UNIVERSITY OF AGDER

Using Computer Technology in Teaching and Learning Mathematics in an Albanian Upper Secondary School

The Implementation of SimReal in Trigonometry Lessons

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Preface

It has been challenging and exciting at the same time to conduct research that deals with an important topic in relation to education in Albania, namely the integration of technology in teaching. Recently, the Ministry of Education embarked on implementing projects which aim to equip every school with computers and internet connection. Also, there are plans intended to meet the necessary conditions for the integration of technology in the teaching and learning of different subjects, as the reformulation of the content of curriculum, based on ICT use, and the training of the teachers to use ICT during the teaching proccess.

From my experience as a student in mathematics and later as a teacher (for a short period) in this field, I believe that technology can be a supportive tool in teaching and learning mathematics. It was interesting to go through the steps of conducting this research, learning the pedagogical reasoning of using computer technology in education and particularly observing how students react to and interact with the program during the lessons.

I hope this study will be a good start for the implementation of computer programs in teaching practices and it will inspire mathematics teachers to think about using these tools in their teaching.

I would like to acknowledge and extend my heartfelt gratitude to the following persons who have made the completion of this research a reality.

Considering that it was my first time of doing a research, it would not be possible without the support of my two supervisors, professor Maria Luiza Cestari and professor Per Henrik Hogstad. Maria Luiza has been my didactic and theoretical guide. She has helped me to develop ideas, to find the relevant theory and provided guidance during the writing process of this research. Per Henrik has helped me with his expertise as a programmer and his experience to develop the program for use as aids in mathematics.

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Special thanks go to my friends and colleagues, especially Suela Kacerja, for her feedback and valuable advice, and Bebwa Isingoma, for his willingness to help me to correct the grammar and to improve my study.

I would also like to thank the teachers in Albania, who supported me in carrying out this research in their school, especially the headmaster, Professor Violeta Volumi, who kindly received me and appreciated my initiative and work.

Finally, I would like to thank my family for their moral support and their encouragement.

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Summary

This study presents an investigation on the use of computer technology in mathematics teaching and learning, focusing on the students' mathematical understanding, their attitudes and opinions in relation to such experience. It provides empirical evidence for the use of the program called SimReal in supporting exploratory and learning related activities of one important part of mathematics, trigonometry.

The participants in this study were students from the second grade of an Albanian private upper secondary school (16 – 17 years old students). This is a new experience in Albanian education. The use of technology and computer programs in the teaching process has recently been recommended and it is one of the standards of the mathematics curriculum in secondary school level (Institute of Education Development, 2010). But for many reasons, such as financial constraints or beliefs related to the effects of technology in the educational process, no steps have been taken to put these standards into practice.

Therefore, the results of this study are important for creating an idea on how the teaching and learning of mathematics can benefit from the use of computer programs, and it can be an inspiration for many teachers to think about the implementation of such tools in teaching mathematics.

An important conclusion of this study is that SimReal can promote better results in students' understanding of trigonometry, according to the comparison of performances in a mathematical test of the experimental and control group. The program can also help them to explore important mathematical features related to trigonometric functions, by interacting with the tool and making dynamic links between the numerical and visual representations. Students reacted positively to the use of the program during mathematics lessons, by expressing appreciation for this way of teaching and learning mathematics.

However, despite these good results, the experimental lessons and data analysis reveal some important issues and limitations related to the implementation of the program. These should be taken into consideration. This case study has two important implications:

- The first one is related to the demostration of the role of technology, specifically of SimReal, in teaching and learning mathematics, so it can inspire mathematics teachers to consider this new practice in their teaching process.

- The second implication is related to some limitations in the introduction of computer programs, which should be taken into consideration before planning to implement such tools in mathematics teaching and learning.

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1 Introduction

This thesis reports results from an experimental study on the use of a computer program in teaching trigonometry in an Albanian upper secondary school. As indicated in the title of the thesis, it has two main directions: the implementation of SimReal in trigonometry lessons and the investigation of the role of of using this computer program in upper secondary school students' learning of mathematics.

1.1 Implementation of SimReal in mathematics lessons

The often individualistic nature of mathematics lessons seems extremely unusual, causing some students to view mathematics classes as 'other-worldly', with no relationship to their own lives and perhaps no connection to other academic areas (Boaler, 2000).

We want to start the introduction of this study with an important pedagogical statement made by Shulman (1987), who emphasised that transforming one's own understanding into forms that help students requires more than knowledge of the subject matter. It is very important how this knowledge is presented and transmited to the students.

Mathematics is closely related to visualisation, and better learning can be achieved by using different representations of mathematical objects and procedures, in order to foster students' understanding of the subject. Even better is when students can interact with these visualisations and can explore on their own new features of the mathematical content.

In relation to the learning of mathematics, The National Council of Teachers of Mathematics (2008, 2011) emphasized technology as an essential tool and important components of a high-quality mathematics education, which can provide access to mathematics for all students. These tools support both the visualisation and the interactive part, helping students to extend mathematical reasoning and sense making by using them for computation, construction, and representation as they explore problems.

From here emerges the theme of this study, which aims to investigate the role of the computer program, SimReal, in teaching and learning mathematics. In relation to the implementation part, it is important to give an overview of the place of technology in Albanian education and in mathematics curriculum in upper secondary school.

Recently, the Ministry of Education started implementing projects which aim to equip every school with computers and internet connection. Also, there are plans intended to meet the necessary conditions for the integration of technology in the teaching and learning of different subjects, as the reformulation of the curriculum content based on ICT use, and the training of the teachers to use ICT during the teaching proccess.

Given these attempts in the Albanian education system to undertake important steps which can influence the quality and the results of teaching, I was motivated to carry out research on the use of computer technology in teaching mathematics.

Also, another motivation emerged from my experience as a student of mathematics. I studied mathematics in Albania for three years for my Bachelor's degree, at the University "Luigj Gurakuqi", in Shkoder. Thereafter, I underwent pedagogical training for one year in teacher education. As part of the pedagogical training, I was involved in teaching practice for 5 months as a mathematics teacher in a secondary school. After that, I had an opportunity to study in Norway for a Master's degree in mathematics education. It was an essential experience for me and my professional background, as I moved from the scientific aspect of mathematics to the pedagogical aspect of it, which is very important for a teacher. Althought I had already realised the difficulties that students have in general with mathematics, it was important for me to analyse them from a didactical and pedagogical point of view.

1.2 The investigation of the role of SimReal used in mathematics lessons

The analysis of the role of technology is examined on the basis of the experience developed in the implementation and evaluation of computer-based visualization for teaching and learning trigonometry. It is a theory-based analysis of the empirical material gathered from this experimental work, which involved the use of SimReal in teaching trigonometry in a second grade class in an upper secondary school in Albania.

The theoretical frame of the research was constructed based on the relation of three components: the subject of mathematics, the use of technology and the cognitive theory of learning. This relation consists in the use of technology during mathematics lessons in order to foster students' understanding from a cognitive point of view. Technology and cognition are related by the principle of cognitive technologies, described by Pea (1987), who suggests that technology should be used as cognitive tools for learning purposes. Mathematics and cognition are related by the construcivist theory of learning, which emphasizes the student-centered model of learning. Finally, the most important relation is the one of mathematics with technology, which, when used during the teaching and learning process, helps students to visualize better the mathematical content, to link different representations of mathematical objects and procedures and promote conceptual understanding through concretisation and real-life context.

This analysis is also based on previous studies which investigated the role of technology in mathematics education from different perspectives. There are many studies which show that it is possible to achieve better learning results using technology in the teaching and learning process. The current study fulfils these results by investigating both quantitatively and qualitatively how students interact with the program and how they use it to learn mathematics. The focus is not on how much knowledge they will acquire. Rather, it is on whether the program will help the students in their understanding of the mathematical content.

An experimental teaching activity was designed and implemented with two groups: the control group, having traditional teaching from the actual mathematics teacher and the experimental group which had mathematics lessons with SimReal. The latter group was taught by the author of this study. During the lessons with the experimental group, trigonometric functions and their variations in different representations were introduced. Before and after the experimental work, the students participated in a pre- and post-mathematical test. Three questionnaires about students attitudes were completed and in the end five students were interviewed. Two questionnaires were completed by both groups before starting the experimental lessons. They consisted of 23 items: 11 items about mathematics self-confidence and motivation to learn this subject and 12 items about computer self-confidence and motivation to use it.

The third questionnaire was completed only by the experimental group, in the end of the experimental lessons, and consisted of 11 items about attitudes toward the use of computers in learning mathematics. The interview had 6 focus questions about the students' opinions in relation to the efficacy and usability of SimReal.

It was found out that the program used during the mathematics lesson had positive effects on the students' performance and also their attitutes were positive, and many of them expressed the desire to continue doing mathematics in this new way. It was aslo found out that are many factors that affect the process of integration of computer programs in classroom, which should be taken into consideration, especially the way of how technology is used during the lessons.

1.3 Research questions

Taking into consideration the description of the theme of this research and its goal (presented in 1.1 and 1.2), we need to state the aims and research questions which were set to guide the study. The main aim is to investigate the role of the computer program, SimReal, in teaching and learning trigonometry in high school. Hence, the research questions are:

Research question 1- What is the role of the program in the learning and understanding of trigonometry?

We aim to find out illustrative information about students using the program interface to construct their knowledge related to mathematical content. This information was obtained from students' solutions of tasks in mathematical tests and class work. Also, there was a quantitative analysis of scores in tests in relation to this research question, comparing the group which had experimental lessons with SimReal with the one which had traditional lessons.

Research question 2 - What are the students' attitutes toward using the program in the classroom?

As the students in the experimental lessons had never experienced this kind of teaching and learning, they were asked to give their opinions and feedback about this new way of teaching and learning mathematics. We answer this research question through quantitave and qualitative analysis of data gathered by means of questionnaires about attitudes, as well as interviews with students.

Research question 3 - What are the potential issues/limitations in relation to the implementation of the program, as a complete new practice in Albanian schools?

After conducting the experimental lessons with SimReal, some limitations in relation to the use of the program during the lessons emerged. These issues are discussed in relation to previous results from other studies.

It is important to add some comment with respect to the third research question. At the beginning of the process of writing, the plan was to have only two research questions: the first and the second one. But after doing the experimental work, during the data collection, we realized that the integration of technology in classroom is associated with difficulties and some very important issues, which should be taken into consideration. And we found it relevant to add a third research question related to these issues, having emerged directly from the experimental lessons of teaching and learning trigonometry with SimReal.

1.4 Structure of the study

The current study is structured in seven chapters: introduction, literature review and theoretical framework, context of the study, methodology, findings, discussion and conclusion and pedagogical implications.

In chapter one, we present the topic of this research, the motivation for conducting it, the research questions and the how the study is structured.

In the second chapter, we give an overview of previous studies related to the integration of computer technology in mathematics and the role it plays in the learning and teaching process of this subject. A framework for the research study has been formulated and presented.

In chapter three, we describe the computer program used and the rationale why it was chosen to conduct this study. We give an overview of the system of education in Albania and the place of technology in the curriculum of mathematics. Then we make a brief description of the school where the experimental work is done and the preparations related to that. Also we present the mathematical context under consideration and the objectives of the subject.

Chapter four introduces how we obtained the neccesary information so as to answer the research questions. It also presents how the data is analysed in the study and how this process brings us to the required conclusions.

In chapter five, we present the results of the study and their interpretation. The analysis in this chapter leads to the discussion and the conclusions of the study in the next chapter.

Chapter six is devoted to the discussion of findings and conclusions. In this chapter, we summarize the results, provide the general conclusions drawn from this study, relating them with conclusions from previous studies. We also offer some suggestions for future studies.

Chapter seven deals with pedagogical implications. It gives reflections about the conclusions of the study and what implications these conclusions have for the researcher and mathematics teachers who plan to use technology in their classes.

2 Review of related literature and theoretical framework

Computer technology and internet are dramatically changing how people communicate, work and play, as well as how they teach and learn. This chapter gives an overview of research on technology development in education, specifically in mathematics education. It also presents the theoretical framework of the study. It is structured in six main sections: 1) technology in education, 2) technology in mathematics education, 3) technology integration in trigonometry, 4) theoretical framework, 5) students' attitudes toward the use of technology in education, and 6) difficult issues and limitations related to technology integration.

Section one begins by showing the potential of technology in daily life and its role in education. It takes into account some pedagogical concerns and focuses on areas that can be improved with the help of technology, specifically with multimedia tools.

The first part of section two focuses on the importance of mathematics and the difficulties that students encouter in learning this subject. Then an overview of the literature related to the integration of technology into mathematics classroom is presented, showing some results and conclusions from previous studies. In addition, we present a historical evolution of technologies used in mathematics education and some of the theoretical frameworks that were considered to be relevant to the issue of technology integration.

Section three deals with the issue of technology integration into the teaching and learning of trigonometry. It pays attention to some problems and difficulties related to this area, followed by an overview of studies which investigate the role of technology in teaching and learning trigonometry.

The theoretical framework is based on the triangulation of three components: mathematics, cognition and technology, and how they can be related to each other in order to promote meaningful learning. This framework aims to describe first the integration of technology into mathematics teaching and learning, based not only on Duval's (1999) theory about semiotic representations, but also on the functionalities of computer programs used as cognitive tools, as suggested by Pea (1987). In addition, the framework aims at describing how the investigation of the role of technology in education can be done, based on the constructivist theory of learning.

Students' attitudes toward the use of technology in doing mathematics represent some results from previous studies which analysed this important aspect, and also how it is related to mathematical and computer attitudes of students.

Finally some limitations related to the process of integrating technology into education are described.

2.1 Technology in education

2.1.1 Potential of technology in daily life

Over the last few decades technology has become a very important tool in everyday life. Computers have become a common tool for communication, text processing, and many other activities, including different forms of media, audio, graphics, videos, and virtual reality. The development of internet and the increase in accessibility have opened a whole new digital world. Children are not only exposed to new information and computer technology (ICT) at school, but also at home. Many children today have computers at home and have access to internet. They use computers and technology everyday for entertainment, communication and education.

Computer literacy is also an essential skill in occupational activities, since technology is widely used in business, economics and many other professions.

2.1.2 The role of technology in education

Many educational institutions have taken into consideration the potential of technology, developing standards related to this new practice in education (Lawless & Pellegrino, 2007), and trying to integrate it into teaching and learning. We take a look at the role of technology in education, considering some pedagogical issues and how the technology integration can affect them.

Pedagogical concerns

Education is no longer about memorizing facts and pictures, but rather, it is about learning where to find this information, and more importantly, it is about how and where the information which has been acquired can be used. Learners must actively construct their own understandings rather than simply absorb what others tell them(Bransford et. al, 2000).

"The traditional learning processes are based on transferring the knowledge directly from the teacher to the students, applying a pedagogy where the learners are passive receivers of knowledge" (Faugli, 2003, p.9). In this case, the content presented in the classroom is disconnected from its real-world context, and this has been shown to have a negative impact on the learning process, affecting in particular learner's motivation (Henning, 1998).

At the same time, real-world learning situated in real-world contexts has been shown to have a positive impact on learning and learners' motivation (Duffy & Cunningham, 1996). Students' motivation is one of the essential keys for a productive learning experience. When students are motivated to learn, they can pay more attention in what they are learning and they can remember better the material.

