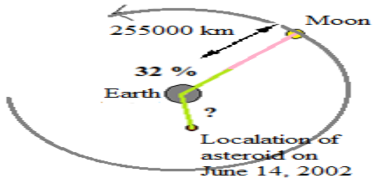
He multiplies and divides

x = 428400 km

Student G multiplies and divides

$\frac{32}{100}$ of 428400 km = 137088 km

From above it is evident that Student G identifies well all given data in the problem statement and builds correctly the mathematical models. However, the numerical results he achieves are incorrect. The external articulations above reveal that a treatment error is the source of the error.

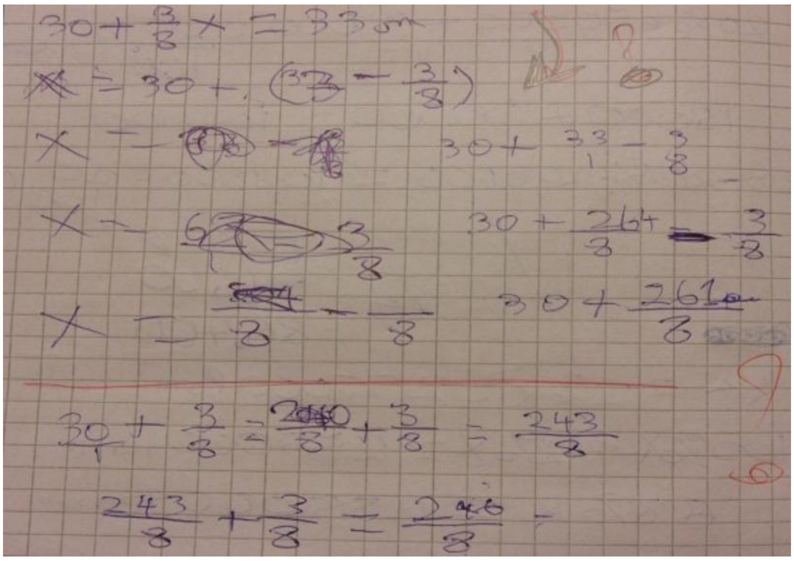


After the student G solved the problem I ask him about the effect of the ready-made diagram (which is presented for him in these moments) and his answer is:

> *Student G: Yes I can link it with the problem. I understand from here (the diagram) that the distance earth-asteroid is thirty two percent of the distance earth-moon, and then the diagram shows that the difference from these is two hundred and fifty five thousand kilometers, so I can find it.*
>
> ……………………………………………………………………………………………
>
> *Student G: Now I solved it but I would use the diagram if it would be present since at the beginning.*

From the student's immediate recognition of the diagram and the positive impression that the diagram makes (because he expresses that he would use it in the case of its presence from the beginning) it appears that it would affect the student’s performance positively.

3Rel) Stalactites Problem
Stalactites are icicle-shaped stone formations found on cave ceilings. They form from minerals deposited by dripping water. Suppose a stalactite is thirty centimeters and is growing at a rate of about three over eight centimeters per decade (ten years). How long it will take for the stalactite to reach a length of thirty three centimeters? (Please explain briefly how you approached this problem)



Student G reads silently the word problem and then he builds a correct mathematical model to find the answer.

$$30 + \frac{3}{8} * x = 33\ cm$$

Brikena: What do you mark with x? What x does it mean?
Student G: The x shows the years of growing up, until it achieves thirty three centimeters.
Brikena: How do you build it (the mathematical model)?
Student G: Thirty centimeters at the beginning plus three over eight x that grows up is equal with thirty three centimeters.

The mathematical model he builds comprises an equation with one unknown variable x. In order to solve the equation (to find the value for this unknown variable x), Student D makes a number of operations as below:

$30 + \frac{3}{8} * x = 33\ cm$

In this step Student G makes an algebraic error because he subtracts 30 from the left side of the equation and then divides the reminder by 3 / 8 while from the right side he adds 30 and subtracts 3 / 8. It means that the equality is no longer valid.

$x = 30 + (33 - \frac{3}{8})$

Student G writes 33 as fraction 33 / 1

$x = 30 + \frac{33}{1} - \frac{3}{8}$

He replaces the fraction 33 / 1 with an equivalent one as 264 / 8

$x = 30 + \frac{264}{8} - \frac{3}{8}$

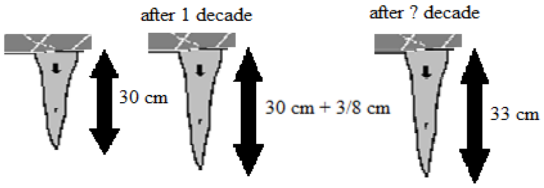
Student G subtracts two fractions

$x = 30 + \frac{261}{8}$

Than in this point he thinks for a few minutes and says:

Student G: I went wrong somewhere
Brikena: Where do you think?
Student G: In operations, but… (He is not finding where)

In this moment of the interview I present in front of him the ready-made diagram and ask the Student G about this diagram.



Student G: With this diagram I would solve it (the problem) without x, thirty centimeters plus three over eight; I would solve it by continued checking until the final result.
………………………………………………………………………………………………
Brikena: Can you explain what do you do until now?
Student G: I started to add for each decade…plus three over eight that grows up for each decade and it is two hundred and forty three over eight. It is not what we are looking for, so I will continue to add and one decade more and so on…

Student G converts 30 in fraction 240 / 8

$30 + \frac{3}{8} = \frac{240}{8} + \frac{3}{8} = \frac{243}{8}$

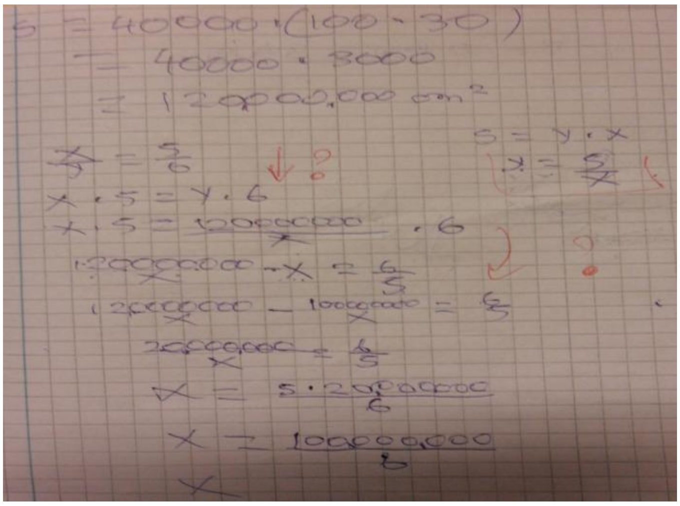
Student G adds two fractions

Brikena: How it is better with or without diagram?
Student G: No, no better with diagram it helped me enough

From the dialogue above it appears Student G understands the ready-made diagram and uses it to find another solution strategy that is not based on the equation. Because (from two items above) it appears that he is not very competent in the solution of equations. Student G is enthusiastic about how the diagram affects his work, suggesting that the diagram has a positive effect in this student' work.

4Rel) "Mother Teresa" Piazza Problem
A construction company has won the tender to fix the floor of "Mother Teresa" piazza in Tirana city, which has a rectangular form. To fix the floor of this piazza the company will use forty thousand rectangular tiles with dimensions thirty centimeters and one hundred centimeters. Can you find the

perimeter of "Mother Teresa" piazza, if its dimensions are in the ratio five over six? (Please explain briefly how you approached this problem)



Student G reads the problem and based on the given data starts to explain how he is building the mathematical models one by one to achieve an answer.

> *Student G: Then we will find the area of "mother Teresa" piazza that is forty thousand rectangular tiles together multiplies by thirty multiplies by one hundred in brackets, because it is rectangle (the tile). It is forty thousand multiplies by three hundred….three thousand is one hundred and twenty million square centimeters.*

S = 40000 * (100 * 30) = 40000 * 3000 = 120000000 cm^2

> *Student G: Then we have to find the perimeter of "Mother Teresa" piazza, if its dimensions are in ratio as five over six (5: 6). Then I mark x the length and y the width so equal with five over six. From here the area is x multiplies y, so we make the operations….and ….and I will find the perimeter*

$\frac{x}{y} = \frac{5}{6}$ And S= y * x → y = S / x

In this step student G multiplies the left side of the equation with 5 * y while the right side with (36 * y) / 5, making so the equality invalid.

x * 5 = y * 6

He replaces y = 120000000 / x

x * 5 = $\frac{120000000}{x}$ * 6

In this step Student G divides the right side of the equation by 6 and then subtracts from the remainder x, while he multiplies the left side with 6 / (25 * x). So, he makes another algebraic error.

$\frac{120000000}{x} - x = \frac{6}{5}$

Student D strangely replaces x with $\frac{100000000}{x}$

$$\frac{120000000}{x} - \frac{100000000}{x} = \frac{6}{5}$$

He subtracts two fractions

$$\frac{20000000}{x} = \frac{6}{5}$$

In this step he multiplies with the same number the both sides of the equation

$$x = \frac{20000000 * 5}{6}$$

He makes the multiplication

$$x = \frac{100000000}{6}$$

He has problems with the solution of equations and he stays in this process for a long time. So at this point, I interrupt his work (forced by the time restriction) and I presented him with the ready-made diagram.

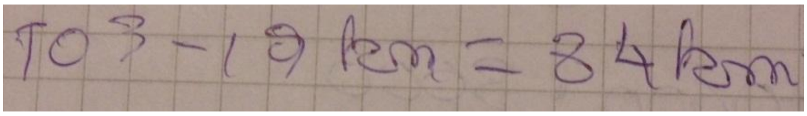
Student G looks it for a while and says:

Student G: It is the same (with and without diagram)…ehe it is the same

It appears that the diagram does not make any impact for the boy and his answer appears so strict.

5Rel) Road Segment Problem

Mira and Arta study in the University of Shkodra. Every Friday after school they take Shkoder – Tirane bus line to go home. Mira lives in a small village situated in the middle of the Shkoder -Lezhe road segment. Arta lives in a village situated in the middle of the Lac – Tirane road segment. The length of the Shkoder - Tirane road segment is one hundred and three kilometers, while the length of the Lac – Lezhe road segment is nineteen kilometers. Can you find how far away from Mira lives Arta? (Please explain briefly how you approached this problem)



Student G reads the problem and starts to reason in order to find its solution.