Another important emphasis advanced by educators on the learning process is to promote meaningful learning for students, helping them to construct their knowledge so as to be able to develop and to apply it appropriately in a range of situations. According to TIME (Technology Integrated in Meaningful Learning Experiences), (in Ashburn & Floden, 2006, p. 8) a meaningful learning experience should create opportunities to achieve deep understanding of complex ideas and enable students to work with complex problems and content that are central to the discipline and relevant to their lives.

In the next section, we present how technology integration can help in promoting productive learning, in keeping with the goals of education, namely real-life context and meaningful learning to help students to construct their knowledge and to be more motivated.

The integration of technology in education

The use of technology in education can enhance meaningful learning better than the traditional classroom instructions. “They can engage a wider range of intelligence, connecting school with real world, supporting interaction, offering dynamic displays, multiple and linked representations, interactive models and simulations and the storage and retrieval of multiply categorized information” (Ashburn & Floden, 2006, p.28). In this way, by integrating technology into the teaching and learning process, educators aim to increase students’ abilities to understand complex ideas and learn challenging content.

There are many studies that have explored the integration and the use of technology in education from a variety of perspectives, emphasizing the advantages of these practices. As noted by Churchill (2005), “technology amplifies our intellectual and physical capacity” (p. 347), and in this context, technology can play an integral role in supporting higher order learning.

Ashburn and Floden (op. cit.), in their book about meaningful learning using technology, point out that technology can be used to demonstrate and to scaffold the development of mental models.“Tools that instantly relate the graphical and symbolic representations and make abstract concepts visible and manipulable can help students comprehend the nature and applications of key ideas” (p.30).

In the next section, we establish a correlation between multimedia instructions tools and Paivio’s (2006) dual coding theory, pointing out the role of these tools from a cognitive point of view.

2.1.3 Using technology for multimedia instructions

One way of integrating technology into education is using multimedia instructions to present and communicate the learning material. In his book about the multimedia learning, Mayer (2001) defines multimedia instructions as presentations involving words and pictures that are intented to foster learning. In this way, information can be processed through both the verbal and nonverbal channels, so the learner has more cognitive paths that can be followed to retrieve the information (Mayer & Anderson, 1991).

This can be related to the dual coding theory developed by Paivio (2006), which emphasizes the concretization of knowledge through imagery and pictures. Specifically, in mathematics the concretization of abstract symbols and relations is important (Skemp in Paivio, 2006, p. 13). From the description made by Paivio (2006), cognition, according to the dual coding theory, involves two distinct subsystems: a verbal system specialized in dealing directly with language and a nonverbal (imagery) system specialized in dealing with nonlinguistic objects and events. The representations are connected to sensory input and response output systems as well as to each other so that they can function independently or cooperatively to mediate nonverbal and verbal behavior. The theory means that both systems are generally involved even in language phenomena. Cognition is the variable pattern of the interplay of the two systems according to the degree to which they have developed.

An important aspect of multimedia instructions is the multimedia principle. In relation to this principle, Mayer (op. cit.) states that “people learn better from words and pictures than from words alone. When words and pictures are both presented, learners have the opportunity to construct verbal and visual mental models and to build connections between them" (p. 280). This means that multimedia learning offers a powerful way for people to understand things which would be difficult to figure out by words alone.

In the following chapter we provide an overview of the role of technology in mathematics education, focusing on the importance of visualizations in the concretization of the mathematical content.

2.2 Technology in mathematics education

One of the major goals in mathematics education is to ensure the success of all students in understanding the subject matter. Mathematics is considered as one of the most challenging and problematic subjects in the educational aspect. But at the same time it is one of the most important areas of science, given that mathematical skills and knowledge are important in everyday life, and there are also many mathematical applications in other subjects and sciences. Christy (1993) states that "mathematics is a basic tool in analyzing concepts in every field of human endeavor" (p. 3).

For these reasons, mathematics is a subject which should be taken seriously. Teachers should focus on fostering the students' understanding of mathematical concepts and they should provide a quality education environment for them. Many students find it difficult to engage with mathematical concepts. For learning to take place, students need to be actively engaged with the explored concepts or objects – whether abstract or concrete (Liang & Sedig, 2010).

To come closer to these difficulties faced by students, an overview of mathematical activities and the process of doing mathematics is offered here. According to Duval (1999), the mathematical activity has two sides: the visible side which are mathematical objects and the cognitive operations, procedures. Mathematical objects are abstract and not amenable to any concrete imagination or manipulation; they are immaterial, not tangible and directly accessible to our thinking like the physical objects (Chiappini & Bottino, 1999).

The cognitive operations are also a difficult part of mathematical activities for students, given that very often teachers attach more importance to the mathematical processes than to their applications to daily life situations or to physical problems. This leads students to solve problems mechanically, by following the algorithm steps without real awareness of their actual meaning (Milovanovic,Takaci & Milajic, 2010).

Given these challenges in doing mathematics, it is teachers' and educators' responsibility to make the learning and the understanding of mathematics easier for students. A very important key for the understanding of mathematics is the use of visualization and representations in the learning and teaching process. As pointed by Duval (1999), "there is no understanding of mathematics without visualization" (p. 13). Vizualisations are intended to be concrete means which allow students to explore more difficult mathematical concepts. Representations and symbols of mathematics establish a semiotic system which is of fundamental importance for any mathematical activity (Chiappini & Bottino, 1999).

In the next section, we review the relevant literature with respect to the integration of technology into mathematics.

2.2.1 Integrating technology into mathematics

Technology is useful for helping students to look at mathematics not only as a set of procedures, but more as reasoning, exploring, solving problems, generating new information and asking new questions. Furthermore, "it helps them to better visualize certain mathematical concepts" (Van Voorst, 1999, p.2). Studies have revealed that activities encouraging the construction of images can greatly enhance mathematics learning (Wheatley & Brown, 1994).

Greeno and Hall (1997) make several observations about the importance of representations, concluding that:

- computer technologies are powerful tools for thinking
- understanding of mathematical concepts and procedures is enhanced when students can transfer understanding among different representations
- they can give learners useful tools for building understanding, communicating information and demostrating reasoning.

Ashburn and Floden (2006) also emphasise the importance of using technology in mathematics, noting that tools that instantly relate the graphical and symbolic representations of mathematical expressions can help make understanding goals more accessible to students. "Simulations that make abstract concepts visible and manipulable can help students comprehend the nature and applications of key ideas" (p.30).

There are many studies which investigated the integration of technology into mathematics teaching and learning of different topic areas (Liang & Sedig, 2010; Milovanovic et al. 2011; Lagrange, 2010; Perjesi, 2003; Lotfi & Mafi, 2012, Gonzales & Herbst, 2009) concluding that technology can help to visualize and represent better mathematics objects and procedures by exploring different graphical representations.

We found it relevant to take a look at previous studies and theoretical frameworks used in studies about the integration of technology into mathematics education.

2.2.2 The evolution of technology and its use in mathematics education

We present this evolution of technology refering to the study of Drijvers et al., 2010. Since the 1960s mathematicians and mathematics educators began to believe that computing could have significant effects on the content and emphases of school- and university-level mathematics. In the past several decades, there have been dramatic changes both in technology development and in the way it is used in mathematics education in an attempt to enhance the teaching and learning process.

Let us take a look at the evolution of technology used in mathematics education during the years 1960s – 1990s (p. 91)

1. Among the earliest applications of technology to mathematical learning in school was CAI – the design of individualized student – paced modules that were said to promote a more active form of students learning (PLATO project).

2. The next texhnology-based approach to mathematics learning involved programming (in Logo and BASIC), taking into consideration the fact that children learn better if we put them to do mathematics rathen than merely learn about it (Papert, 1972).

3. During the 1970s there emerged the development of more specialized pieces of software, some of which were specifically created for mathematics learning (Cabri), and others adopted for use in mathematics classrooms (spreadsheets).

4. Lately, we have seen the developent of microcomputers and graphing calculators which consist in using multiple representations of mathematical objects.

Theories on the role of technology in the teaching and learning of mathematics during the 1980s

As examples of the first steps in theorizing on the use of technology in mathematics education, we describe briefly the Tutor-Tool-Tutee notions, the White Box–Black Box idea, the notion of Microworlds and Constructionism, and the Amplifier–Reorganizer duality.

The theoretical beginnings focused on specific issues related to integrating technology into education. The notions were related to specific types of software and not to more general theories on learning.

Tutor, Tool, Tutee . With the arrival of the microcomputer and its increasing proliferation, a new framework was developed, which classified educational computing activity according to three modes or roles of the computer: tutor, tool, and tutee (Taylor, 1980). To function as a tutor, "the computer presents some subject material, the student responds, the computer evaluates the response and from the evaluation, determines what to present next" (p. 3). To function as a tool, according to Taylor, is to have the possibility to use the computer in a variety of ways. The tutee mode is to tutor the computer, by using the programming language to talk to it.

White Box – Black Box. A theoretical idea that focused on the interaction between the knowledge of the learner and the characteristics of the technological tool was the White Box/Black Box (WBBB) notion put forward by Buchberger (1990). According to Buchberger, the technology is being used as a white box when students are aware of the mathematics they are asking the technology to carry out; otherwise the technology is being used as a black box.

Microworlds and Constructionism. This frame was based on the theory developed by Papert and Harel (1991) in relation to the notion of constructionism: "learning-by-making" (tutee mode). They provided examples of microworlds, such as turtle geometry and defined them as worlds where ideas can be developed by exploring their properties.

Amplifier – Reorganizer. Pea (1987) re-elaborated the psychological notion of cognitive tools for the case of technology in education. Computers have the potential for both amplifying and reorganizing mathematical thinking. However, Pea argued that the one-way amplification perspective, whereby tools allow the user to be more efficient and to increase the speed of learning, misses the more profound two-way reorganizational possibilities afforded by the technology.

Current developments of theoretical frameworks

This section provides an overview of some current theoretical approaches in relation to studies on mathematical teaching/learning and technology.

1) Theoretical approaches adapted from existing theories in mathematics education:

- *Situated abstraction* describes how learners construct mathematical ideas reorganizing previously constructed mathematics in a new mathematics structure (Noss & Hoyles, 1996).
- *Theory of Didactical Situations* (Brousseau, 1998): within this framework the learning outcomes results from the use of an instrument at the practical level.
- *Perceptuo-Motor Activity* (Nemirovsky, 2003)

2) *Instrumental approach* is one of the most dominant frameworks while considering the role of technology in the teaching and learning of mathematics, especially for the understanding of student-CAS interactions and their influence on teaching and learning. The theoretical foundations of this framework are both the cognitive theory (Verillon & Rabardel, 1995) and the anthropological theory of didactics (Chevallard, 1999).

3) *Semiotic mediation* is based on the semiotic approach, which is focused on the role of signs and symbols and their use or interpretation (Saenz-Ludlow & Presmeg, 2006). Through actions and tasks accomplished with artifacts, mathematical meanings are presented through different kinds of representatives – words, gestures, drawings – (Radford, 2003), disclosing the semiotic potential of the artifact.

In our research about the role of the computer program SimReal in teaching and learning mathematics, we decided not to use any of these theoretical approaches directly. However, we constructed a theoretical framework inspired by these frameworks and based on functionalities of the program used as a cognitive tool (Pea, 1987), related to the construction of mathematical meaning through different representations (Duval, 1999).

Before introducing the theoretical framework, we need to make a review of related literature about the use of technology in trigonometry.

2.3 The use of technology in teaching and learning trigonometry

Research on the teaching and learning of trigonometry, with or without technological aids, lags behind research conducted in other domains of mathematics education (Ross, Bruce & Sibbald, 2011). In addition, Davis (2005) notes that little attention has been given to trigonometry and the various ways it has been representated in classroom teaching.

It is unfortunate that this topic area has been neglected, yet it is an important course in high school curriculum, and knowledge of trigonometry is crucial to success in many college programs. Understanding trigonometric functions is a pre-requisite for understanding topics in Newtonian physics, architecture and many branches of engineering (Weber, 2005). In the United States, the standards of the National Council of Teachers of Mathematics (NCTM, 2000) highlight the importance of trigonometry in the study of functions, particularly periodic functions and emphasize trigonometry's utility in investigating real-world phenomena.

2.3.1 Difficulties in the learning of trigonometry

Trigonometry has been described as the hardest part of high school mathematics curriculum, and students find it very challenging and difficult (Takaci, Herceg & Stojkovic, 2005). There are two main reasons related to these difficulties:

First, it is a topic area in which students meet trigonometric functions, not as direct numerical manipulations of the functions' argument as they used to do before, but by definition, as the ratio of the lengths of sides in a right angled triangle (Pritchard & Simpson, 1999). The mathematical difficulties are related to the requirement of the ability to move flexibly between abstract, visual and concrete representations of mathematical objects (Ross et al., 2011).

The second difficulty is related to the way this topic area is explained. Trigonometry often is taught as a completely mechanical series of routines, without engaging students in any non-routine mathematical thinking. Also, using rough sketches of triangles may give the impression that the numerical procedures are the only way to get accurate results causing a possible schism between the use of pictures and numerical procedures (Blackett & Tall, 1991) .

Next, we take a look at what previous studies suggest in relation to the use of computer technology and simulations to reduce the difficulties encountered by students in learning trigonometry.

2.3.2 Aims of teaching trigonometry with the computer

Regarding the understanding of trigonometric functions, Park (1994) points out the role of simulations, which can be used to highlight how a numerical output is linked to certain unknown symbolic representations through a graphical approach. This is because motion can bring to the students' attention the critical features and their relation to other components that might not be easily grasped in an abstract system. Simulations can also illustrate procedural relationships. For example, while transforming trigonometric curves that involve four transformations, students can see the sequential steps to achieving the end result.

Jonassen (2000) also suggests that students will understand trigonometric functions better and more conceptually if they are able to inter-relate numerical and symbolic representations with their graphical output. With respect to the way trigonometry is taught, Blacket and Tall (1991) point out the advantages of the computer approach comparing to the traditional approach, stating that it can allow students to manipulate the picture and relate its dynamically changing state to the corresponding numerical concepts, having the potential to improve understanding. They call this ability to use the computer to carry out certain arduous constructions whilst the student can focus on specific relationships 'the principle of selective construction', considering it as one of the most powerful educational principles for the use of new technology.

In relation to these aims of using computer programmes in teaching and learning trigonometry, we can express the advantages of such practices according to the categorisation made by Wilson (2008):

- Promoting drill and practice - facilitating routine computations
- Enabling the exploration of the variation of parameters - linking representations of changes and transformations
- Fostering conceptual understanding

2.3.3 Previuos studies about the integration of technology into teaching and learning trigonometry

Many studies which incorporated technology into the teaching of trigonometry have demonstrated largely positive effects on student achievement (Blackett & Tall, 1991; Wilson, 2008; Choi-Koh, 2003; Ng & Hu, 2006; Steckroth, 2007).

Blackett and Tall (1991) employed a computer program that draws the desired right triangles to facilitate students' exploration of the relationship between numerical and geometric data. The results of the study show that computer representation enabled students to make this exploration in an interactive manner. They were encouraged to make dynamic links between visual and numerical data, which is less apparent in a traditional approach. The authors conclude that even the least able became adept at using the computer and, though they had some difficulty writing down their results, they had few difficulties with visualization.

Wilson (2008) studied the role of dynamic web tools in trigonometry lessons, and he concluded that there was improvement both in the quality of students work and in their interest toward the subject. He points out that these tools provide excellent vehicles to monitor drill and practice and to foster conceptual understanding in many situations.

Choi- Koh (2003) investigated the patterns of one student's mathematical thinking processes and described the nature of the learning experience that the student encountered in trigonometry as he engaged in independent explorations within an interactive technology environment. He concluded that representations offer students an opportunity to explore and conjecture mathematics which fosters a balance between procedural and conceptual knowledge. Students can effectively use a graphing calculator as an instructional tool to help them understand the connection between graphical and algebraic concepts, and not use it just to get quick answers. He also emphasized the role of technology in enhancing students' attitudes to mathematics learning.

Dynamic features enables the software to illustrate mathematical changes that might not be otherwise visible and helps students visualize a dynamic model containing trigonometric relationships that are difficult to depict with static images (Ng & Hu, 2006).

Steckroth (2007) found that software that included animation and visualization produced greater learning than software limited to graphing functions.

Lesser and Tchoshanov (2005) presented evidence that students need to be taught abstract, visual and concrete representations to develop 'function sense' (the ability to integrate and flexibly apply multiple representations of functions). They found that the optimal sequence for introducing representations in trigonometry is to present the abstract first; the visual and concrete became meaningful only after the abstract had been learned.

The sequence of teaching trigonometry in technological environments is also studied by Ross et. al (2011), who concluded that the better learning is promoted when technology is used after the teacher explains the content.

2.4 Theoretical framework

The theoretical framework constructed for this research study is divided into two main components: the integration of the computer program in mathematics teaching and learning, and the investigation of the role of such practices in students' achievement. The first component is related to the use of multiple representation in mathematics teaching and learning (Duval, 1999) and the use of the program as a cognitive tool (Pea, 1987). The second component is based on the three types of understanding of mathematics (Skemp, 1979) and the constructivist theory of learning.

2.4.1 Multiple representations

The technology approach involves actions and perceptions, and it produces learning based on doing, touching, moving and seeing (Chiappini & Bottino, 1999). As argued by Mayer (2001), using multimedia instruction to communicate information involves more than one presentation mode, given the way the material is represented: with words or pictures.

In this study we rely upon the notion of registers of representations from Duval (1999), who stresses that a mathematical object is generally perceived and treated in several registers. There are no other ways of gaining access to mathematical objects, but to produce some semiotic representations, which are named by Duval as registers of representations.

He distinguishes two types of transformations of semiotic representations: treatments and conversions. A treatment is an internal transformation inside a register. A conversion is a transformation of representation that consists of changing a register of representation, without changing the objects being denoted. It is important that students recognize the same mathematical objects in different registers and they should be able to perform both treatments and conversions.

We bring here a conrete example from students' activities in learning trigonometry. Students have to switch from algebraic settings, where trigonometric functions can be treated symbolically by their expressions and numerical tables, to the geometric register, which are the graphical representations for these functions. As explained by Lagrange and Chiappini (2007), working in the geometric setting, students would understand the problem and the objects involved, and after switching to algebra, this understanding would help them to make sense of the objects and treatments in the algebraic setting. Mastering these expressions as well as treatments, and flexibly changing register, are important for students' ability to handle functions and acquire knowledge about this notion.

Research suggests that using multiple representations, in both teaching and learning, supports the development of mathematical understanding (Byers, 2010). In fact, when students learn multiple representations, they are preparing for the kinds of activities common to those who use mathematics in their professional work (Greeno & Hall, 1997) – activities that require selecting an appropriate representation or set of representations for a particular situation (Byers, 2010).

Being able to mathematically connect different representations or generate new representations of the same object has been shown to be a strong indicator of a college student's mathematical knowledge and ability (AMATYC, Kessel & Linn, in Byers, 2010, p. 4).

The following scheme is based on Dreyfus' (1995) organization of activities which involves going from mathematics structure to mental image:

MATHEMATICS STRUCTURE
1 ↓
VISUAL REPRESENTATION
2 ↓
MENTAL IMAGE

1.Going from the first activity to the second, involves the use of multiple representation of some aspects of the mathematical structure static or dynamic external – graphs, drawings – to engage the perceptual and conceptual reasoning: "seeing the unseen".

2.Going from the second activity to the third one is related to the construction of mental images of the content involved in the mathematical avtivity with the help of visualization. It also involves register change to transfer knowledge in other contexts.

Connection between registers makes up the cognitive architecture by which the students can recognize the same object through different representations, and can make objective connections between deductive and empirical mathematics. Students who perform register change can transfer their mathematical knowledge to other contexts different from the one of learning (Duval, 1999).

According to Dreyfus (1995), visual imagery is the use of mental images with a strong visual component. Such mental images derive from external visual information. He states that it is the visual imagery, rather than the diagram, which directly influences our reasoning process. It may or may not allow to flexibly switch between different representations for the same concept or process (p.6). It is important to know how static and dynamic diagrams and operations on diagrams influence the construction of visual image. Students may not see in a diagram what is obvious to their teacher, because the teacher already has a principal interpretation, while students have to construct theirs (p.14). The teacher's responsibility is to help them to construct their visual images.

2.4.2 Visualizations, animation and interactivity

Nowadays, the centrality of visualization in learning and doing mathematics seems to become widely acknowledged. Visualization is no longer related to the illustrative purposes only, but it is also being recognized as a key component to support exploration and learning of mathematical concepts (NCTM, 2000).

The fundamental meaning of the term visualization is to form a mental image. The definition given by Zimmermann and Cunningham (1991) is about the transformation of the symbolic into the geometric. "Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding" (Zimmermann and Cunningham, 1991, p. 3)

Vision is central to our biological and socio-cultural being. The biological aspect is described well by Adams and Victor (cited in Arcavi, 2003, p. 215): "the faculty of vision is our most important source of information about the world" (p. 207). A definition which characterizes visualization and its importance, is that "visualization offers a method of seeing the unseen" made in the study of McCormick, DeFantim and Brown (1987, p. 3).

There are many researchers who conclude that interactive technologies, especially visualization tools are emerging to be powerful instruments that can engage students in meaningful learning (Beynon, Nehaniv, & Dautenhahn; Jonassen & Carr; Lajoie; Sedig & Liang, mentioned in Liang & Sedig, 2010, p. 973). This makes interactive visualizations an important aspect for the learning process. Also linking multiple visualizations can bring different cognitive benefits, helping students to estabilish relationships among different representations.

Technology used for educational purposes includes also animations, which are dynamic and flexible, so they can help to promote a better understanding, as students need to see things moving to understand and to process information (Hogstad & Brekke, 2010).

In the end it is important to mention also that the aim of using visualisation in mathematics teaching and learning is not to replace the formal definitions and theorems, but rather to complement them (Perjesi, 2003). The same determination stands also for the purposes of technology integration in education: it should be an aid tool which helps the traditional teaching.

2.4.3 Cognitive technologies

Cognitive technologies have transformed how mathematics can be done and how mathematics education can be accomplished. Each of these technologies makes mathematical activities newly accessible to students. A cognitive technology can be an amplifier, extending the existing curriculum, or it can be a reorganizer, changing the fundamental nature and arrangement of the curriculum (Pea, 1987).

A purpose function of cognitive technologies is knowledge for action – a condition to promote mathematical thinking. These tools provide functional environments in order to see applications immediately (interactive model). They help students and motivate them to think mathematically by providing activities whose purposes go beyond "learning math". There are also three process functions mentioned by Pea:

- *Developing conceptual fluency* – helping students become more fluent in performing routine mathematical tasks.

- *Mathemmatical exploration* – the computational discovery learning environment provides a rich context that helps students broaden their intuition. Students can make conjectures about different mathematical objects (medians, angles, bisectors). They can explore the properties of triangles and discover theorems on their own.

- *Integrating different mathematical representations* – linking different representations of mathematical concepts, relationships and processes. They help students to understand the relationship between different ways of representing mathematical problems, for example, change the value of a variable in an equation and observe the changes in the graph. Rapid interactivity and representational tools create a new kind of learning experience.

We are in need of a ‘cognitive technology’ (Pea, 1987) as “any medium that helps transcend the limitations of the mind: in thinking, learning, and problem solving activities”(p.91). Such ‘technologies’ might develop visual means to better ‘see’ mathematical concepts and ideas.

There are many studies which aimed to investigate how education can derive benefits from the use of technology, arguing that these tools should be used as cognitive tools for knowledge construction. Jonassen (2000) has argued that computer technologies, when used as cognitive tools or mindtools, represent a departure from traditional thinking about technologies (also see Jonassen & Reeves, 1996). Jonassen (1994) critically contends that students cannot use cognitive tools without thinking deeply about the content that they are learning, and that the tools will facilitate the learning process.

2.4.4 Different types of understanding

Skemp (1979) categorizes understanding of mathematics into: instrumental, relational and logical understanding.

Instrumental understanding is the ability to apply an appropriate memorised rule to the solution of a problem without knowing why the rule works. The rule or the procedure used can be applied only for certain tasks and the mental structures (schemas) built through the instrumental understanding are short-term and cannot be easily modified.

Relational understanding is the ability to deduce specific rules or procedures from more general mathematical relationships. So, one knows “how” and “why” (p. 38). The goal of relational understanding is the construction of relational schemas, which means making a connection between newly encountered concepts and the appropriate (relational) schemas. During this process the schema itself has undergone further development. Also, another goal may be the deduction of specific methods for particular problems, or specific rules for classes of tasks. Yet another kind of goal is improvement of existing schemas, by reflecting upon them to make them more cohesive and better organized, and also more effective for the first and the second kind of goal. Relational understanding requires the student to choose, change and apply data, formulas and principles in new situations.

Logical understanding is closely related to the difference between being convinced oneself, for which relational understanding is sufficient, and convincing other people. This type of understanding involves the ability to connect symbol signifiers in mathematics in relevant mathematical ideas and connecting the ideas into a scheme. Students with logical understanding can use their understanding to influence other students or prove mathematical statements. Logical understanding also involves efforts to demonstrate what is stated according to logic or proving that a statement is true (p. 43-44).

Learning is a dynamic process not a static one and it should not be based on memorization but upon mathematical understanding. The process of creating complex concepts starts from the connections between ideas, facts and procedures to form basic concepts, followed by the process of connecting these basic concepts. From this process is expected the development of relational and logical understanding, and not just instrumental understanding.

According to the Learning Principle (Stylianides, 2007), learning with understanding is related with these importand aspects:

- close interrelation between factual and procedural competence (Bransford et al., 2000; Hiebert & Carpenter, 1992)

- prior knowledge and experience always facilitate subsequent learning (Hiebert & Carpenter, 1992)
- knowledge transfer from one situation to another - making connections among ideas which can facilitate the transfer of prior knowledge to novel situations.
- learning and knowing can only be understood when considered in the broader cultural context (Davis et al., 2000, p. 69).

In the following section, we examine the aspects of the learning process according to the constructivist theory and how technology integration can support such a process.

2.4.5 Investigating the role of technology integration in relation to learning theories

Researchers on cognition and pedagogy emphasize constructivist approaches as an important learning theory. This theory emphasises the cognitive process of learning, focusing on the connection between prior and new knowledge.

Grabingar and Dunlap (1995), in their study about supporting constructivist learning by multimedia, characterize constructivism by three different aspects. In the following paragraph we take a look at these three aspects and analyse how technology practices can help in this learning approach:

1. Learning is an active and evolving process

Constructivist models of learning strive to create environments where learners are active participants, in a way to help them construct their own knowledge, rather than having the teacher interpret the world and insure that students understand the world as they have told them. The constructivist model is defined as a student-centered model and it is better aligned with the goals of the school reform. Another important principle related to active learning is the one developed by Dewey, "learning by doing" (Dewey, in Faugli, 2002, p.16), which was later defined by Bruner (cited in Faugli, 2002, p17) as "discovery learning". Since technology supports this kind of learning model by offering students interaction with the tool, its importance in the education process is obvious. It allows students to work more meaningfully with tasks and engages them in knowledge construction rather than knowledge reproduction (Jonassen, 1994).

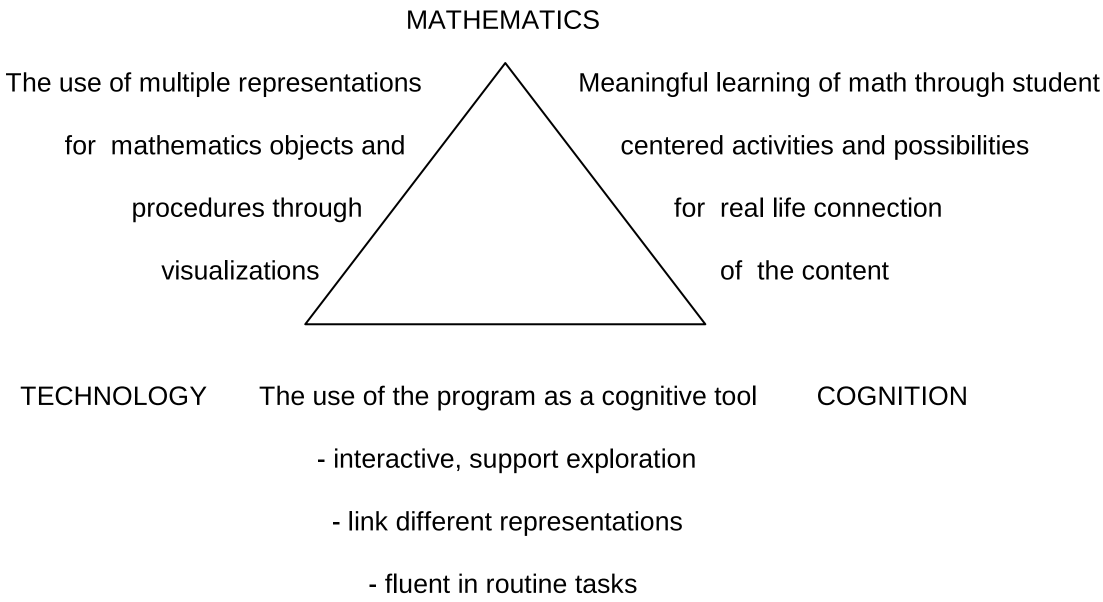
2. The context is significant in the building of knowledge

It is very important for students to build their knowledge in rich environments, related to situations where they can apply and try what they learn.What students learn should not be separated from how they learn it (Brown et al., in Faugli, p. 16). As mentioned above, Ashburn and Floden (op. cit.) emphasize the role of technology by showing how it can help for a better learning experience, connecting school with real world. In this way, students will be able to transfer the knowledge constructed during the lessons to other situations out of the classroom.

3. The social context in which the learning takes place is of great importance to conceptual development and takes place by sharing ideas and testing ideas with others.

This can be related to the social-cultural theory developed by Vygotsky (1978), which states that students learn through socially mediated activities where they discuss and discover concepts from different perspectives under teacher's facilitation. As technology supports interaction and collaborative learning (Ashburn & Floden, 2006), it helps to promote meaningful learning and makes it easier for students to engage in their knowledge construction.