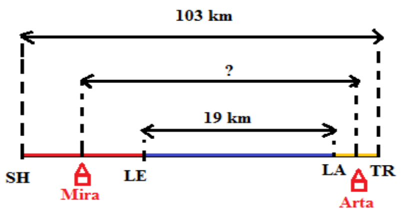
Student G: It (the problem statement) says that the length of the Shkoder - Tirane road segment is one hundred and three kilometers. And then if we subtract from the length of the Shkoder - Tirane road segment, the length of the Lezhe –Lac road segment, it is…. eighty four kilometers …

Brikena: What is eighty four kilometers?

Student G: The length of the Shkoder - Lac road segment, eighty four kilometers…

Brikena: What are you thinking?
Student G: I think…I do not think…

It appears that his thoughts are separated from the real world context (Shkoder-Tirane minus Lezhe –Lac is Shkoder –Lac) but he is guessing to achieve just a numerical answer. In this point, I give him the ready-made diagram and I wait for his reaction.



Student G: …………eighty four kilometers is Shkoder-Lezhe plus Lac-Tirane
Brikena: but you said Shkoder-Lac, perhaps the diagram is not correct or?
Student G: no, no it is correct
Brikena : if we start from Shkodra then is Lezha then is Laci then is Tirana?
Student G: yes
Brikena: but do you know it?
Student D: yes
Brikena: but you do not think to use it (the information from real world)?
Student G: no

He does not achieve a solution for the problem but at any moment his thoughts are based on this diagram.

Brikena: do you score better in the presence or in the absence of the diagram?
Student G: In the presence, because this diagram is a very good guide, it helps me so much.

From above it appears that the diagram causes him to reflect on the information that is involved in this word problem. Moreover the source of why the Student G is not taking into account this real information appears to be the school situations that they are used to meet the word problems.

At the end of the interview I asked him for the difference between two tests he completed. At the beginning he answers *"It is almost the same"* but then he starts to make comparisons and he concludes *"the problems of the last test (class test) are clearer"*. However, I need to recall that it is just his opinion because his performance is almost the same in both tests (the same types of errors). He also confessed that the test that they meet usually in their mathematical lessons is *"this one (the class test)"*.

It needs to be emphasized that the above detailed analysis supports the main part of general conclusions that have been emerged by the interviews analysis. However, the remainder of general conclusions remains unsubstantiated, because it is impossible to choose just one student who performed exactly as the trend.

4.4 Summary

This chapter has presented overviews of students' performance in the test and task based interviews, followed by detailed accounts of individual students' performance in order to make evident the way the general conclusions of this study are achieved. The chapter has also presented an overview of students' opinions about the test and their performance.

The results of this study reveal that students have difficulties to solve word problem. They made both conversion and treatment errors to word problem solution process. However, it appears that the majority of students' errors arise in the cognitive processes of conversion, which appear to be more complex and difficult processes for students. The students' responses to both real and abstract context word problems exposed a tendency to avoid engagement with the context of the problem statement. Therefore their performance in both abstract and real world contexts appear to have little difference. Certain difference arises in the interaction between the real word contexts and diagrams. It appears that in some cases the real world context interferes with students drawing a meaningful diagram or their use of the ready-made diagram. Whereas, the students' responses to both word problems with and without diagrams exposed some difference on their performance that might arise from the provision of the diagram.

In the next chapter the reported results about students' difficulties, the effect of real world and abstract context, and the effect of the ready-made diagram in word problem solution process will be discussed in the light of the study design, the theory and previous research.

5 Discussion

In the present dissertation students' difficulties in mathematical word problem solution processes exposed through their engagement with four different types of word problems (abstract context with diagram, abstract context without diagram, real world with diagram and real world without diagram) are reported.

In Chapter Three I explained that to collect data several methods were employed: participant observation with four Grade 8 classes of an elementary school, a pencil paper test with a total forty one students in one class, a questionnaire also with these forty one students, and task-based interviews with a total of eight students selected from the class of forty one.

The reader will recall that this study addresses the research questions reproduced below:

1. What evidence of Grade 8 Albanian students' conversions and / or treatments is exposed in their solution of mathematical word problems?
   1.1 What is the evidence of influence of the inclusion of problem statements in abstract or real world context on students' solutions?
   1.2 What is the evidence of influence of the inclusion (or omission) of a diagram in problem statements on students' solutions?
2. What are the students' opinions about the test and their performance?

The Data Analysis chapter presents the analysis of the data in terms of Duval's (2006) cognitive approach, the theoretical framework adopted for this study. In this present chapter, the results (i.e. the outcome of the data analysis) will be discussed in the light of the study design, the theory and previous research set out in Chapters Two and Three.

This chapter is divided into four parts:

- Students' difficulties in word problem solution process

In this part of the chapter the reported results about students' conversions and treatments are discussed in terms of Duval's (2004, 2006) assertions.

- Real world and abstract context

In this part of the chapter, the reported results about the influence of the problem statement context (real word and abstract) to students' performance to word problems, is discussed. The discussion is structured according to the design of the study, the theory and previous research such as De Bock, et al. (2003); Duval (2004); Greer (1997), etc.

- Interaction of the context and diagram

This part of the chapter is based in the design of the study and previous research such as Booth and Thomas (2000); De Bock, et al. (2003); Duval (2004); Greer (1997); the results about the interaction of the context and diagram are discussed.

- Ready-made diagram

In this part of the chapter, the reported results about the influence of the ready-made diagram to students' performance to word problems are discussed. The discussion is structured according to the design of the study, the theory and pervious research such as Diezman and English (2001); Pantziara, Gagatsis and Elia (2009); Pantziara, Gagatsis and Pitta-Pantazi (2004); etc.

5.1 Students' difficulties in word problem solution process

The results of the present study, according to students' responses to word problems in the pencil and paper test, reveal that 18% of students' responses were correct and 36% were missed (blank). The students' responses in the interviews demonstrate that 38% of their responses were correct and 8% were missed (blank). The above provides further evidence that students have difficulties to solve word problems. The above evidence also reveals that students performed better in the interviews than in the pencil and paper test. Two explanations might explain this difference. The first is related to the study design. I chose eight students with average and higher mathematical competences to be interviewed, while in the test students with low mathematical competences also participated (see Chapter Three). The second explanation arises to the circumstances in which students performed. The presence of camera and the dialogue with me during interviews appear to motivate students' attempts to achieve correct solutions for the problems.

The results of the present study demonstrate that the majority of students' errors, during word problem solution process, arise in the cognitive processes of conversion. Concretely, 72% of students' errors in the test and 68% of students' errors in the interviews arise generally to errors in the first or last conversion. However a considerable proportion of students' errors (28% in the test and 32% in the interviews) arise in the cognitive processes of treatment. These findings support the assertions of Duval (2004, 2006) (discussed in Chapter Two) who considers the two types of transformation (conversion and treatment) as belonging to different cognitive processes that often are the sources of incomprehension in mathematical learning. Furthermore, Duval (2004, 2006) notes that conversion is a more cognitively complex process because for students the passage from one semiotic system (register) into another (without changing the denoted objects) is a difficult cognitive transition to be overcome.

5.2 Real world and abstract context

Earlier in this dissertation (Chapter Two and Chapter Three), I expressed my assumption that the real world context and the ready-made diagram would be positive factors, which could help students to improve their performance on word problems. Contrary to my conjecture, but in agreement with the results of other researchers in the field such as Boaler (1994); De Bock, et al. (2003); and Movshovitz-Hadar and Shriki (2009) (discussed in Chapter Two) the real world context did not produce the expected improved performance.

It appears that in the tests and task-based interviews, the real world context yielded no positive effect on students' performance. To explain the unexpected results a conjecture can be made over the way the real world context was implemented in the present study. It is mentioned in the Chapter Three that students faced for the first time the five real world problems in the test or in the interview. Establishing an analogy to the notion "*didactical contract*" (Brousseau, 1984 as cited in Greer, 1997, p. 298), Greer (1997) defines the notion *"experimental contract"* (p. 305) as a system of norms, rules and expectations that affect the performance of participants (participants' thoughts and actions). Therefore the time restriction of the test or of the interview' duration might have put a pressure on students. Consequently students might be inclined to avoid the real world contexts of the problems, thus they did not benefit from the advantages of the real world context. Additional support for this hypothetical explanation appears to emerge from the interviewed students' opinions. Students remarked that the real word and abstract problems resembled each other, but it was easier to extract data from the abstract problems than from real world problems. These opinions lead to suspicions that students did not try to enter the situation in which the problem was set, but attempted to achieve the solution as quickly as possible. They appear to focus on extracting the mathematical content from the text of the problem (problem statement).

Another explanation for the unexpected results of the real word context in students' performance is based on Hart (1996). This focuses on the real word situations used to frame problem statements into real world context. In Chapter Three it is explained that these real word situations were chosen with the intention that they would be familiar and interesting for students. However I was not exactly involved in the participant students' interests and preferences but, I chose them using my experience as an Albanian student and student-teacher; my knowledge about Albanian school context; Albanian curriculum; and the contact with the teacher. Therefore these real world contexts might not necessarily be familiar and interesting for them.

5.3 Interaction of the context and diagram

It appears from the test results, that the real world context in which problem number 3Rel, (Stalactites Problem) is framed has a negative effect. It is evident, especially from students' reflections on this task that the context information interfered to their drawing a meaningful diagram. Furthermore, this context also interfered with students' use of the ready-made diagram. The negative effect of this real world context in relation to the construction of a diagram also emerged in the interviews. However in the interviews there appears to be no effect due to the context in relation to the usage of the ready-made diagram. The conjecture which possibly explains the emergence of this negative effect is that the difference in performance in the task arises from the students' unfamiliarity with this real world context. This unfamiliar context might cause confusion and therefore students might fail to imagine the situation in which the problem is set. Consequently it appears that students combine mechanically the mathematical elements involved in the problem statement to achieve a solution for the problem. They did not attempt to construct a diagram to visualize the real world context.

It is interesting to note how this context does not have the same effect in relation to the usage of the ready-made diagram in the test, wherein appears that students' performance is not affected by the ready-made diagram and in the interview, wherein the ready-made diagram has improved students' performance and in some cases it has affected students to generate a new solution strategy to solve the problem. A number of factors such as the presence of the camera, the dialogue with me, etc. might have influenced the interviewed students to try harder to understand the ready-made diagram and to connect it with the problem statement. Another possible explanation might be based on Booth and Thomas' (2000) conclusions, the students' spatial competencies affect the way they identify the structure of the problem in the presented diagrams. Therefore a possible explanation such as the higher spatial competencies of the interviewed students in comparison to the tested students (who faced the 3Rel problem) cannot be excluded.