Inspired by cognitivist theories, the following scheme has been formulated to clarify the structure of the theoretical framework:



2.5 Students' attitudes toward the use of technology in classroom

2.5.1. Relationship between students' attitudes toward the use of technology in teaching and learning and confidence in learning mathematics and using computers

Students' attitudes affect the impact of technology, especially students' perceptions of their abilities to solve mathematical problems and attitudes to using technology to learn mathematics (Moos & Azevedo, 2009). As stated by Galbraith and Haines (1998) it is a crucial step in understanding how the learning environment for mathematics is affected by the introduction of computers and other technology gaining insights into students' attitudes and beliefs. Attitude represents an emotional reaction to an object, to beliefs about an object, or to behaviour towards the object. Attitudes toward teaching and learning mathematics with technology are also related to students' attitudes toward mathematics and the use of computers, expecially to self-confidence and motivation.

Mathematics confidence. Students with high mathematics confidence do not worry about learning hard topics, expect to get good results, and feel good about mathematics as a subject. Students with low confidence are anxious about learning new material, they feel they are naturally weak in mathematics, and worry more about mathematics than any other subject.

Mathematics motivation. Students with high mathematics motivation, enjoy doing mathematics, stick at problems until they are solved, continue to think about puzzling ideas outside class, and become absorbed in their mathematical activities. Those with low motivation do not enjoy challenging mathematics: they are frustrated by having to spend time on problems: they prefer to be given answers rather than left with a puzzle, and they cannot understand people who are enthusiastic about mathematics.

Computer confidence. Students demonstrating high computer confidence feel self-assured in operating with computers: they believe they can master computer procedures required from them; they are more sure of their answers when supported by a computer, and in cases of mistakes in computer work they are confident on resolving the problem themselves. Students with low computer confidence feel disadvantaged at having to use computers; they are anxious about learning new computer-based procedures; they do not trust computers to produce correct answers, and they panic if errors occur when using a computer program.

Computer motivation. Students demonstrating high computer motivation find the approach to learn more enjoyable; they like the freedom to experiment and enjoy testing out new ideas. Students with low computer motivation avoid using computers; they believe their freedom is eroded by program constraints and cannot understand how others become absorbed by such activity.

The following result has emerged from a previous analysis of relationship between computer and students' mathematical attitudes and their attitudes in computer-related activities in mathematical learning, computer attitudes are more influential than mathematical attitudes in facilitating the active engagement of computer-related activities in mathematical learning" (Fogarty et. al, 2001).

In relation to trigonometry, students' attitudes have been rarely investigated as an outcome of technology-supported trigonometry instruction. Ross, 2011. Choi- Koh….

the affective reactions of students toward the presence of computers in teaching and learning process might be a critical factor to explore in successful implementation of this technology.

2.5.2. Thematic analysis of representations of successful use of technology expressed by students reactions

In this section, we present an analysis of teachers' accounts of successful use of technology in the teaching and learning of mathematics (Ruthven, 2002). The successful use of technology is considered in relation to the influence of this practice on students' attitudes in class.

Ruthven (2002) divided the themes which represent the use of technology in two groups: success and operational themes. The initial analysis focused on the *success themes* which were conceptualised in terms of three related priorities, concerned with securing and enhancing of:

- the *participation* of students in classroom work in terms of overcoming disengagement and demotivation on the part of students;
- the *pace and productivity* of such work in terms of time saved and pace maintained and work produced;
- the *progression* in learning arising from it, framed in terms of ideas being formed and embraced, and in more general terms of development and learning.

From the analysis made by Ruthven, ten *operational themes* emerged:

♦ *Ambience enhanced*: associates technology use with change, difference or variety in working ambience. At one level this is often a matter of change of working location -from ordinary classroom to computer classroom- and correspondingly of work organisation.

♦ *Minimization of constraints* which inhibit the participation of students in classroom work. Technology use is often associated with a reduction of the writing demands -physical and intellectual- of much conventional classwork; demands which may challenge some students. It also changes the status of mistakes, not only by facilitating their correction, but by removing evidence of them which might attract unwelcome attention from the teacher.

♦ *Tinkering assisted*: focuses on how the provisionality of many technology results assists forms of tinkering to improve them.

♦ *Motivation improved*: associates technology use with the motivation of students toward classroom work. This idea has been present in many of the quotations which have already been used to illustrate earlier themes. Teachers have commented on what students 'love', 'like' and 'enjoy' in relation to using technology; likewise on what 'motivates them', on what they 'respond well to', and on what they are 'quite taken by'.

♦ *Engagement intensified*: associates technology use with deeper and stronger student engagement in classroom work. Clearly this theme is closely related to *Motivation improved*.

♦ *Routine facilitated*: associates technology use with facilitation of relatively routine components of classroom activity, allowing them to be carried out more quickly and reliably, with greater ease, and to higher quality.

♦ *Activity effected*: associates technology use with securing and enhancing the pace and productivity of classroom activity as a whole.

♦ *Features accentuated*: associates technology use with the provision of vivid images and striking effects through which features of mathematical constructs -or relations between them- are accentuated.

♦ *Attention raised*: associates technology use with reducing or removing the need for attention to subsidiary tasks, and with avoiding or overcoming related obstacles, so as to better focus students'attention on overarching ideas and processes.

♦ *Ideas established*: associates technology use with the formation and consolidation of ideas. In earlier quotations the sub-themes of ideas being 'seen', 'understood', 'accepted' and 'remembered' by students have already arisen.

2.6. Issues and limitations related to technology integration

2.6.1. Barriers in technology integration

Integrating new technologies into everyday teaching and learning of mathematics has proven to be a slow process that involves multiple challenges for teachers and students (Hohenwarter & Lavicza, 2007, p.49). In spite of the considerable promise that technology provides for the reform of mathematics education, there are potential barriers to the fulfillment of that promise. The incorporation of technology into mathematics generates a list of objections and concerns identified by Heid (1997):

1. *Finance and equity* – this concern is related to the possibilities of the institution to afford the financial support for technology: the resource materials and the maintenance for the hardware and the software.
2. *The nature of technology use* – it is a concern related to the integration of technology in mathematics education and which revolves around how students will use technology, if they will overuse it or use it for inappropriate topics. They can develop a false security, not being aware of the computer limitations or calculators results. Also students can shift attention from the mathematical activity to the tool.
3. *Students' Learning* – a third set of concerns points toward what students would not learn if technology is incorporated into the teaching of mathematics. Computers and tools can become a crutch, replacing students' mathematical thinking, reasoning, mental computational abilities and basic skills.
4. *Curriculum balance and implementation* – two major areas of concern arise with respect to curriculum: what is the proper balance of computer use with other practices, and what is required from the students to do and to know? When students are presented with rich problems and powerful technology, they are likely to take a variety of directions in approaching the problems This is because teachers and students begin with problems instead of topics, making them focus on many different topics at the same time (Romagnano, 1994). Many of the technology intensive curricula require more time both in and out of the classroom (Schmidt & Callahan, 1992; Solow, 1991).
5. *Teacher preparation* - One of the most pervasive concerns about the integration of technology ino the teaching of mathematics is a concern about the adequacy of

teacher preparation, if they will be appropriately prepared to engage their students in using the technology as an aid in their mathematics learning.

6. *Public perception* – this is a concern about how the changes brought on by technology should be communicated to the public (the parents, school board members), so as to understand the promise of technology in education.

2.6.2. Difficulties around visualization

In this section, we present two important obstacles mentioned by Guzman (2002), which are related to the use of visualization in teaching mathematics:

1) Visualization leads to errors

The author argues that an incorrect use of the visualization can lead to errors in different ways. Some of these uses are related to the incorrect interpretation of the figures. In these cases our intuition can lead to false conclusions.

2) Visualization is difficult

It is true that an image is worth a thousand words, but we should add the important condition that the image should be understood. Otherwise, it is worth nothing. The correct performance of visualization requires previous preparations and the knowledge to interpret what it represents.

These two difficulties, formulated by Guzman, are not directly related to the use of technology, but they represent obstacles in using visualization in mathematics teaching. However, the possibility that visualization can lead to errors should not be an argument againts its efficiency in different processes in mathematics activity (Guzman, 2002).

3 Context

The context of the study consists of five sections:

- the description of the tool used during the experimental lessons to teach trigonometry, including the rationale why this tool has been chosen and some information related to the programmer and the development of SimReal.
- information for the Albanian education system, standards of using ICT in education and the place of technology in the curriculum.
- research context: preparation for data collection
- the description of the school where the data were collected,
- the mathematical content under consideration in the experimental lessons.

3.1 SimReal program

3.1.1 The development of SimReal as a part of parAbel project

The tool used in this research, SimReal, was programmed by Per Henrik Hogstad, professor at the University of Agder, in Grimstad. It aims to be a supplement to traditional teaching, using visualization and interactivity to explain difficult topics in physics, mathematics and computer science (Hogstad & Brekke, 2010). This program started as a part of a project known as parAbel, developed in 2001 by Agder University College (AUC), Heriot-Watt University, Scotland, Norsk Interaktiv (Norwegian Interactive), and a consortium of high schools and education centres in Norway.

ParAbel consisted of e-learning courses for upper level students (from 16 – 19 years old) in mathematics and physics, based on interactive computer materials which include a high integration of animations and simulations. The main goal of this project was to attract young people to mathematics and physics through high-quality instructional modules that make the subjects understandable, exciting and linked to real life. As the project developed, SimReal was programmed, as a set of advanced simulations and a computer-based graphic calculator.

3.1.2 SimReal properties

SimReal includes simulators, interactive animations, movies, games and a calculator. Interactive animations and simulators are the most important elements in e-learning. Simulators recreate elements from the real world. In order for a simulator to respond, it needs information from the user. The strength of a simulator lies in its ability to explain events hidden from view. Movies are useful for the introduction of subjects, i.e. scenes from a real situation or history. Games can help students improve their skills interactively. They will improve abstract and creative thinking. Playing is a valuable educational method which offers variety and fun in the learning process.

Advanced technology creates a flexible system based on intelligent objects. Combinations of different types of objects make it possible to construct almost all kinds of tasks linked to real events. New multimedia technology gives the opportunity to create variations. Students especially liked working with interactivity, simulations and problems-solving.

With SimReal it is developed also SimVideo, which is an interactive learning tool, and contains videos of lectures, simulations, problem solving and applications. But in the current study, only SimReal is used.

3.1.3 Results from the use of parAbel and SimReal at the University of Agder

SimReal was used for six years to teach physics mathematics at the University of Agder in Grimstad, and it was concluded that it is a very positive and fulfilling experience. By integrating computer-based work into the classroom students were more satisfied, spent more time studying, and most importantly performed better. The feedback of students using the program has been very positive. They appreciated variation and different techniques learned during the course. Many students try out, on their own many other simulations that are not part of the curriculum (Hogstad & Brekke, 2010; Brekke, 2009).

Feedback from users

User feedback was gathered and used to improve the quality of parAbel, where 35 schools, approximately 50 teachers and 1000 students took part in the test.

Feedback from teachers:

- parAbel is a good supplement to traditional teaching.
- <u>Missing animations about the unit circle, sine and cosines functions (now avaiable in SimReal)</u>
- It’s very difficult to write mathematic symbols.
- We need some more experience with parAbel

Feedback from students:

- parAbel is good fun, really good with something different. It is different from the text book and varied
- Can’t use it at home
- <u>I miss an online calculator. Still have to use paper and calculator on the desk.(already present in SimReal)</u>
- Easy to find subjects. The folder system is good
- Different from traditional teaching, would like to use parAbel more often

3.1.4 The rationale for choosing SimReal

Having as a focus to study the implementation of technology in mathematics lessons, we decided to use this program for the following main reasons:

- It can be defined as a cognitive tool, according to the description made by Pea (1987)
- Easy to have access to it, easy to use, to be learned, easy to add new content (with the help of the programmer)
- It has already shown to be successful in improving students’ achievement and their attitudes toward learning (Hogstad & Brekke, 2010; Brekke, 2009).

3.2. Education in Albania

3.2.1. Education system in Albania

The Albanian school system has changed in 1999 – 2000. The old system consisted of four years of Elementary level, four years of Lower secondary level and four years of Upper Secondary level. During the academic year 2009-2010 the new secondary school structure was implemented, as it is presented in the table 1:

Table 1. Education System in Albania

Education	School level		Length of the program	Age level
Pre-school			1–4 years	2 - 6
Primary	Elementary school		5 years	6 -11
	Lower secondary school		4 years	11 - 15
Secondary	Upper secondary school		3 years	15 - 18
	Vocational	Vocational school	2 years	15 - 17
		High technical school	5 years	15 - 20
Tertiary	Bachelor		3 years	18 - 21
	Master degrees		1.5–2 years	
Quaternary	Doctoral		3 years	

Primary education lasts for 9 years following a non-mandatory period spent at pre-school. Students must pass graduation exams at the end of the 9th grade and at the end of the 12th grade in order to continue their education. Although education is only compulsory for the first 9 grades, most young people stay on through to grade 12.

Secondary education known as ‘regular’ takes a further three years to complete. The focus is on academic teaching and preparation for university. Many schools have recently been rebuilt and are being equipped with modern technologies. Most schools are public and financed through the government, but recently several private schools of various levels have been opened. There are about 5000 schools throughout the country.

Vocational education is an alternative to ‘regular’ school and takes between 2 to 5 years depending on whether a simple diploma or a full trade qualification is desired. Considerable effort by the state in this direction is adding muscle to a growing economy.

There are a significant number tertiary institutions or universities in Albania, both public and private, and these are well dispersed in the major cities. The University of Tirana was the first and was founded in 1957. Today, it has a student population approaching 15,000 and nearly 900 teaching staff.

3.2.2. Standards related to the use of ICT in education

The Ministry of Education and Science has set as a priority the integration of ICT in teaching / learning. Education Development Institute has started the design of standards of using ICT in teaching and learning for teachers and school administrators.

Recently, the Ministry of Education started implementing projects which aim to equip every school with computers and internet connection. Also, plans are in place to ensure that the necessary conditions are met for the integration of technology in the teaching and learning of different subjects, as the reformulation of the curriculum content based on ICT use, and the training of the teachers to use ICT during the teaching proccess. (According to the annual analysis of the Ministry of Education, 2011).

3.3 Scutari upper secondary school

Scutari is a private three-year school in Shkodra, Albania. It is a new school, which started its regular work in september 2004. In the first year of its operation, it had only one class with 16 students and ten teachers. Then, the following year, two new classes were opened,with a total number of 61 students and 7 new teachers.

In the academic year 2006-2007, there were 7 classes, three for grade 9, 2 for grade 10 and 2 for grade 11 (with 150 students in total and 8 new teachers, thus increasing the number of teachers to 21). In this year, the school was declared by the Ministry of Education as one of the best 10 schools in Albania.

In the academic year 2007 - 2008, there were 9 classes with an enrollment of 191 students and 25 teachers. 34 pioneer students graduated in this year, with 100% passing rate and the annual grade point average was over eight (ten is the highest rating).

In the academic year 2008-2009, there were reforms in the education system, namely the duration of the primary cycle was changed from 8 years to 9 years, so there were no new students, and the school had only three levels: grades 10, 11 and 12.

Currently (2011- 2012), there are six classes, 2 profiles for each level, one scientific and one social. The average number of students for each grade is 29, and the total number of students is 181. The average of student's level is rated eight (10 is the greatest degree). The total number of teachers is 17, and 3 of them are mathematics teachers. The school has 10 classrooms, a laboratory for chemistry and one for informatics. The school is run by a coordinator and a principal, who is also a teacher in this school.