From students' reflections on problem number 5Rel (Road Segment Problem); it appears that the real word context information (in which the problem statement is framed) interfered with students' effort to draw a meaningful diagram. The conjecture raised in the above section, about students' attempts to extract data from the problem statement without being involved in the context, appears to take place also here. The real world context in which this problem is set might introduce an amount of "noise" that makes it harder for the students to extract the information. Another possible explanation is established on De Bock, et al. (2003) (discussed in Chapter Two) who conjecture that such results arises because students are used to solve word problems with traditional approach. This conjecture is supported by the results of the questionnaire, wherein it appears that from the students' point of view abstract word problems are more similar to the word problems they solve in the classroom.

In this case the real world context (in which this problem is set) is familiar for students. However contrary to Duval (2004) (discussed in Chapter Two), it appears that students do not call on their experience and mental representations to comprehend the situation (to involve information from real life in the mathematical problem solution). Another possible explanation is related to the status of this problem. It was a voluntary problem, therefore when students realized that the context of the problem was complex (because to draw a correct diagram it requires locating the cities in a correct geographical sequence) they have not persisted to work on it. This real word problem was the fifth problem in both the test and in the interview. Therefore other conjectures, such as time restriction or students' tiredness might be valid, based on Greer's (1997) notion of "experimental contract". Students might not have had sufficient time or energy to think about of the cities location in a correct geographical sequence (see p. 40 for the problem statement).

5.4 Ready-made diagram

My hypothesis about the positive effect of the ready-made diagrams is partly supported by the results reported. I use the word "partly" because the positive effect of the ready-made diagram is not evident in word problems number 1Abs/ 1Rel (% Bigger & % Smaller Problem/ Stadiums Problem) and 4Abs/ 4Rel (Rectangles Problem/ "Mother Teresa" Piazza Problem) (in both test and interviews results).

A potential explanation that appears to stand for the first problem (1Abs/ 1Rel) is related to the students' partial understanding of the concept of the percentage of a number. Therefore, since the diagram represents the components of the situation and their organization (Diezman & English, 2001), it appears to have no power to enable students to overcome this difficulty (to understand the concept of the percentage of a number).

The insignificant effect of the ready-made diagram in problem number four calls for explanation. Generally in problem number four (4Abs/ 4Rel) students appear to have problems in finding a right way to connect the area of the rectangle (piazza) to the ratio of their sides in order to find the perimeter of rectangle (piazza) (see p. 41 for the problem statement). A possible reason that might cause this problem for students can be the diagram's construction. Perhaps the relation between the area of the rectangle or the piazza and the ratio of their sides is not shown clearly by the ready-made diagram. However another possible explanation might be based on Pantziara, Gagatsis and Pitta-Pantazi (2004) (discussed in Chapter Two) suggestion about the development of the diagrammatic literacy before using diagrams. A further possible explanation is that problem number four is difficult because there is no direct way for students to find the dimensions of the rectangle (piazza) given the ratio of the sides. Therefore the diagram fails to aid students to overcome this difficulty.

The positive effect of the ready-made diagrams appears to be evident in word problems two, three (in abstract context) and five. The results of the present study, in terms of ready-made diagrams' effect on students' performance on above word problems, appears to support the suggestions of several authors from the mathematics education community such as Booth and Thomas (2000); Diezman and English, (2001); Dirkes (1991); Gagatsis and Elia, (2004); Pantziara, Gagatsis and Elia (2009); and Pantziara, Gagatsis and Pitta-Pantazi (2004) who asserted that the diagram as a useful tool for thinking in the solution process of a word problem. Other support for the positive effect of the ready-made diagrams appears to emerge from the questionnaire results, wherein the majority of students who worked with word problems with ready-made diagrams admitted that they used the diagrams and that the diagrams helped them to find an answer. Moreover, students who worked with word problems without diagrams are expressed more disagreement about the fifth statement (I consider this as an easy test) than the other students.

5.5 Summary

This chapter has discussed in the light of the study design, the theory and pervious research the reported results about students' difficulties and the effect of two factors such as the problem statement context and ready-made diagram in students' performance to word problems.

The results of this study reveal that students have difficulties to solve word problems. In accordance with Duval's (2004, 2006) assertions, students made both conversion and treatment errors in the word problem solution process. However, the majority of students' errors arise in the cognitive processes of conversion, which appear to be more complex and difficult processes for students. The students' responses exposed that their engagement with the real world problems did not improve students' performance on word problem. To explain the unexpected results conjectures are raised about the way these real word contexts are chosen and implemented. Some negative effect of the real word context arises in the interaction between the real world context and diagram. To explain the reported results some conjectures about the unfamiliarity of the context, students' spatial competencies, problems with traditional approach with which students are familiar, etc. are discussed. The positive effect of the ready-made diagrams is partly supported by the reported results. To explain why it is not completely supported conjectures are raised over the notion of the diagram, the diagram's construction and the development of diagrammatic literacy before using diagrams.

Based in the achieved results, in the next chapter, issues such as: trustworthiness of the study results; how close to the intention was the implementation of the methodology; implication of findings from perspective of theory, future research, teacher and policy makers; and the research effect in my development will be discussed.

6 Conclusion

This chapter is divided into five parts:

- Summary of the Dissertation

This part of chapter presents an overview of the study, wherein is described the aims of the research, the methods used to collect data, the study context and the attained results.

- Trustworthiness of study results

The elements that can affect the trustworthiness of the study results are discussed in this section.

- Intention and Implementation of Methodology

This part of the chapter describes how close to the intention was the implementation of the methodology.

- Implication of Findings

This part of the chapter discusses what the study results show in terms of: theory (Duval's (2006) cognitive approach), future research, teachers' perspective and curriculum planers and policy makers' perspective.

- The research effect on my development (as a researcher and as a teacher)

The gained knowledge during the study conduction in terms of my development as a researcher and as a teacher is described in this section.

6.1 Summary of the Dissertation

Word problems have an important part of the Albanian school mathematics curriculum. The study presented in this dissertation set out to explore Albanian students' difficulties in the mathematics word problems' solution processes. In particular, it explores the role of two factors such as the context in which the problem is framed, and the provision of ready-made diagrams which represents the components of the problem statement and the relation between them.

This research was a case study conducted in one Grade 8 class of an elementary school in a small city in the north of Albania. Forty one students (13-14 years old) participated voluntarily in the study.

Initially relevant literature provided an overview in the field of inquiry. Theory and previous research guided the work. To identify students' difficulties on word problems, I used Duval's (2006) cognitive approach. In this theory, conversions between different registers and treatments within the same register are considered as two types of cognitive processes (transformations) that are independent source of incomprehension in mathematics learning.

The research questions addressed are:

1. What evidence of Grade 8 Albanian students' conversions and / or treatments is exposed in their solution of mathematical word problems?

1.1 What is the evidence of influence of the inclusion of problem statements in abstract or real world context on students' solutions?
1.2 What is the evidence of influence of the inclusion (or omission) of a diagram in problem statements on students' solutions?

2. What are the students' opinions about the test and their performance?

To answer the research questions it was necessary to collect data from the students' engagement with word problems. Four methods to achieve this goal were combined, these were:

- Participant observation- I engaged in participant observation for one week with four Grade 8 classes. The main goals of this week were: the familiarization of students with my presence in the classroom, the identification of the classrooms milieu and the choice of one Grade 8 class to be part of the rest of the study.
- A pencil and paper test with word problems- The purpose of this test was to clarify the characteristics of the students' performance and to pick eight students for the task-based interviews.
- A questionnaire - The questionnaire was used to gather information about the students' opinions about the content of the test and their performance.
- Task based interviews- The purpose of the task based interviews was to expose the students' thinking processes during their engagement with word problems.

The results of the study reveal that students have difficulties to solve word problems. The majority of students' errors arise in the cognitive processes of conversion, however a considerable part of students' errors arise in the cognitive processes of treatment. Thus, showing that both cognitive processes (conversion and treatment) are sources of difficulty in mathematics word problem' solution process. Furthermore, consistent with Duval's (2004, 2006) explanation, the cognitive processes of conversion appear to be more complex and difficult processes for students. The students' responses to both real world and abstract context word problems exposed that the context of the problem statement appeared to have little differences on students' performance. Certain differences arise in the interaction between real world context and the diagram. It appears that in some cases the real word context interferes with the students' effort to draw a meaningful diagram or to use the ready-made diagram. Whereas, the students' responses to both word problems with and without ready-made diagrams exposed some difference in their performance that might arise from the provision of a diagram. One implication that can be drawn from these observations is that students will benefit from instruction in the use of information provided in word problem statements.

6.2 Trustworthiness of study results

The study results might be influenced by a number of elements such as:

- The word problems which students faced during the test and the task based interviews. The reader should recall that these word problems (in different contexts and with and without ready-made diagrams) are produced by the researcher. In Chapter Three it is explained how these word problems are designed by following a set of criteria such as: the theoretical notion of a word problem, the theoretical notion of a diagram, students' mathematics abilities, the textbook content, the Albanian context, etc. In addition, in the same chapter it is also explained that the design reason of those problems was to see the students' performance in each types of word problem. Thus, the reader has the possibility to evaluate the design of these problems.

- The methods used to collect data. The reason for the combination of four methods used to collect data, their advantages and limitations and the implementation of these methods are explained in the Chapter Three. Furthermore, how close to the intention was the implementation of these methods will be described in the present chapter. These enable the reader to reach a judgment about the chosen methods and their implementation.

- The data analysis is conducted using Duval's (2006) cognitive approach. The way in which this theoretical framework is applied to analyze data is explained in Chapter Three and also in the Chapter Four. Thus creating, it is hoped, a clear picture of the use of this framework and giving the reader the possibility to evaluate and to criticize its usage. The application of Duval's (2006) framework proved challenging at first and this was one of the parts of this study from which I learned a lot about the processes and objectivity of research; this is explained in section 6.5

6.3 Intention and Implementation of Methodology

Chapter Three presents detailed explanations of: the methods chosen to collect data (participant observation, a pencil and paper test with word problems, a questionnaire that asks pupils to reflect on the test, and clinical interviews with a small sample of students); the purposes of each method; the advantages and limitation of each method; the reasons for combining four methods; etc. The methods implementation took place in the expected way. In addition, I believe, the implementation of each method fulfills the projected goals, thus giving access to the required data. However, one exception is the case when one student voluntarily agreed to participate in the interviews but then he hesitated to speak during the interview. By his limited answers it appears that he was not expressing openly his thoughts and he was looking forward to finish the interview. Therefore this interview is not included in the data analysis within the study.

Chapter Three also explains in detail the design process of problems and in addition the goal to set word problems into real world contexts which are familiar and interesting for students. From students' responses to the real world problems it appears that the real world contexts were not necessarily interesting to them, since the students appeared to avoid their involvement in the context. However, from students' opinions expressed in the interviews, they do find working on real word problems interesting. Therefore, it remains to be confirmed by the future studies if the real world contexts were not interesting for students or if the implementation way of these real world contexts has prevented students to identify the interesting and beneficial part of the contexts. I was surprised by students' apparent unfamiliarity with the real word context of problem number three (3R, the 'stalactite problem'). I had assumed that students had learned about stalactites in their geography courses (a compulsory course in the Albanian elementary schools) that they followed in the previous years of school. From this outcome I learned that previous years' syllabuses are not a very reliable way of judging what students know.

6.4 Implication of Findings

The study results reveal that both Duval's cognitive processes (conversion and treatment) are useful as approaches to analyze students' responses to word problems. In other words, both these types of transformations (conversion and treatment) have potential to identify possible sources of students' difficulties in the word problem solution process and to understand causes (reasons) of these difficulties. In this way, the study results confirm that Duval's

(2006) cognitive approach can be used as an effective theoretical framework to analyze students' performance to word problems.

The findings of the present study can be helpful for (Albanian) teachers to equip them with information about where students' experience difficulties in solving word problems. In this way, by knowing the sources of incomprehension, teachers can help students to overcome their difficulties. Other helpful information for teachers can be derived from the present study in terms of context and ready-made diagrams. The results of the study suggest that teachers might support students by finding appropriate ways to introduce the real world context and ready-made diagram into word problems. Students might benefit from instruction about how to use the real word context and the ready-made diagrams in order that they can make use of each of them.

The findings of the present study highlight new ideas for further inquiry. Since this is a case study, it cannot be claimed that the findings have general validity beyond that Grade 8 class (which participated in this study). Therefore, to estimate the general validity of these findings the same study would need to be conducted with a larger and representative sample. If the study were to be replicated then it would be necessary to take care to carefully match the context, if similar findings were to be exposed. An expansion of this study could be developed by including in its design a new instrument that implements teaching activities through which the researcher or the teacher (instructed by researcher) can teach students to use effectively the real word context and the ready-made diagram to solve word problems.

Another idea to develop the present study in the future can relate to the choice of the real word contexts. While I was designing the problems my aim (as a researcher) was to set the problem statements into real world situations that I presumed would be familiar and interesting for students. It needs to be emphasized that I designed the real word problems before having contact with students (for the reason of time restrictions of the field work) and I tried to achieve my intention by using my experience as an Albanian student and student-teacher, by using my knowledge about Albanian school context, Albanian curriculum and the contact with the teacher. Therefore, a future study could investigate whether increasing the duration of the participant observations, in the way that the researcher to be able to identify the real world contexts that are interesting and familiar for students and to use these contexts to construct the real word problems (for the test and the interviews) would produce different outcomes.

In Albania research in the field of mathematics education is limited. Therefore the present study can contribute in the development of this research field in Albania. Furthermore the results of this study (even though it is a case study) can provide useful information for the Albanian curriculum planers and policy makers.

6.5 The research effect in my development (as a researcher and as a teacher)

The first step that I made to conduct the present study was studying the relevant literature in the field. I read several studies conducted by different authors. At the beginning, the way these authors designed their studies did not appear difficult from my point of view. However when I started to develop the design of my study I understood that it was not easy. On the contrary, I have to think twice for every decision to involve a method, a problem, a context, a diagram, the way of a method implementation, etc. I have to consider their role in my study, their effectiveness, their advantages and limitations, their relevance in the study context, etc.

The available time for the study design and for the fieldwork was limited therefore I tried to make the best possible choices within that time. However, the data analysis results showed that I could make better choices, for example the involvement of another method as teaching activities (to instruct students to solve real world problems and to use the ready-made diagram). In other words during this part of study I (as a researcher) learned to consider several factors such as the choice of the study components and also to be critical over choices made.

The data analysis is another part of study by which I (as a researcher) learned a lot. At the beginning, my only intention was to find a relevant theory for the study and to personalize this theory in order to analyze the available data. However, when I fulfilled that purpose (I found a relevant theory for my study, the Duval's (2006) cognitive approach) I understood that the theory does not define a strict "recipe" for each case that was presented during the data analysis. Therefore, often based on the present evidence I should raise assumptions for the cause of: the error, the student' behavior, etc. Another innovation that I learned during this part of the study was the necessity to maintain the objectivity of these assumptions. As a final conclusion I would say that the research has made me (as a researcher) gain experience in this field, which I wish to follow in the future.

This study affected also my development as a mathematics teacher. Two years earlier, during my practice as a teacher (one year practice) I identified (just) that students meet difficulties in word problems. The present study has provided me with better knowledge to understand students' difficulties in word problems, in which part of the solution process these difficulties stand, and how the implementation of some factors such as real world context and ready-made diagram might improve students' performance. In other words the present study has allowed me to see how the development of my awareness of theory can have an important influence on my practice.

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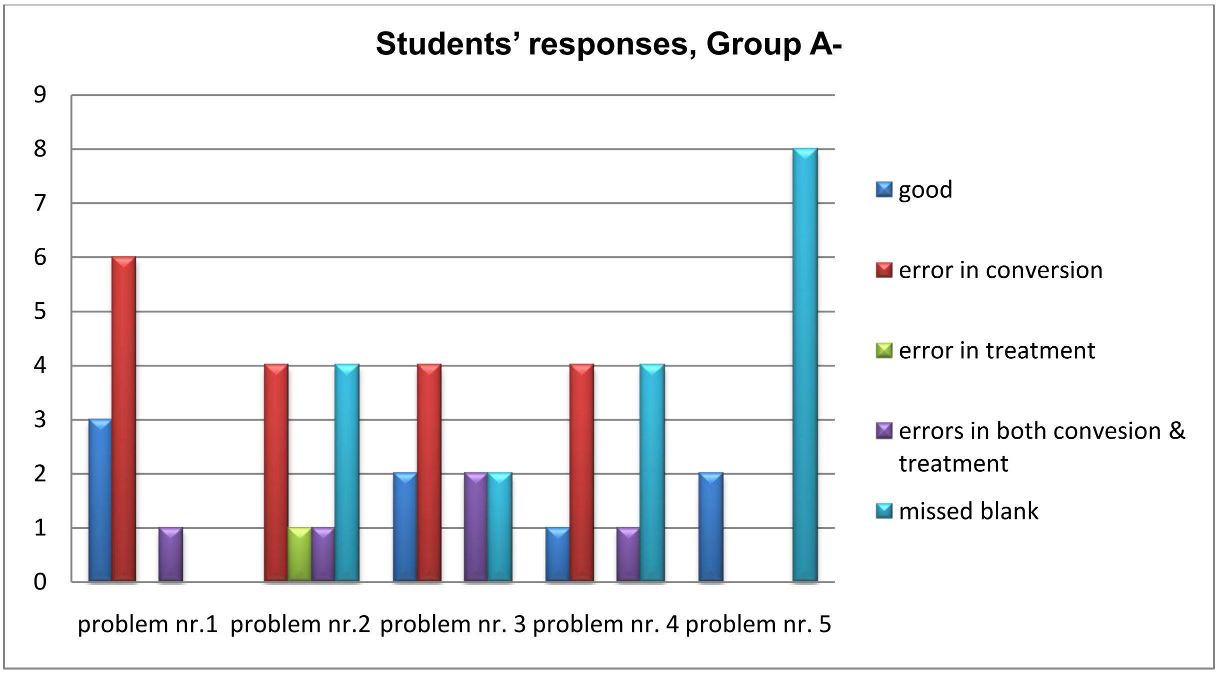
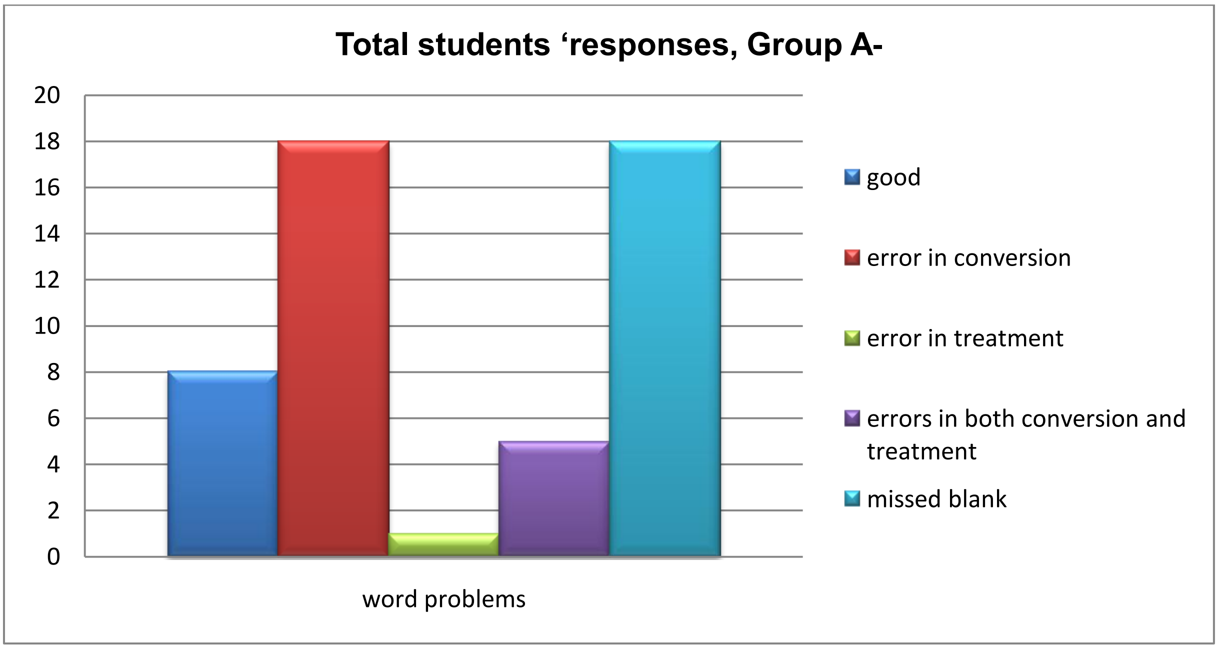
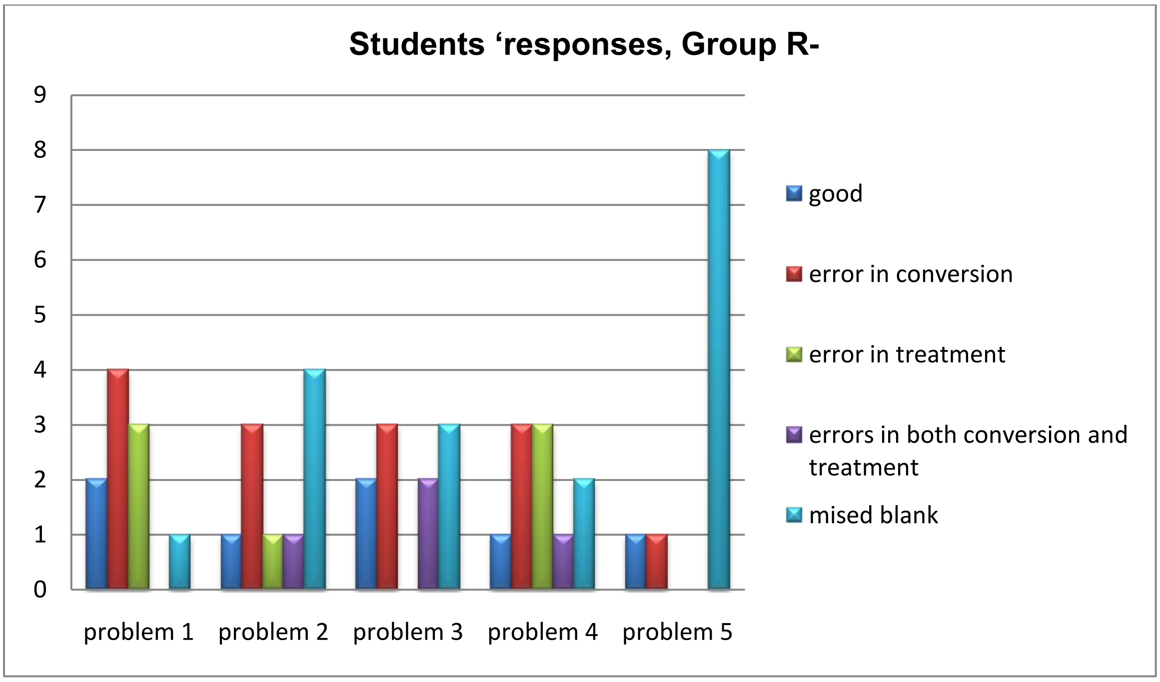
Test Appendices