3.4 Research context

The design phase included:

- the learning and preparation of the SimReal, with the help of the programmer.
- the design of students activities and tasks
- design of teaching arramgements: organisation of the lessons and the way the teacher would deal with the theory, tasks and tool

The first step after the decision related to the theme of the research was to communicate to the principal of the school in Albania, informing her about the study as well as asking her if it was possible to collect data from her school.

After receiving a positive answer, we chosed the grade and the mathematical content where we wanted to experiment with the program. As we were interested in a topic area where we could use the program widely, we chose trigonometry as the mathematical content to focus on. As for the grade, we chose one of the second grades of the school, specifically the one with the scientific profile.

At this stage it became imperative to work with the programmer of SimReal so as to learn how to use it and to prepare the program with new aids related to trigonometry (suggested by the researcher).

The next step was the preparation for data collection, including the laboratory where the teaching was to take place, testing the camera and making lesson plans. Before starting the lessons, I participated in a mathematics hour, to introduce myself to the class, to inform them about the project and to divide the class into groups.

Given that SimReal is a program which requires a browser, it was important to confirm internet availability and the qualitiy of the connection. Making use of the computers and the program during mathematics lessons depended also on gaining access to the computer science classroom. The school has a specific classroom for computer science lessons, which contains 13 computers, a blackboard and a projector which is connected with the main computer (at the teacher's desk).

3.5 Mathematical content

The mathematical content chosen to conduct this research is trigonometric functions and their variations. With the aim to investigate the role and the effectiveness of using SimReal in mathematics teaching and learning, the study focus on:

- The definition of the trigonometric functions represented in the unit circle
- Variations of trigonometric functions and graphical representations
- Reduction formulas and relation with the periodic nature of the functions
- Applications of trigonometric functions
- Solving equations that invlove trigonometric functions

The type of mathematics curricula in Albania is spyral and the content lines between grades remains the same, getting extended each year.

4 Methodology

4.1 Research design

4.1.1 Methods

The main aim of this study is to investigate the role of the computer program, SimReal, in teaching and learning trigonometry in upper secondary school. In order to achieve this aim, a mixed method research design has been used (Johnson & Onwuegbuzie, 2004), including a number of data collection instruments, such as: pre- and post- tests, self-completion questionnaires, direct observations (videotaped) and interviews (audiotaped).

Johnson and Onwuegbuzie (op. cit.) define mixed methods research as “the class of research where the researcher mixes or combines quantitative and qualitative research techniques, methods, approaches, concepts or language into a single study” (p.17). They note that mixed methods research uses “multiple approaches in answering research questions, rather than restricting or constraining reseachers’ choices” (p.17). They state the advantages of using this form of research over using a purely quantitative or a purely qualitative method:

> It is an expansive and creative form of research, not a limiting form of research. It is inclusive, pluralistic, and complementary, and it suggests that researchers take an eclectic approach to method selection and the thinking about and conduct of research. What is most fundamental is the research question— research methods should follow research questions in a way that offers the best chance to obtain useful answers (Johnson & Onwuegbuzie, 2004, p.17-18).

Both quantitative and qualitative approaches offered beneficial attributes in this study to investigate the effectiveness of technology implementation. The quantitative approach helped to assess students’ understanding and learning of the concepts explored with SimReal according to the scores earned. The qualitative approach afforded the opportunity to analyse a wide variety of data, including interviews and feedback of students in order to define their attitudes and to make a qualitative analysis of their work (through a test and class work), comparing solutions between the two groups.

4.1.2 Participants

22 students participated in this study, i.e. 6 boys and 16 girls. They were from the second grade (16-17 years old) of a private upper secondary school in Shkodra, Albania. The participation in the experimental lessons was optional and not compulsory. All the students were informed beforehand about the procedure and the purpose of the experiment, and only those who agreed participated in it. They were also informed about their privacy and that the data would be used for research purposes only. The videos were only to be used to conduct this research, and the names on tests and questionnaires were to be coded with numbers.

4.2 Data collection

4.2.1. Procedure

The class was divided randomly in two groups: control and experimental group. This division was made according to the list of students in alphabetic order, where the students belonging to the odd numbers of the list were selected for the control group and the others for the experimental group.

However there were students in the experimental group who preferred to stay in the control group. There were three girls, relatively good in mathematics, who thought that changing the teacher of mathematics during the year would affect their learning. This reaction was totally expected, given that it is a new and an unknown way of doing mathematics in Albania. In fact,this could have affected the results of the experimental group compared to the control group, because better students remained in the control group.

The control group had lessons with their mathematics teacher, with 3 years of experience in this school. There were 11 students (8 girls and 3 boys) for each group. By listing the names of each group in alphabetical order, and starting from the students in the experimental group we coded the names “student 1”, “student 2” and so on until the last in this group, i.e. “student 11”. Then the first student of the control group was coded “student 12” and the last one “student 22”.

In table 2, we present the instruments used as well as the description of activities and the data collected during these activities. The instruments are listed according to the chronology of application.

Table 2. Presentation of activities and data collected according to the instruments used

Instruments	Description of activities	Data collected
1. Likert scale questionnaires	Both groups completed two questionnaires about their attitudes toward and self-confidence in mathematics and computers.	
2. Mathematical pre-test	The whole class participated in the test.	Points of tests
3. Experimental lessons using SimReal	For 6 weeks the two groups had separate lessons for the same mathematical content. The control group had traditional lessons, with their actual mathematics teacher. The experimental group was taught by the author in the informatics laboratory, using SimReal during the lessons. Some of the lessons were videotaped to see students' interacion with the program.	-Lessons plans -Videos -Students' work
4. Mathematical post-test	At the end of the topic the whole class participated in a test.	Points of tests

5. A likert scale questionnaire	The experimental group completed a questionnaire about their attitudes toward using SimReal in mathematics lessons.	
6. Interviews	Five students in the experimental group were asked to participate in an interview about their impression in relation to SimReal and its importance and utility during the mathematics lessons	Audio registrations of interviews with students in the experimental group.
7. Field notes	After the lessons, students in the experimental group, were asked to give their opinions and feedback in relation to the lessons.	Students' feedback about lessons

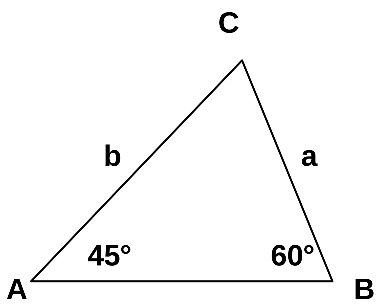
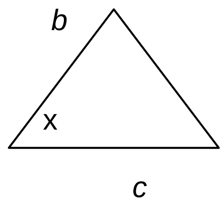
4.2.2. Instruments used for data collection

The pre- and post- mathematical tests were set by the author, in consultation with the mathematics teacher who works at the school where the data were collected. The tasks in both tests were taken from a textbook authored by Lulja, Babamusta & Bozdo, 2010.

The pre-test was formulated according to students' prior knowledge about the mathematical content acquired in their previous grade. The post-test was about the mathematical knowledge taught during the lessons. The pre- and post-test had the same number of tasks, and approximately the same kind of questions. In tables 3 and 4, we present the pre- and the post-test and the points for each task.

Table 3. The content and the distribution of points in the mathematical pre-test

PRE-TEST			
Task	**The question**	**Description**	**Points**
1	What is 1 Radian?	The definition of the angle measured 1 Radian.	10
	Turn into radian the angles: 20^0, 30^0, 270^0, 3240^0.	For each angle 2 points, and 2 points for the formula used to find these values.	10
2	Give the definitions of the trigonometric functions in the right angle triangle ABC.	Students have to draw a right angle triangle and show which side represents the sine function and which side represents the cosine function. (10 points) Also, they have to write the formulas of these functions and the formulas for the tangent and cotangent function. (10 points)	20
3	Let side **a** of the triangle ABC be 10 and the angles be $\alpha=45°,\beta=60°$. Find the length of side **b**.	Students have to use the Sine Theorem to find the side b.	20

		Sine theorem: $\frac{a}{sinA} = \frac{b}{sinB}$ They can also divide the triangle in two right angle triangles and use the Pythagorea's Formula. 	
4	sin(90°-α) - cos(180°-α) + tan(180°- α) + sin(-α)	Students have to use the Reduction Formulas to make these transformations, 5 points for each of them. sin (90°- α) = cos α cos(180°-α) = - cos α tan (180°- α) = - tan α sin(-α) = - sinα	20
5	Can you find the area of a triangle if you know the lengths of sides b and c and the angle *x* between them?	$A = b \cdot c \cdot \sin x$ 	10
	If we can change the angle *x*, for which value of *x* will we have the largest area?	$\sin x \leq 1$ The greatest value of the sine function is 1, for the angle *x* = 90°. So the right angle triangle will have the largest area.	10

Each task in the pre-test is scored out of 20 points. Table 3 also shows a description of what is expected from students in terms of answers. The content of the pre-test is related to students' prior knowledge:

- Radian concept
- Definition of trigonometric functions in right angled triangle
- Sine and Pythagorean theorem
- Reduction formulas (for angles that have the sum 90^o or 180^o)
- The area of triangle

In the following, we present the content of the mathematical post-test and how it is set, compared to the pre-test.

Table 4. The content and the distribution of points in the mathematical post test

POST TEST			
Task	**The question**	**Description**	**Points**
1	What is an angle 1 radian?	The definition of the angle 1 radian	10
	Find the values in degree of the angles: 3π, 3π/2, 4π/3, 7π/6	For each angle, students have to calculate how many degrees correspond to the given value in radian (2 points), and also they have to write the formula used to find these values (2 points).	10
2	Draw a unit cirlce and take an angle x. Show the sine, cosine and the tangent of the angle x. Which of these functions are limited? Find the period of each function and draw the graph of sinx.	For an angle x students have to determine the sine and cosine of this angle, showing them in the unit circle. (10 points) They also have to write some properties of these functions, as the limits, the periods (5 points). In the end they have to draw the graph of sine function (5 points) .	20
3	Given the equation of motion of a point with the function y = 2cos(2x) + 1, find the amplitude, period and the frequency of the motion. Draw the graph of this function.	3 points for each property found,: amplitude, period, frequency. 11 points for the graphical representation of the given function, which is drawn based on the graph of the function: y = 2cos(2x).	20
4	cos (90°-α) – sin (180°-α) + tan (90°+ α) + cos (-α)	Students have to use the reduction formulas for angles that have the sum 90^o or 180^o (2.5 points for each transformations)	10
	1/(1-cosx)+1/(1+cosx)	Studetns have to use the main trigonometric formula for this transfomation and also some elementary arithmetic transformations	10
5	Solve the equations: sinx = - cosx	This type of equations has been done in class with the help of SimReal. They have to solve it algebraically.	10

	$2\cos^2 x - 5\cos x + 3 = 0$	This is an equation of second degree where the replacement cosx = t should be used.	10

From table 4, it can be noticed that the distribution of points is the same as in the pre-test: 20 points for each task. The theoretical tasks (1 and 2) are approximately the same as in the pre-test. In this way, it would be possible to see the improvement of students in understanding the concepts. The tasks 3 and 4 are problems which were dealt with during the lessons by both groups. It is important to mention that in the experimental group these problems were elaborated with SimReal. In this way it would be easier to indicate the role of the program in students achievement. The following points show the focus of the post-test content:

- The radian definition
- Trigonometric functions in the unit circle
- Properties of trigonometric functions: amplitude and period
- Reduction formulas and the main formula of trigonometry, which is based on the Pythagorean formula
- Solving trigonometric equation

Notice the similarity in the structure of the pre- and post-test and how they are related to each other. However, the content in the post-test was a little more extended, given that the content in the textbook for the second grade is wider compared to what is covered in the first grade. Moreover, it involves some new knowledge. But the aim was to make it possible for students to relate the current knowledge with what they learned a year ago, and to have the possibility of comparing the performace of each student in the pre- and post-test.

Questionnaires

The three questionnaires used in this study are taken from the article by Fogarty, Cretchley, Harman, Ellerton and Konki (2001). In that study these three questionnaires were validated and used to measure mathematics confidence, computer confidence and attitudes toward the use of technology for learning mathematics.

The first questionnaire about computer confidence has 12 items and the other two have 11 items each. Not all the items are positive; they are combined, and in the end they have been evaluated according to the points of each item. The positive item has 5 points for “totally agree”, and 1 point for “totally disagree”. The opposite is for the negative item, which has 5 points for “totally disagree” and 1 point for “totally agree”.

The interview

The interviews were conducted at the end of the practice work, and they focus on students’ perception of the utility and usability of the computer program during the mathematics lessons. The questions were about :

- evaluation of utility of the program in helping the learning process;
- concrete examples of mathematical concepts understanding which are facilitated by the computer program;

The questions for the interviews were formulated at the end of the practice work.

Questions for the interview:

1. Was it helpful for you to use the program SimReal in mathematics lessons?
2. Which part of the program was more useful for you: the simulations, the interactive part, or doing calculations? (functionalities of SimReal)
3. Do you think it is easier to understand the lessons explained with SimReal, or it is better to explain before the lesson on the blackboard and then to look at the program? (sequence of teaching)
4. Is SimReal easy to use? Do you think more practice was necessary to learn how to use it better? (Easy to use?)
5. Can you bring any concrete example where SimReal helped you to better understand the mathematical content?
6. Did you use the program after the lessons?

4.3. Qualitative and quantitative data analysis

We decided to present the data analysis in one section without dividing the presentation into quantitative and qualitative analysis. These two approaches complement each other and are used together in the analysis of this study.

4.3.1. Test scores

The test scores for each grade were planned to be analysed statistically with SPSS, but given the small size of the sample, we realised that it would not be a meaningful analysis. So, we decided to present the results showing the mean, the standard deviation and a graphical representation for each group.

A comparison between the groups' results in the pre- and post-test is done, and for each student there is a comparison of results in the pre- and post-test in order to establish individual performance before and after using the program.

4.3.2. Points of questionnaires

As already mentioned, the items of the questionnaires are formulated using a Likert scale with five options. For each of them, we calculated the points gained according to the scales (1 to 5) of the items evaluated by the students. The results are calculated in percentage and presented according to table 5 (Bryman, 2008) :

Table 5

Excellent	>85%
Very good	70-85%
Acceptable	60-69%
Barely acceptable	50-59%
Not acceptable	<50%

The results from the questionnaire about the attitudes toward the computer program are used in the analysis of students' attitudes toward using computers in mathematics, as previous studies show that there is a strong correlation between confidence and motivation in using computers and reactions of students toward doing mathematics with computers.

The last questionnaire was completed by the experimental group only, and it shows how the students reacted to the use of the program. The results from that questionnaire are used to answer the second research question in relation to students' attitudes toward the learning of mathematics with computers.

4.3.3. Experimental lessons using SimReal

The activites during the experimental lessons are analysed and organised in relation to the purposes of using the computer program: promoting drill and practice, enabling the exploration of the effects of variation of variables, and fostering conceptual understanding (Wilson, 2008). A correlation is established between these three purposes and the pedagogical benefits and concrete examples from the use of SimReal.

During the lessons the students of both groups solved two tasks in relation to the trigonometric functions, drawing graphs, and comparing two graphs when a variable is changed.