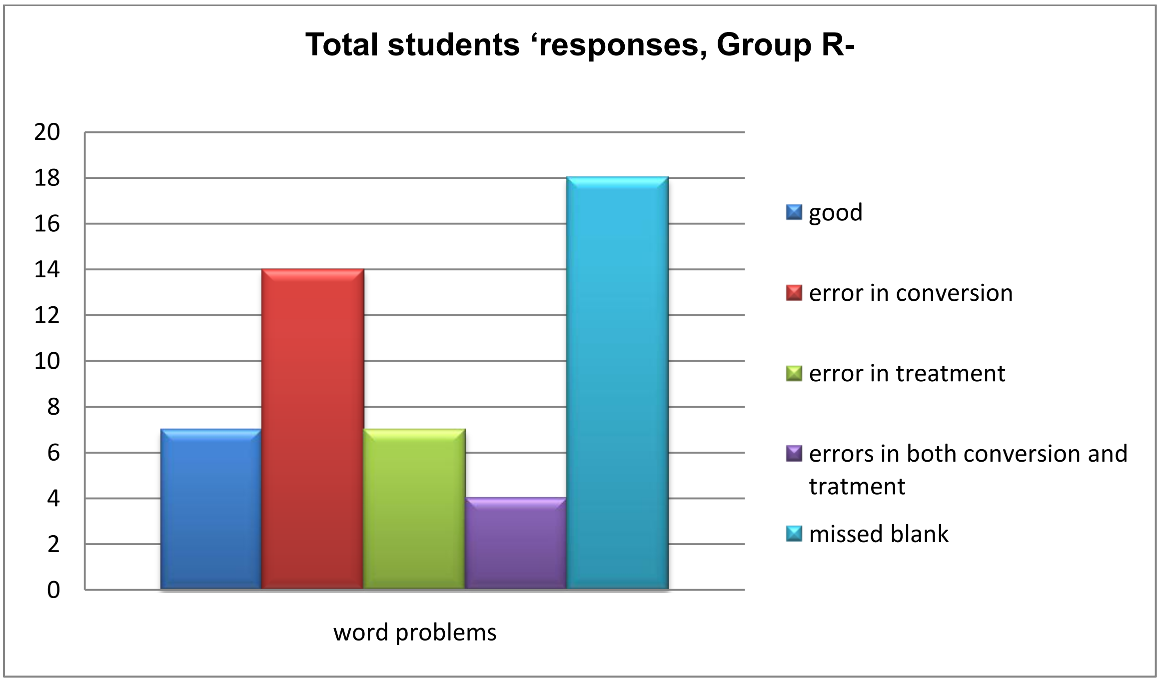
Appendix 1: (Total) Students' responses, Group A-



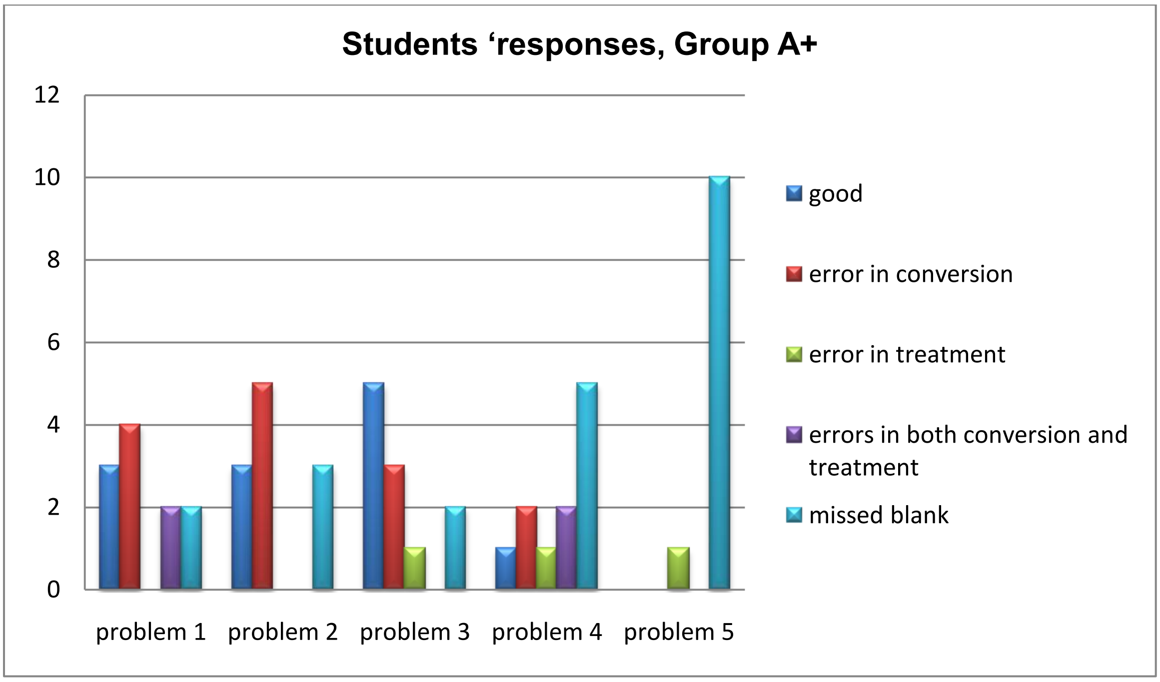


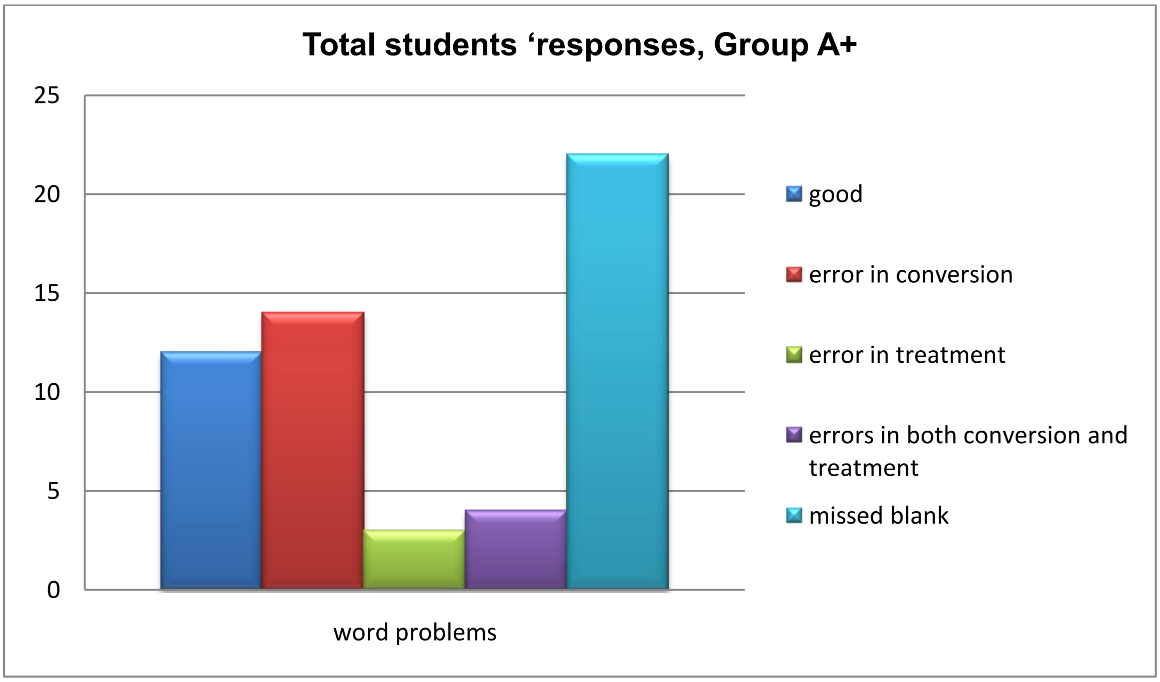
Appendix 2: (Total) Students' responses, Group R-



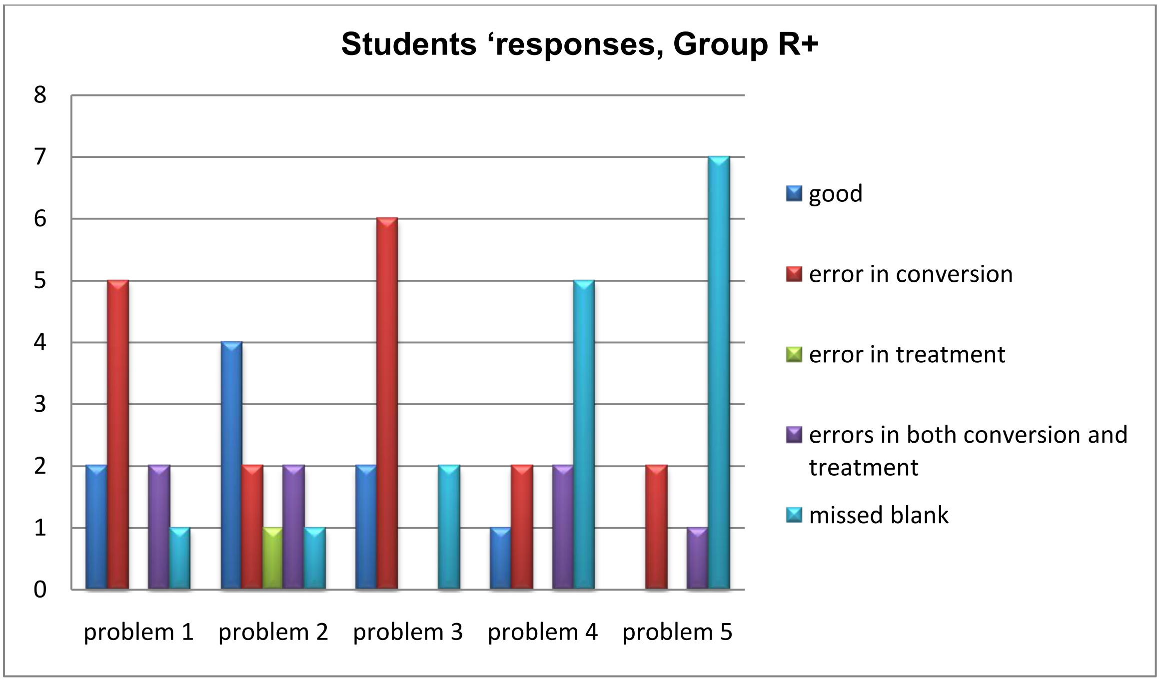


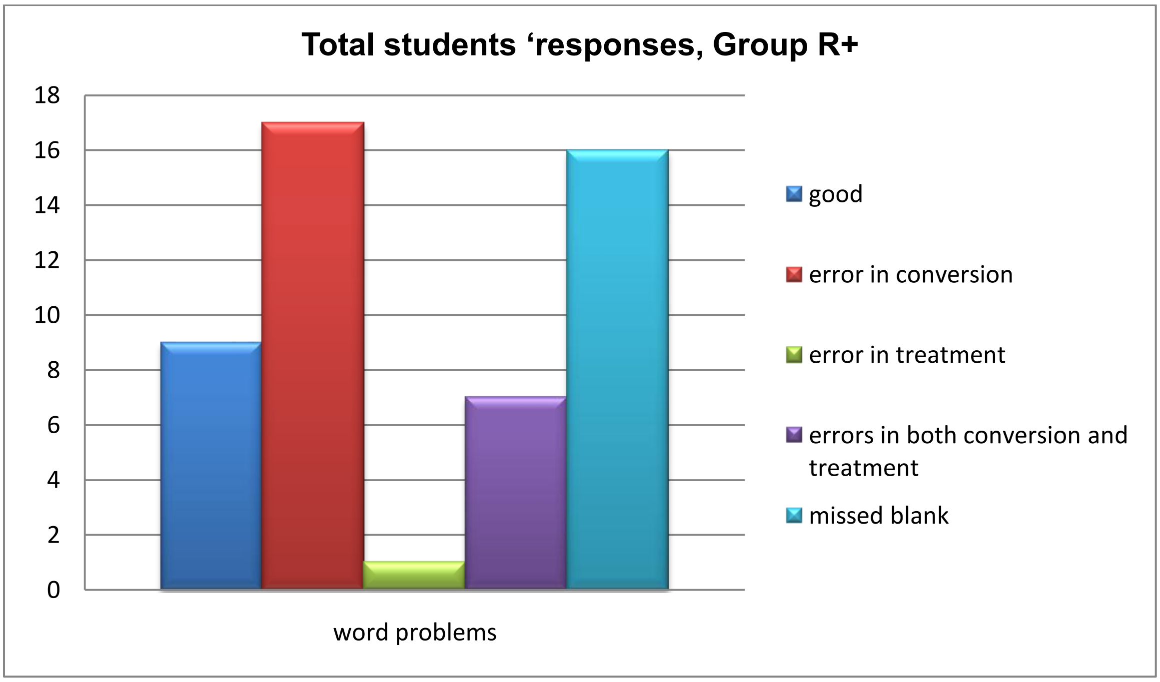
Appendix 3: (Total) Students' responses, Group A+