Task 1.Draw the graph of the trigonometric function y = tgx and then the graph of y = - tgx.

Task 2.Draw the graph of the trigonometric function y = cosx and then the graph of y= 1/2cosx

The students' work were analysed and categorised according to the way they solved the task.

4.3.4. Interviews

All the interviewees were given exaclty the same content of questions in order to ensure that their answers could be aggregated and categorised according to the responses in relation to the practical work with the program (Bryman, 2008). We planned to have a semi-structured interview, given that the students could have positive or negative responses with respect to the usefulness of the program during the lessons.

The interview had five specific questions:

- Starting with the first question, we aimed to determine the students' opinion regarding the role of SimReal. However, during the interviews, all the students (5) gave a positive response to the first question, so the rest of the interview questions were the same for all.
- The second question aimed to find out whether students prefer to interact with the program, or whether it is better for them when the teacher uses it to explain the mathematical content.
- The third question is related to the sequence of actions during the explanation: blackboard – program or program – blackboard.
- The fourth question is about the use of the program, i.e. whether they found it easy to learn, or they would prefer to have more time to learn and explore it better.
- The next question asks the students to show some concrete mathematical content that they think they understood better by using the program.

- The last question of the interview is about the use of the program outside of the classroom.

4.3.5. Field notes

Students from the experimental group were asked to make comments in relation to the program and to give feedback about the lessons with SimReal. This feedback from students was very important, since it was an opinion given immediately after the lesson. It was a kind of discussion, different from the interview, where the students felt a little under pressure, giving mostly positive responses about the program.

Students gave different kinds of responses in relation to the practice with SimReal, which are analysed and grouped according to some thematic representations of successful use of computer programs, insipired by Ruthven (2002), (ref the section in chapter 2), who formulated these themes from the analysis of what practioners conceive as successful use of computers to support mathematics teaching and learning. These themes are divided into five components:

- Participation of students
- Productivity of lessons
- Students' progression
- Engagement during the lessons
- Students' attention

As we can see, these themes are aslo related to each other, and it is important to mention that the interpretations for each group are based on the students' perception and their opinions about the use of SimReal during mathematics lessons.

5. Findings

The current chapter presents the results from the analysis of the data. It is organised in three main sections, in keeping with the aims of the research:

1) investigation of the role of SimReal in teaching and learning trigonometric funtions;

2) analysis of students' attitudes toward using SimReal in mathematics lessons;

3) presentation of some limitations on the use of the computer program, SimReal, in teaching and learning mathematics.

As the experimental work was based on the use of SimReal during mathematics lessons, we start by, presenting a summary of activities developed in order to point out how SimReal has been used during these lessons to teach trigonometry. After that we present some of students' solutions of tasks set during classwork, focusing on a comparison between the two groups. Also we show the results of the mathematical tests, comparing the performance for each student in the pre- and the post-test, and comparing the scores of the post-test between the experimental and control group.

The second section deals with students' attitudes toward using SimReal during mathematics lessons. It consists of the presentation of scores of the questionnaire completed by the students of the experimental group at the end of the practice. An analysis of students' responses to the interview is made so as to show their opinions in relation to some advantages and disadvantages of using SimReal. Also, we present a categorisation of field notes gathered from students' feedback related to the practice with SimReal.

The third section presents the difficulties that emerged from data analysis and during the experimental work in relation to the use of SimReal during the teaching and learning of mathematics.

5.1 The role of SimReal in teaching and learning trigonometric funtions

5.1.1 Experimental lessons using SimReal

The experimental lessons took 13 hours, including the two hours (the first and the last) of the pre- and post-test, in a period of six weeks, namely from 9th of November to mid-December 2011. In the annexes, we present a schedule of the lesson plans developed during the experimental work, with dates and themes from the trigonometry topic.

We organised the activities for the use of SimReal during the lesson according to three main purposes of using a tool in the teaching and learning process (Wilson, 2008), relating them to some important pedagogical benefits. These activities are shown in table 6. We also demonstrate some of the SimReal interfaces where these activities took place.

Table 6. Presentation of activities using SimReal in relation to the pedagogical purpose of its use

	Purpose of using the tool	Trigonometry lessons activity using SimReal	Pedagogical benefits
1	Promoting drill and practice	Conversion between alternate measurement of angles : radian – degree	-Reduction of time required to practice basic skills
		Calculation of the values of trigonometric functions for different angles	-All students engaged during the lessons
		Drawing the graphs of trigonometric functions	
2	Variation of parameters	Changing parameters of trigonometric functions and looking at the changes in the graphical characteristics	-Immediate response is effective to make the connection between the parameters in trigonometric
		Exploring and discovering the meaning of amplitude and period, changing the values of parameters in the algebraic expression of the trigonometric functions	functions and helps to construct a mental model of how a variable affects the graphical representation
3	Conceptual understanding	Doing and presenting the same problem (equation) in the algebraic and graphical representation	Emphasis on understanding instead of memorizing the algorithm for solving equations

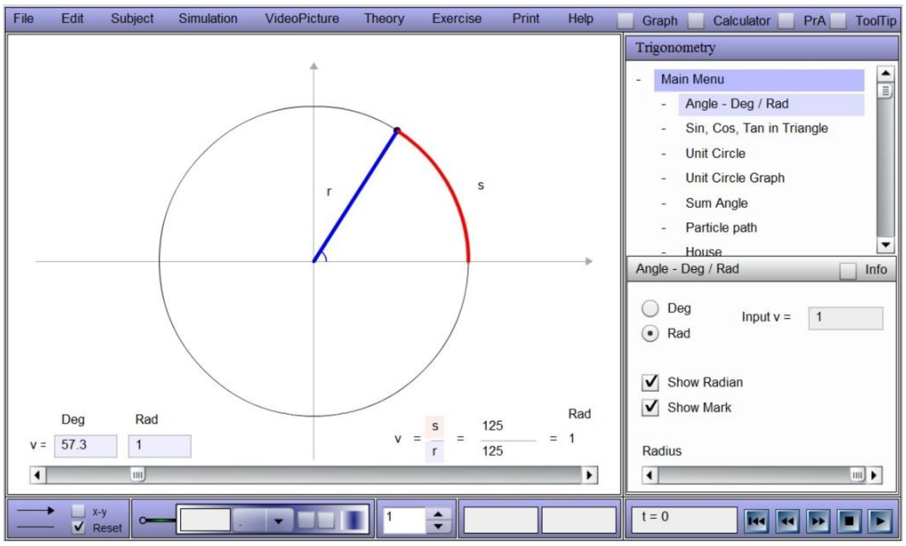
As it can be noticed from table 6, the pedagogical benefits are related to the three important principles of cognitive learning:

- active learning of students,
- exploring and discovering by themselves functions properties,
- meaningful learning through concretization.

We can see some of these activities developed in the program. In the following figures there are examples from the program for each of the activities mentioned in the table 5.

1. Drill and practice

Figure 1. The angle 1 radian in the unit circle represented with SimReal



In figure 1, the interface of SimReal is represented, showing the angle 1 radian in a unit circle. The radius of the circle is shown in blue, which in this case is 1, while the length of the arc which corresponds to the angle 1 radian is shown in red. Below is the definition of the angle 1 radian: *"the length of a circular arc corresponding to an angle of 1 radian is equal to the radius of the arc"*.

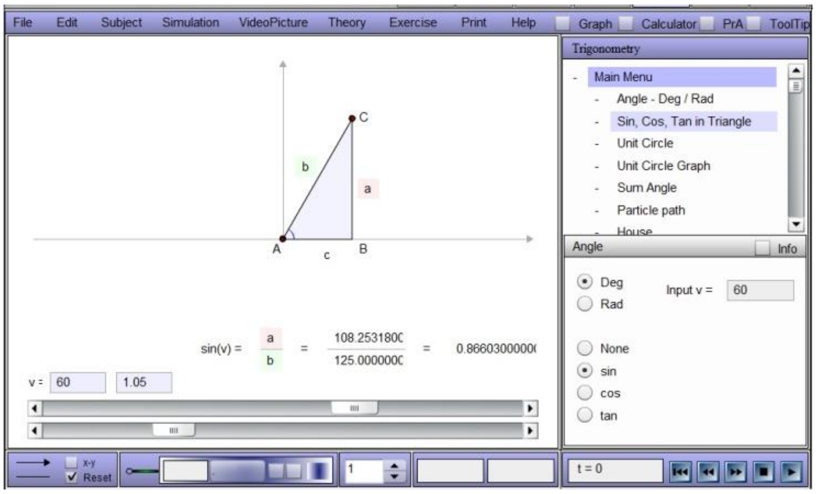
In this case, the length *s* is equal to the radius *r*. We also notice from the picture the formula to convert the degrees in radian: *s/r*, where the length *s* can be calculated with the formula of the arc length for a given angle. We can also see that the angle 1 radian corresponds to the angle 57.3°.

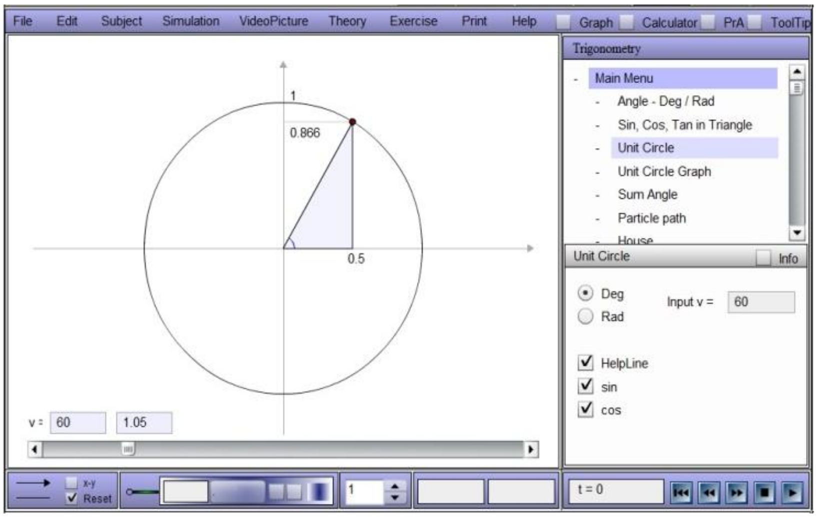
The program helps students to define the meaning of radian (demonstration part), to compare it with the degree, and also to convert angles from one measure to another (calculation part).

They can give an input, in degree or radian, convert it in another measure, and in the same time they can look at the angle represented in the unit circle.

In figure 2 we represent how the introduction to the trigonometric functions in the unit circle has been made, by connecting it with prior knowledge, given that the students already knew the trigonometric functions as ratios of sides in the right angled triangle.

Figure 2. The definition of trigonometric functions in the right angle triangle and in the unit circle



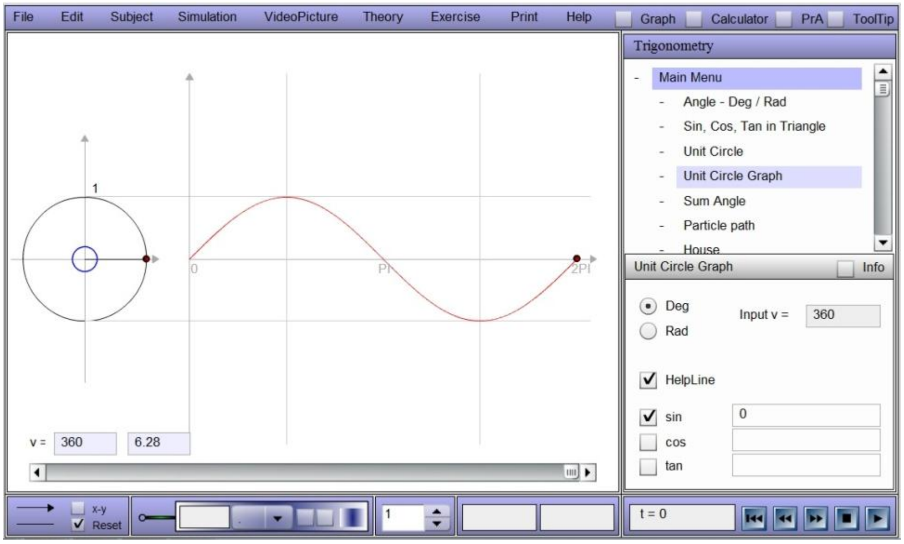


The first picture shows the definition of the sine of the angle A as a ratio *a/b*. In this interface students can interact with the program (interactive part), by changing the angle A and the side c (with the two scrolls). In this case the angle A is 60°, and we can see that the sine is 0.866.

In the second picture, the sine of the angle 60° is represented in the unit circle. We can see that is the same definition: as the ratio of sides in the right angled triangle inside the circle, but in this case we say that the sine of the angle is the side in front of the angle, as the hypotenuse of the triangle is 1 (as the radius of the unit circle). This interface makes it possible to calculate the sine and the cosine for every value of an angle. Students just give an input, in degree or in radian, and take an output, represented visually in the unit circle

Another important functionality of the program related to trigonometry is the graphical representation of functions, in order to help students to understand how the values of the angles in the unit circle can be represented graphically (figure 3).

Figure 3. Drawing the graph of sine function with SimReal

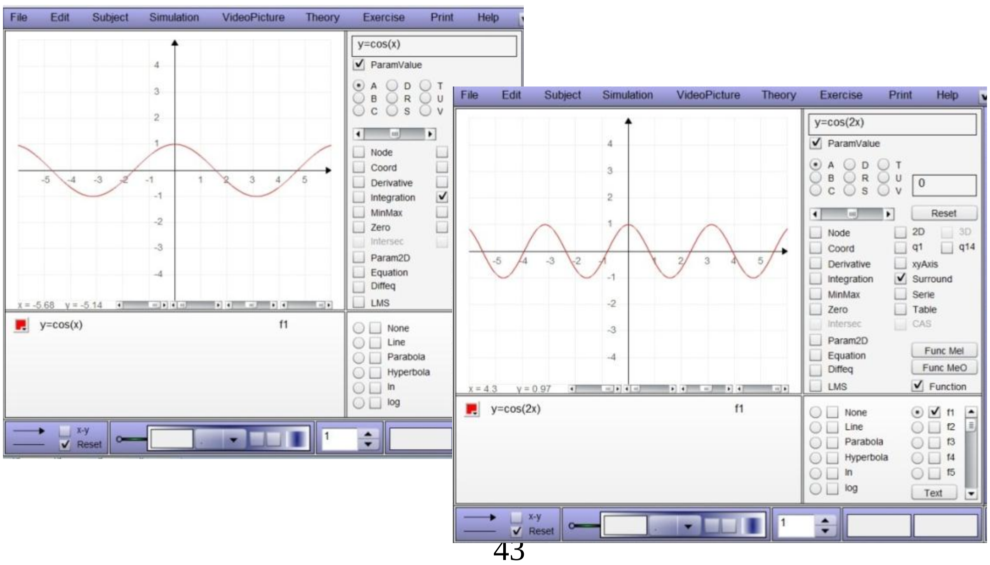


In figure 3 we can see the unit circle and the graphical representation of the sine function, in the case when is given the input v = 360º. Automatically is calculated the value of sine for this angle, which in this case is 0, and also the angle is converted in radian, 6.28. At the same time, the red point shows the angle in the unit circle and the graphic which correspons to it. Students can give the input in the box (in degree or in radian), or they can change the angle using the scroll.