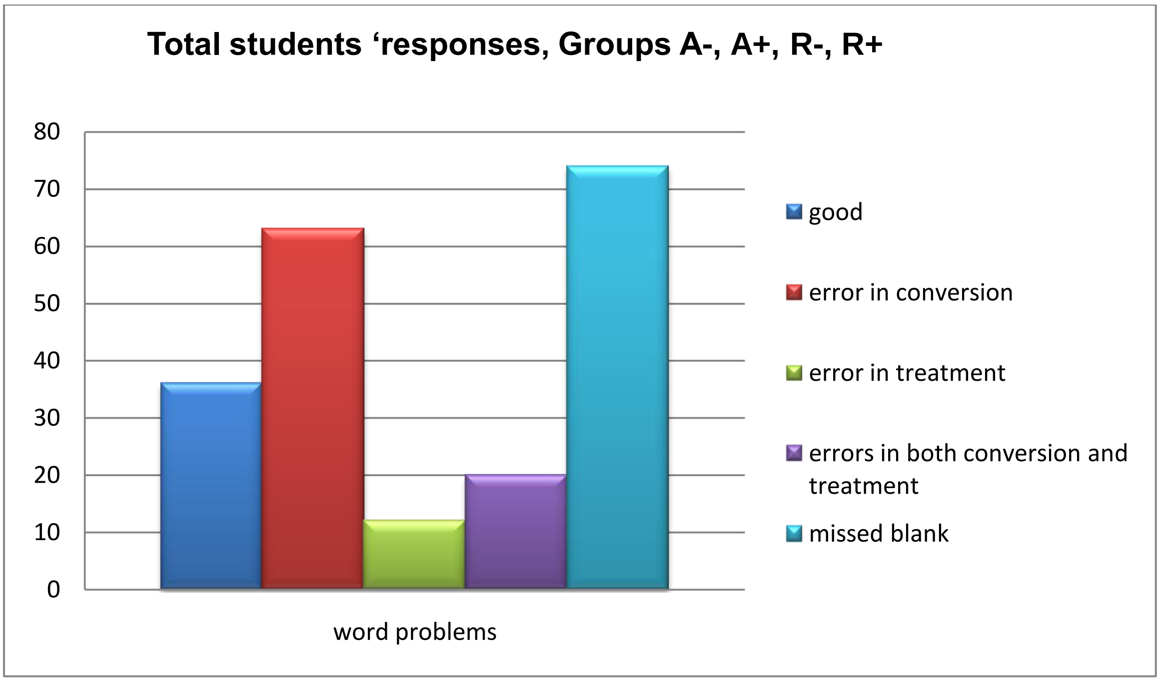


Appendix 4: (Total) Students' responses, Group R+

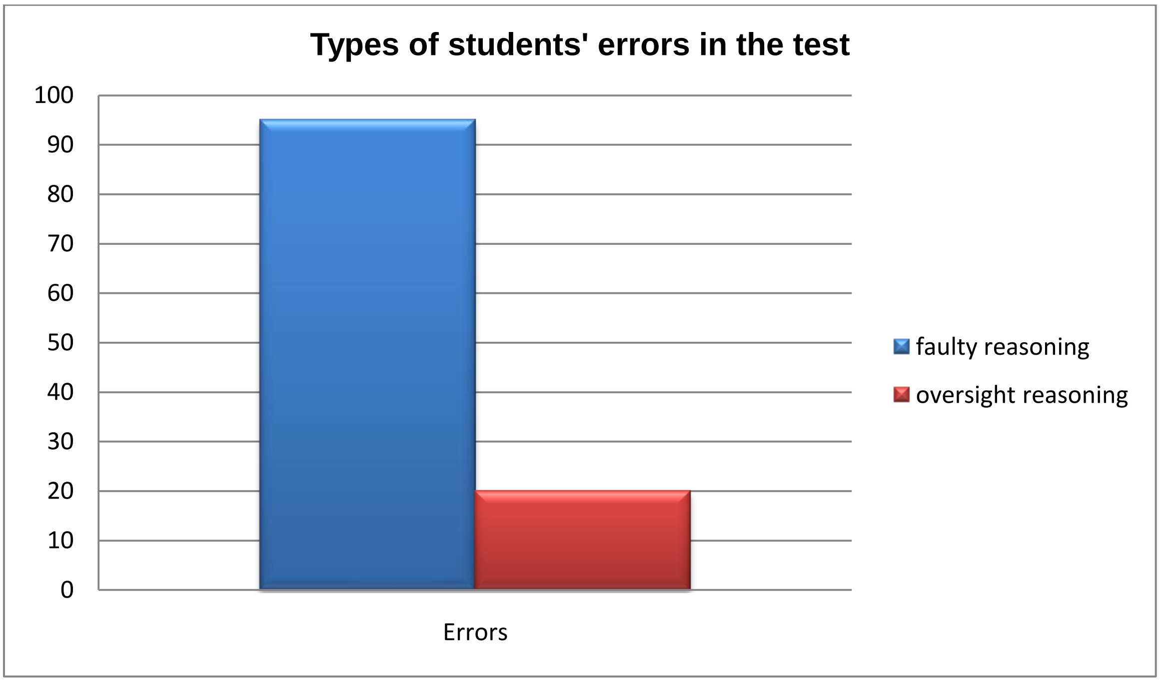




Appendix 5: Total students' responses, Group A-, A+, R-, R+

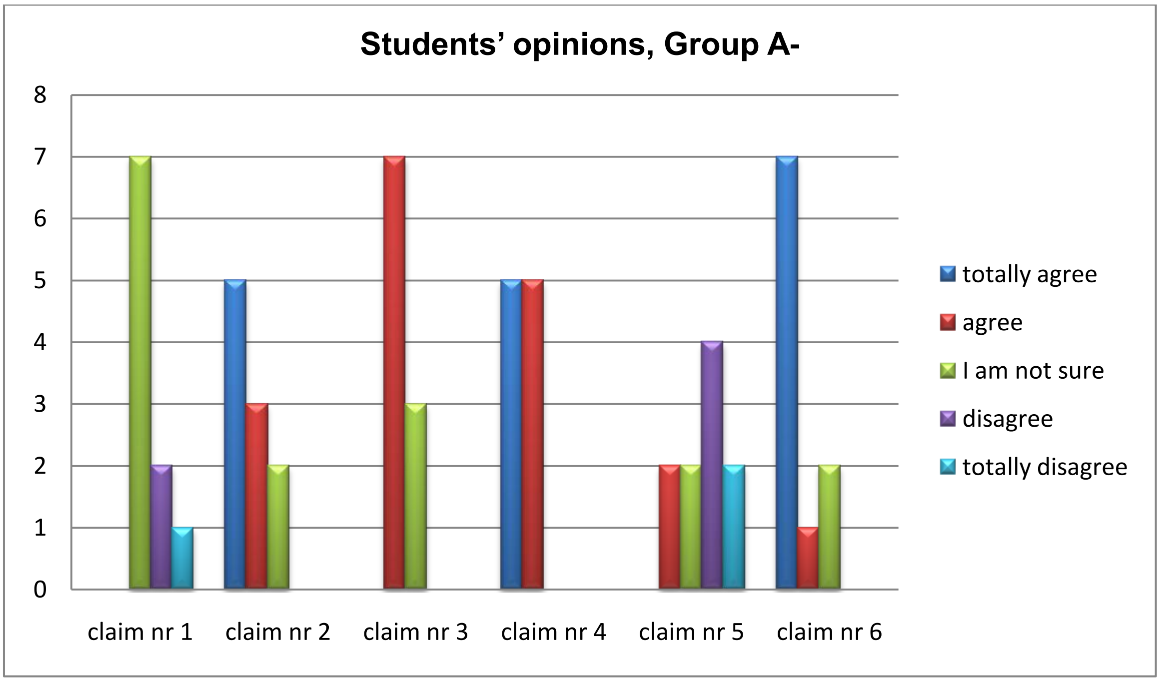


Appendix 6: Types of students' errors in the test

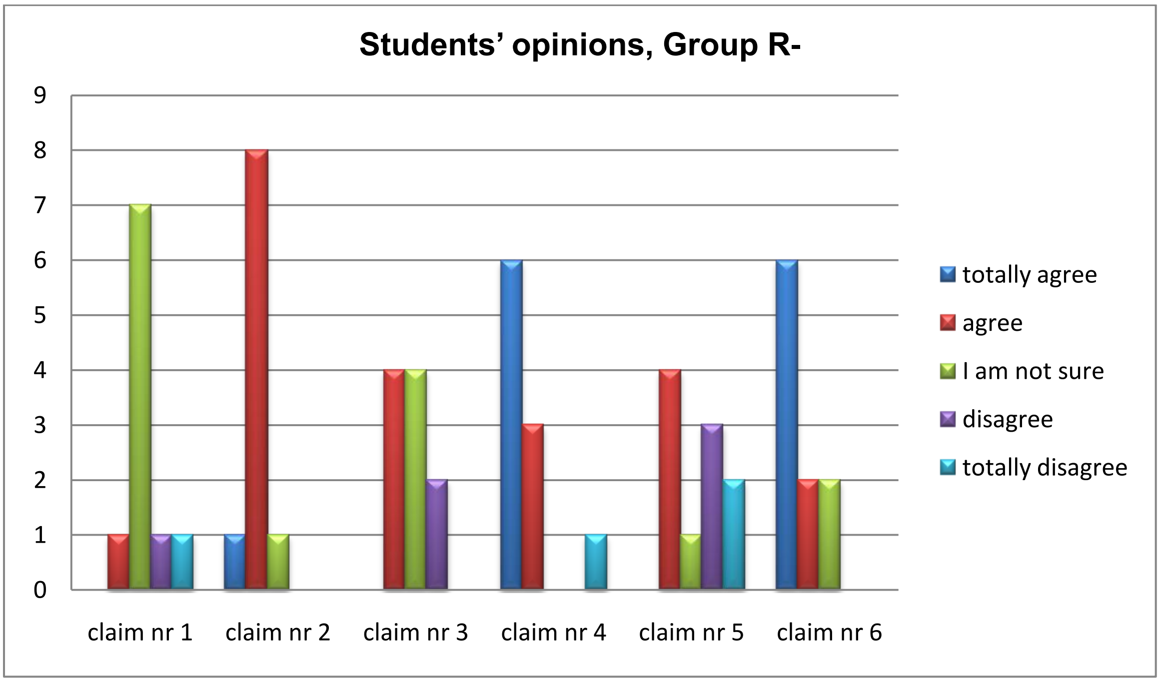


Questionnaire Appendices

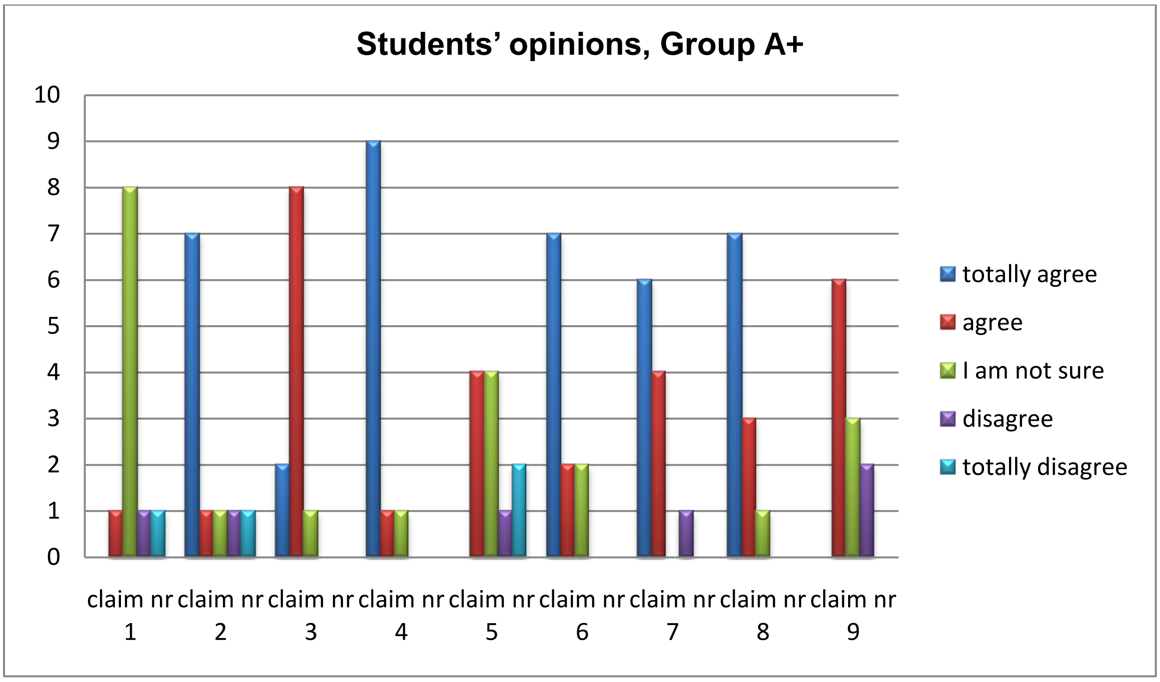
Appendix 7: Students' opinions, Group A-



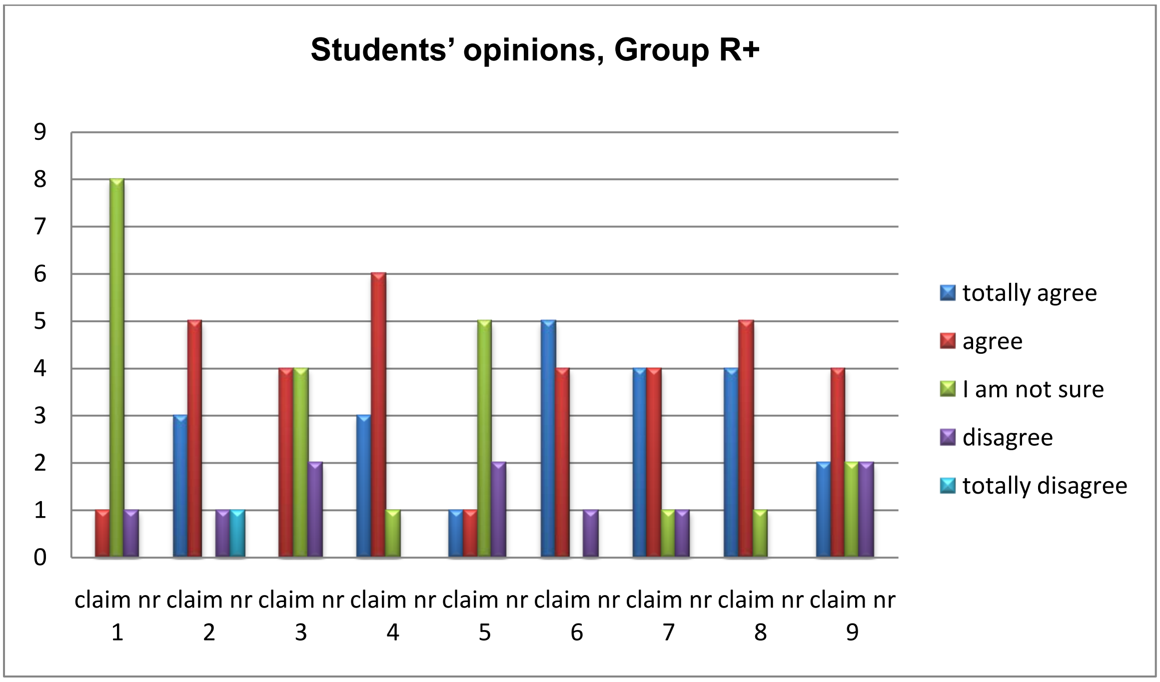
Appendix 8: Students' opinions, Group R-



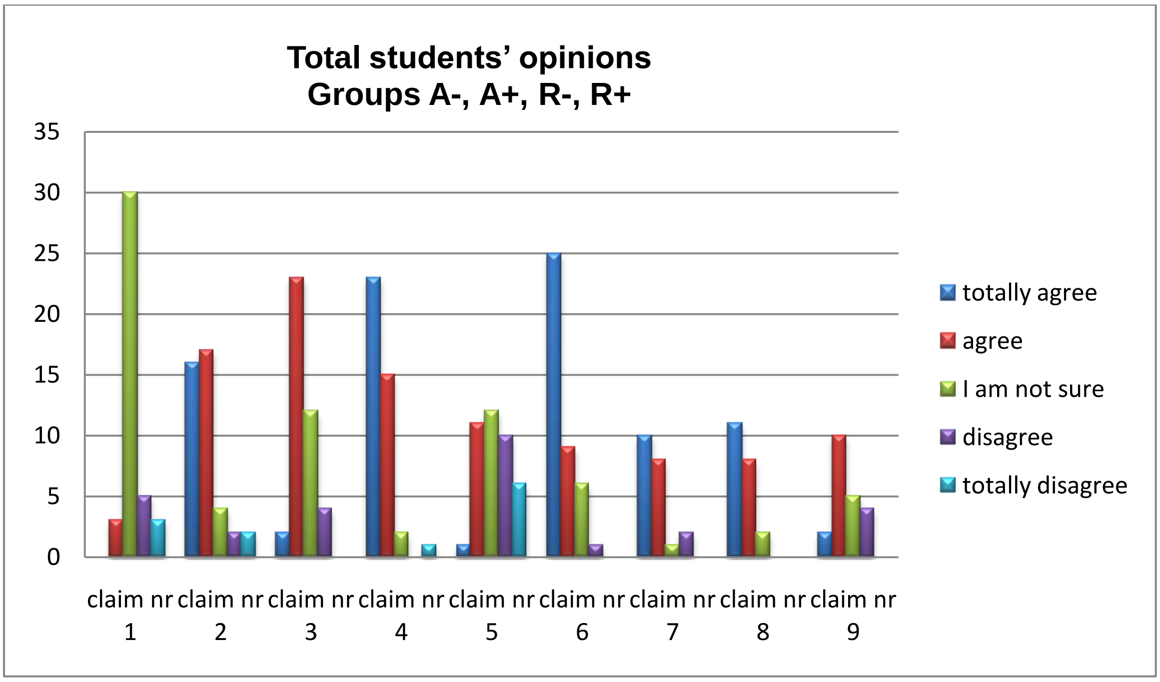
Appendix 9: Students' opinions, Group A+



Appendix 10: Students' opinions, Group R+

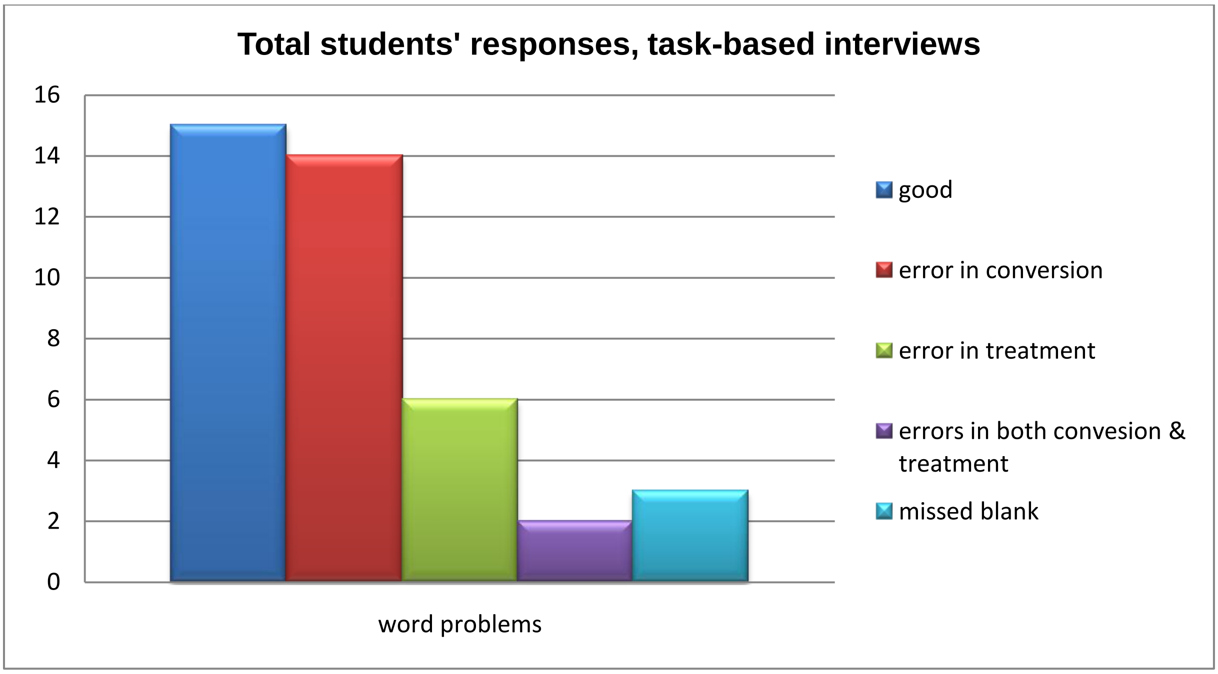


Appendix 11: Total students' opinions, Group A-, A+, R-, R+

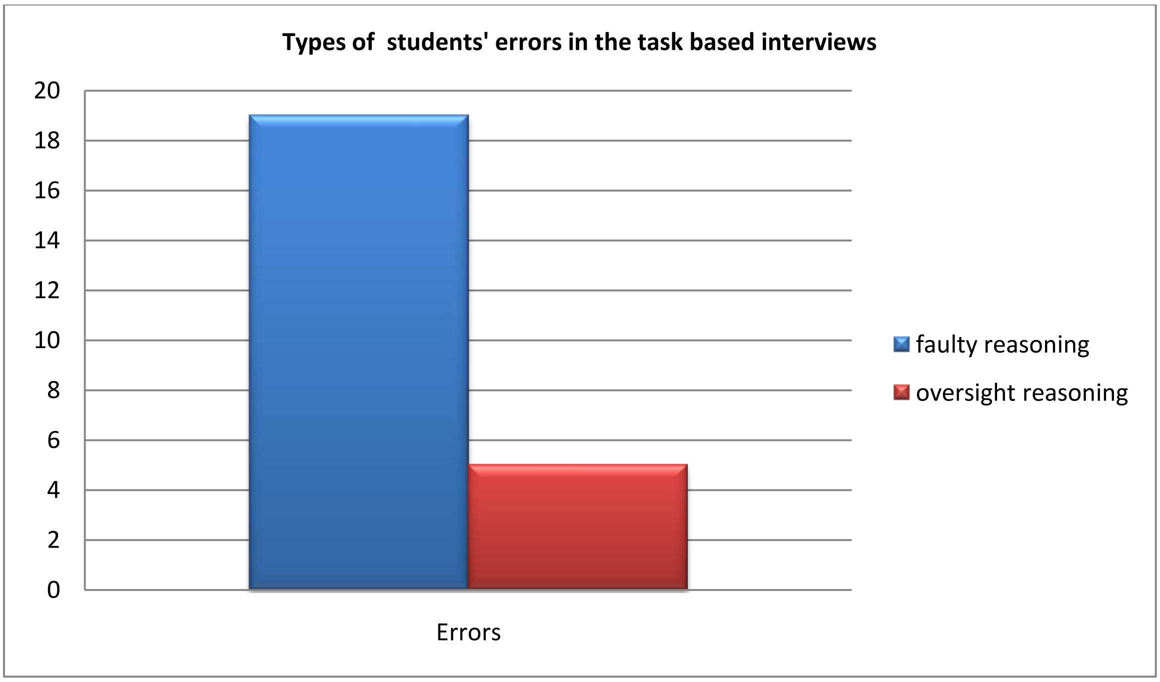


Task-Based Interviews Appendices

Appendix 12: Total students' responses, task- based interviews



Appendix 13: Types of students' errors in the task- based interviews



X^2 test Appendices

Appendix 14: X^2 test, Pencil and paper test results

Total Responses

	Correct responses	Incorrect responses	
Groups A- and A+	20 $\boxed{18.4}$	85 $\boxed{86.6}$	105
Groups R- and R+	16 $\boxed{17.6}$	84 $\boxed{82.4}$	100
	36	169	205

$X^2 = 0.139 + 0.018 + 0.145 + 0.031 = 0.333$; df = 1; $\alpha = 0.05$

Total Responses

	Correct responses	Incorrect responses	
Groups A-	8 $\boxed{9.5}$	42 $\boxed{40.5}$	50
Groups A+	12 $\boxed{10.5}$	43 $\boxed{44.5}$	55
	20	85	105

$X^2 = 0.237 + 0.055 + 0.214 + 0.051 = 0.557$; df = 1; $\alpha = 0.05$

Total Responses

	Correct responses	Incorrect responses	
Groups A-	8 $\boxed{7.5}$	42 $\boxed{42.5}$	50
Groups R-	7 $\boxed{7.5}$	43 $\boxed{42.5}$	50
	15	85	100

X^2 = 0.033+ 0.006 + 0.033 + 0.006 = 0.078; df = 1; α = 0.05

Asteroid Problem

	Correct responses	Incorrect responses	
Groups R-	1 $\boxed{2.6}$	9 $\boxed{7.4}$	10
Groups R+	4 $\boxed{2.4}$	5 $\boxed{6.6}$	9
	5	14	19

X^2 = 0.985+ 0.346 + 1.067 + 0.388 = 2.389; df = 1; α = 0.05

Asteroid Problem VS. % Difference Problem

	Correct responses	Incorrect responses	
Groups A-	0 $\boxed{2.6}$	10 $\boxed{7.4}$	10
Groups R-	1 $\boxed{2.4}$	9 $\boxed{6.6}$	10
	1	19	20

X^2 = 0.5+ 0.026 + 0.5 + 0.026 = 1.052; df = 1; α = 0.05

Appendix 15: X^2 test, Questionnaire results

Claim nr. 2: I liked to work on this test

	Totally Agree	Agree	
Groups A- and A+	12 $\boxed{7.8}$	4 $\boxed{8.2}$	16
Groups R- and R+	4 $\boxed{8.2}$	13 $\boxed{8.8}$	17
	16	17	33

X^2 = 2.261 + 2.129 + 2.129 + 2.004 =8.523; df = 1; α = 0.05

Claim nr. 3: The problems are similar to the problems we solve in the classroom

	Totally Agree + Agree	Not Sure	Disagree	
Groups A- and A+	17 $\boxed{12.8}$	4 $\boxed{6.2}$	0 $\boxed{2.1}$	21
Groups R- and R+	8 $\boxed{12.2}$	8 $\boxed{5.8}$	4 $\boxed{2}$	20
	25	12	4	41

X^2 = 1.378 + 0.781 + 2.1 + 1.446 + 0.835 + 2 =8.54; df = 2; α = 0.05