In the same way it can be shown the graphical representation of cosine and tangent function. Students can also choose to view the three graphs in the same interface, by selecting all the three functions (tick in the boxes).

2. Variation of variables

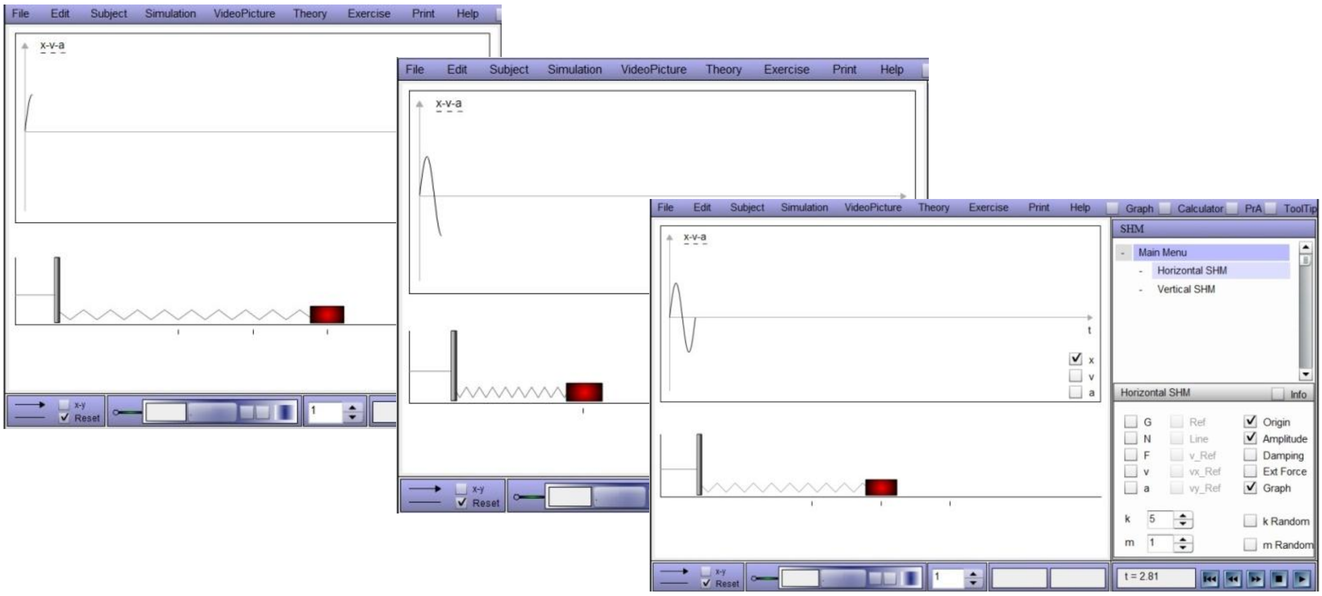
Figure 4. Representation of the graphs in SimReal when we change the variables in the expression of the cosine function from cos(x) to cos(2x)



In figure 4, we show how the program can help students to see the changes in the graphical representation of the cosine function during the variations of the parameters in the algebraic expression of the function. In this case we go from the graph of the function cos(x) to the graph of cos(2x), and we can see that the period of the function becomes two times smaller. This is related to the number 2 before the variable x, and from here students create an idea of how to define the period of a given trigonometric function.

The next example (figure 5) is related to the application of the trigonometric function in physics, it shows the harmonic motion of a given object.

Figure 5. Demonstration of the harmonic movement of the object as an application of the variation of the sine fuction



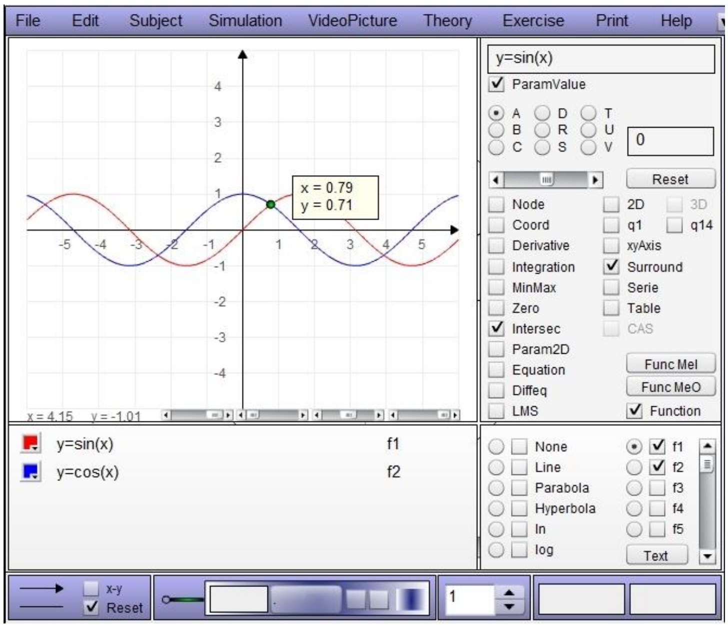
This example helps to explore the meaning of the amplitude in a trigonometric function, in relation to the movement of the object, which can be defined as the distance between the edges where it can move. Also students can realise that the motion of the object is described by a sine function.

3. Conceptual understanding – Solving equations

The last example (figure 6) is related to the demonstration of how an equation can be solved graphically.

During the lessons, the equations were solved first algebraically on the blackboard, and after that students looked at the geometric solutions in SimReal.

Figure 6. Presentation of the graphical solution of the equation sinx = cosx



We have the given equation sin(x) = cos(x), and after presenting the graphical representations of both functions (sinx with red line, cosx with blue line), we chose the option to find the intersection of these two lines, which represents the solutions of the equation (the green dot). This way of solving the equations helped them to understand better the procedure, not just to remember the steps and to give the solutions.

5.1.2 Analysis of students' work in class

After six hours of lessons, students in both groups had an hour of exercises. During this hour students had to work on two tasks:

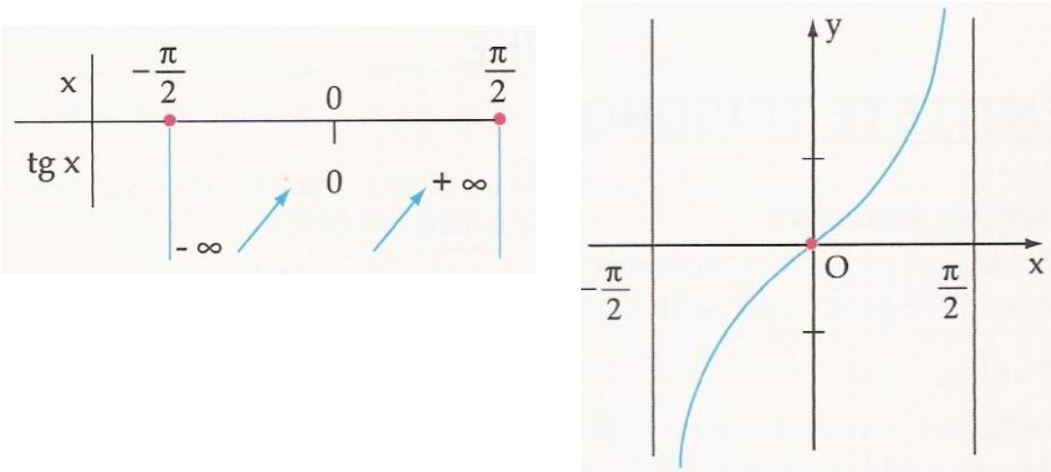
Task 1*: Draw the graph of the trigonometric function y = tgx,*
and then the graph of y = - tgx.

Task 2. *Draw the graph of the trigonometric function y = cosx,*
and then the graph of y= 1/2cosx

These two tasks involve two main activities related to the use of the program (according to table 6): *drill and practice*, i.e. the students had to draw the graph of the tangent and cosine functions, *variation of parameters*, as they had to draw the graph of *–tgx* and ½ *cosx* based on the graphs of *tgx* and *cosx*.

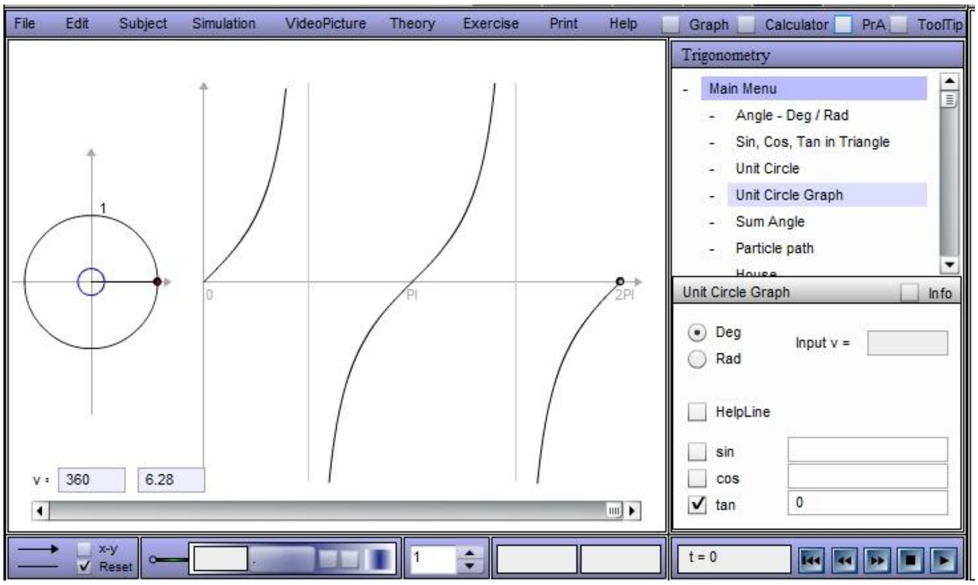
In figure 7 and 8 we present how the graph of tangent has been introduced in the textbook and in SimReal. We want to compare these two representations as they present the graph in two different intervals of values.

Figure 7. The graphical representation of the tangent function from the textbook



As we can see from the table of values, the textbook refers to the interval from -90° to 90°. The main reason for this reference is to define the periodic nature of the tangent function, and if we take values of angles greater than 90°, the graphical representation will have the same shape as represented in this interval.

Figure 8. The graphical representation of the tangent function from SimReal



In figure 8 we see the graphical representation of the tangent function in SimReal. In this case the values in the graph start from 0° up to 360°, giving the students the possibility to discover themselves that the shape of the graph is the same after the 180°, which means that the tangent function is a periodic function, with a period π.

In table 7, we show how we have organised the solutions of task 1 in three categories: correct, partially correct and wrong solution.

Table 7. Categorisation of students' solutions of task 1 during classwork

Category			Description	C	E
1.Correct			The table of values for the function and the values in the graph are correctly represented for both functions	2	4
2.Partially correct	Both graphical representation are described accordingly	Not related to the values of the table	The graphical representations of both functions are correctly done, but not related to the values on the table	1	1
		Imprecision in the representation	Both graphical representations are correct, absence of values and numbers		
	Only one graphical representation is described accordingly	The table of values is correctly represented	Values represented in the table are correctly represented graphically as well	1	1
		Not related to the values of the table	The graphical representations of the function is correctly done, but not related to the values on the table	1	
		Imprecision in the representation	The graphical representation is correct, absence of values and numbers		3
	Only the table of values is represented correctly		The values in the table are correctly represented for the function, the graphical representation is not done	3	
3.Wrong	Some work is done	Table and graph	Table values and graphical representation are partially done, however incorrect	1	
		Graph	Graphical representation are partially done, however incorrect		1
		Tables value	Table values are partially done, however incorrect	1	
	Nothing is done		The task is not done	1	1

These three categories are also divided in subcategories, according to the students' work. The category 'partially correct' includes the cases where the graphs are done both correctly but there is imprecision in presenting the values in the graphs, or when the values in the table are not presented accordingly in the graph. Also this second category includes the cases where only one graph is presented correct, or when only the table of values is correctly calculated.

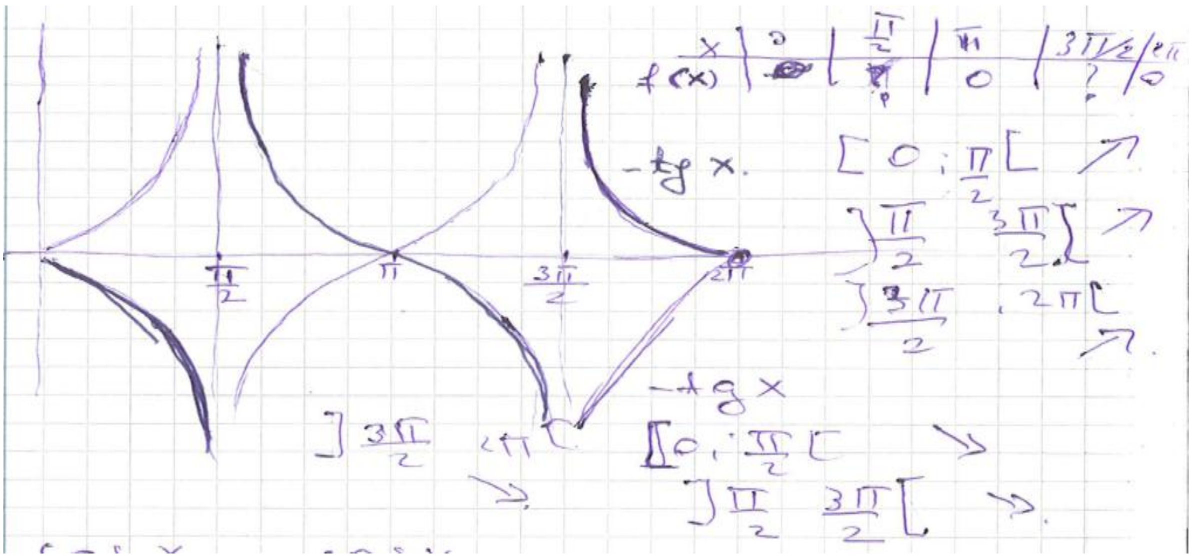
The category of wrong solutions is divided in two subcategories: the first one includes the cases where the students presented some values on the table, or make a graph, but they did not give a correct solution. The second subcategory is about the students who did not do the task and did not present anything in their work.

For each category and subcategory we present the number of students in the experimental and control group. As it can be observed, there are two students in the control group and four students from the experimental who solved the task 1 correctly.

It is interesting to analyse also the case of the partially correct solution, where three students from the experimental group had imprecision in the presentation of their work. And also it is the case where three students from the control group presented only the values on the table and not the graphical representation of the functions.

In the figure 9 we present a correct solution from one of the students in the experimental group (student 5), and after that we can compare it with a correct solution from the control group (student 20), presented in figure 10.

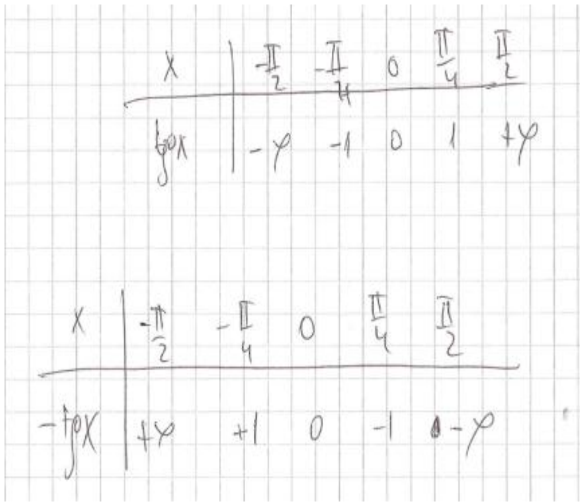
Figure 9. Correct solution of the task 1 of the student 5 in the experimental group

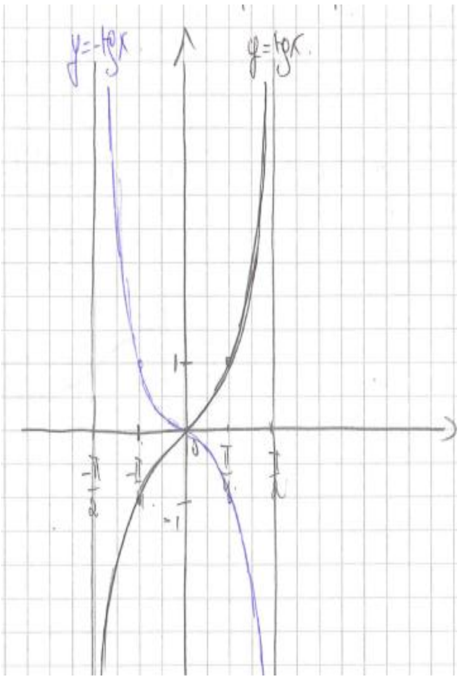


In this work we can see how the students represent the graph of tangent according what is presented in SimReal, angle values starting from 0^o and extended to 2π (360^o).

In figure 10, we present a solution from one of the students in the control group, which is in accordance with the variant of the textbook.

Figure 10. Correct solution of the task 1 of the student 20 in the control group





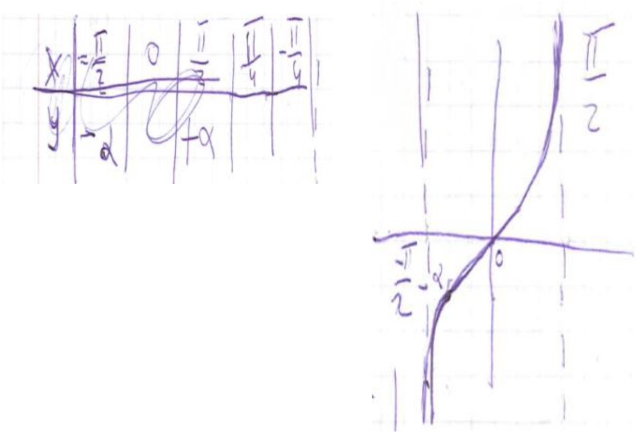
In the solution made by the student in the control group we notice that the interval of values presented on the table starts from -π/2, π/2 (-90° to 90°), and also the graphical representation belongs only to this interval.

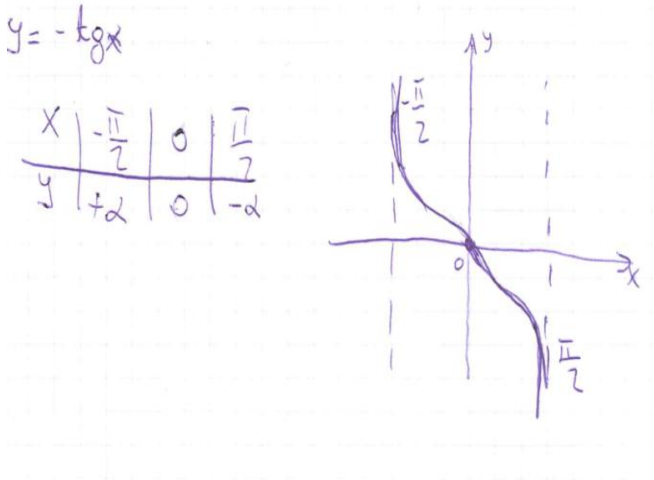
Compared to the correct solution of the student in the experimental group, we categorised both as correct solutions, but the representation made by student 5 shows that he knows what happens with the graph if we extend it after the value 90°.

The other correct solution from the student in the control group is similar to the one presented in the figure 10. In the experimental group, from the four correct solutions, three are similar to the one presented in figure 9. One student (student 11) solved the tasks in two variants, according to the textbook and to SimReal. We present the both solutions in the figure 11a,b:

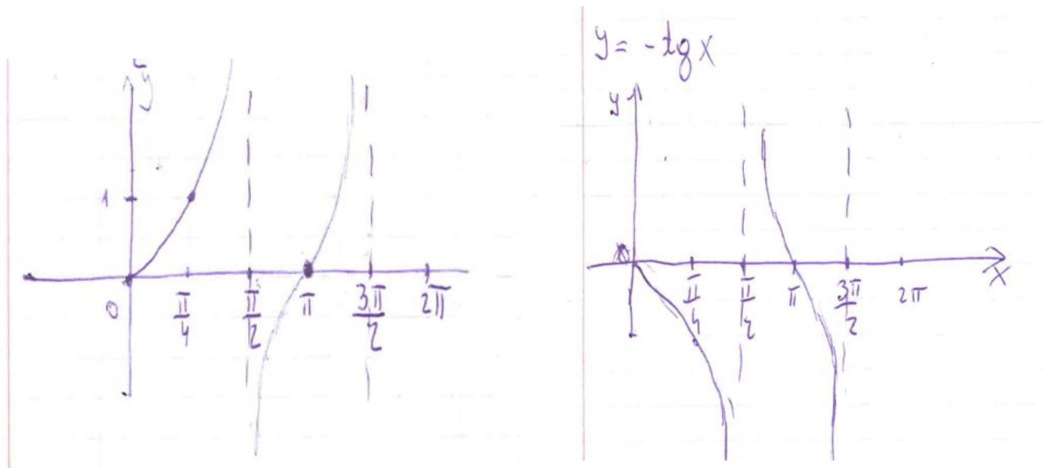
Figure 11. Correct solutions (a,b) of the task 1 of the student 11 in the experimental group

(a) solution of the task refering to the textbook





(b) solution of the task refering to SimReal



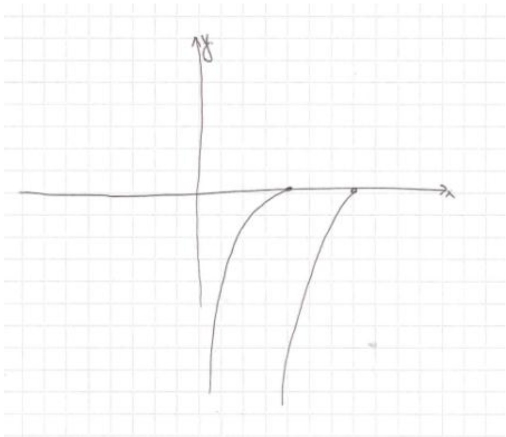
In figure 11(a) we can see how he drew the graphs of tg(x) and – tg(x), as they are presented in the textbook (refering to the figure 7). After that, he asked the teacher for some instructions to learn how he can use the program to draw these graphs. So, he drew the same graphs, as are presented in the figure 11,b, this time according to SimReal (refering to the figure 8).

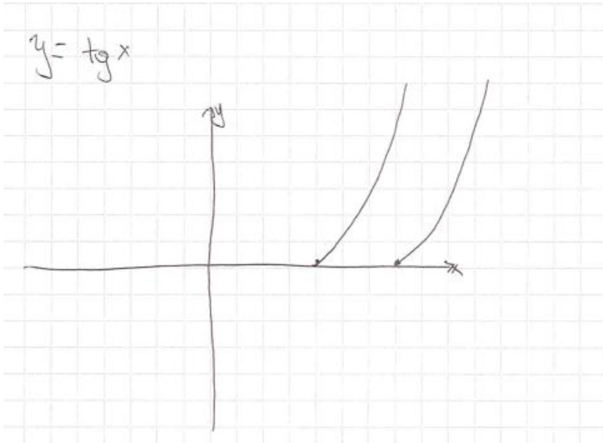
This fact can lead to an important indication related to the sequence of learning using SimReal, as students prefer to refer first to the textbook and to the explanation of the teacher on the blackboard, then they use the program. We will discuss this issue later, in the results related to interview responses of students.

Now we are going to show some results for the category of partially correct solutions. It is important to mention the case where the student drew the graph of tg(x) but it was an imprecision in the representation and a case of absence of values and numbers in the graph. And this was done by three students in the experimental group.

In figure 12 we can see a solution from one of the students in the experimental group, who drew only the shape of the graph, without referring to the numbers.

Figure 12. Partially correct solution of task 1 of the student 8 in the experimental group

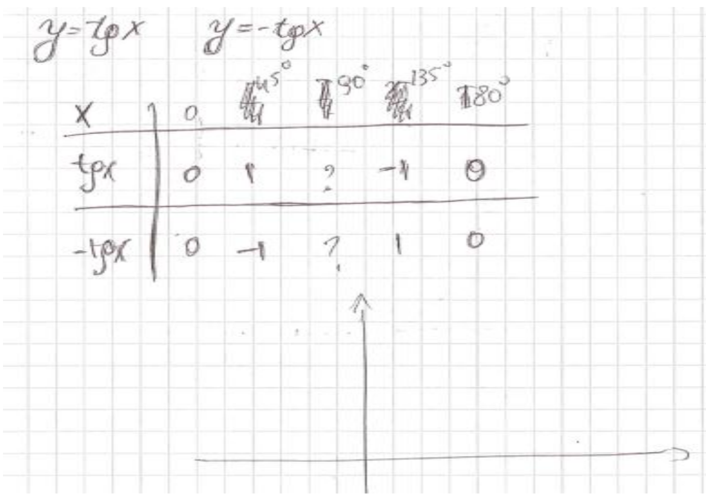




From here emerges one of the issues related to the use of the program during mathematics lessons, as students can focus only in the visualization part, without relating it with the mathematical knowledge which stands behind that presentation (discussed more in the section on limitations of using SimReal).

Now let us look at an example from the category of partially correct solution, done by the control group. There are three students in this group who presented only the table of values for the tangent function. In the figure 13 we show the solution of student 17, from the control group:

Figure 13. Partially correct solution of task 1 of student 17 in the control group



We can observe that the values in the table are calculated correctly, but there is no graphical representation. If we compare the partially correct solutions in the work of students in the two groups, we can indicate that the students in the experimental group had a tendency to work more with the graphical representation, while students in the control group worked more with tables of values. In relation to this fact, may have influenced the use of the program, helping students in the experimental group to focus on the graphical representation of trigonometric functions.

Looking at the category of wrong solutions, we can see that there are two students who did not do anything in their work, one for each group. And in the category of work where something is presented, one student from the experimental group presented something graphically and one student from the control group wrote only the calculations for the values of the table. We can indicate the tendency of the students in the experimental group, even the ones with a low level, to present something graphically, in relation to what they have focused on from the visualizations of the program.

Task 2. *Draw the graph of the trigonometric function y = cosx and then the graph of y= 1/2cosx*

From the analysis of students' work, this task was found to be easier for them comparing to the task 1. In figures 13 and 14, we present the graphical representation of cosine function as it is in the textbook, and how it is treated in the program. The advantage of using the program here is that students can see at the same time both graphs of cosx and cos(x)/2 and compare them.

Figure 14. The table of values and the graph of cosine function from the textbook

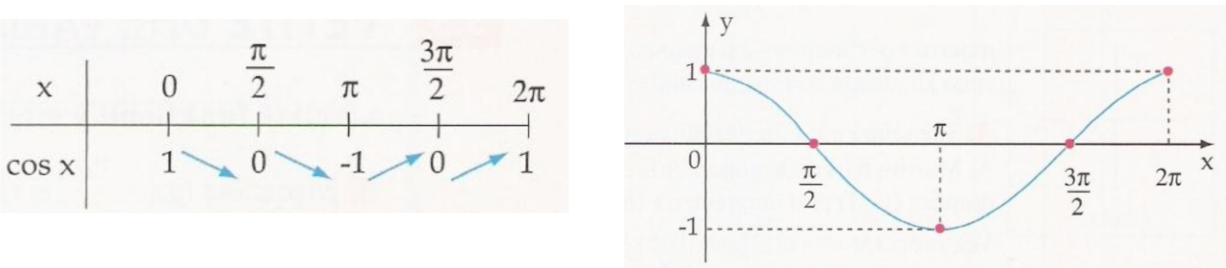


Figure 15. Graph of cos(x) and cos(x)/2 represented in SimReal

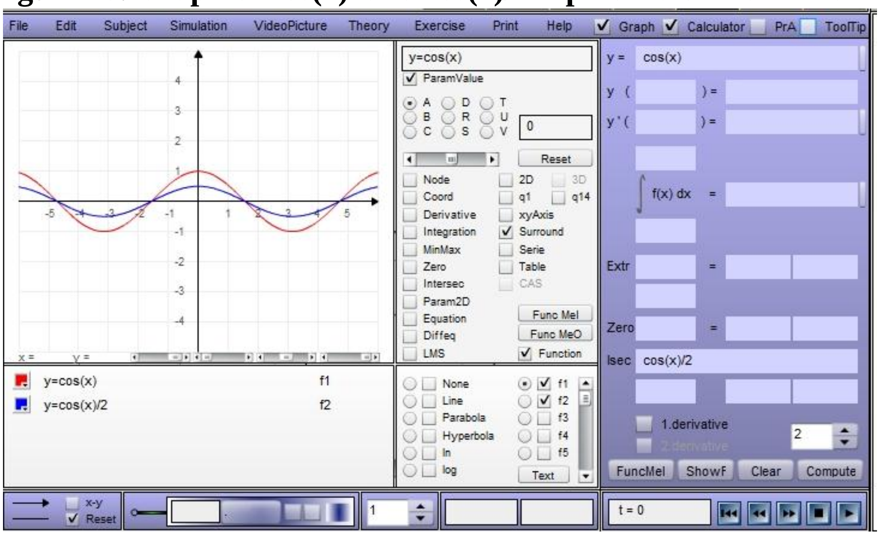


Table 8. Categorisation of students solutions of task 2 during classwork

Category			Description	C	E
1.Correct			The table of values for the function and the values in the graph are correctly represented for both functions	4	5
2.Partially correct	Both graphical representation are described accordingly	Not related to the values of the table	The graphical representations of both functions are correctly done, but not related to the values on the table		
		Imprecision in the representation	Both graphical representations are correct, absence of values and numbers		
			The graph is drawn not according to the numbers		1
	Only one graphical representation is described accordingly	The table of values is correctly represented	Values represented in the table are correctly represented graphically as well	1	1
		Not related to the values of the table	The graphical representations of the function is correctly done, but not related to the values on the table	2	
		Imprecision in the representation	The graphical representation is correct, absence of values and numbers		2
			The graph is drawn partially correct, not according to values	1	
	Only the table of values is represented correctly		The values in the table are correctly represented for the function, the graphical representation it is not done		
3.Wrong	Some work is done	Table and graph	Table values and graphical representation are partially done, however incorrect	1	1
		Graph	Graphical representation are partially done, however incorrect		
		Tables value	Table values are partially done, however incorrect		
	Nothing is done		The task is not done	2	1

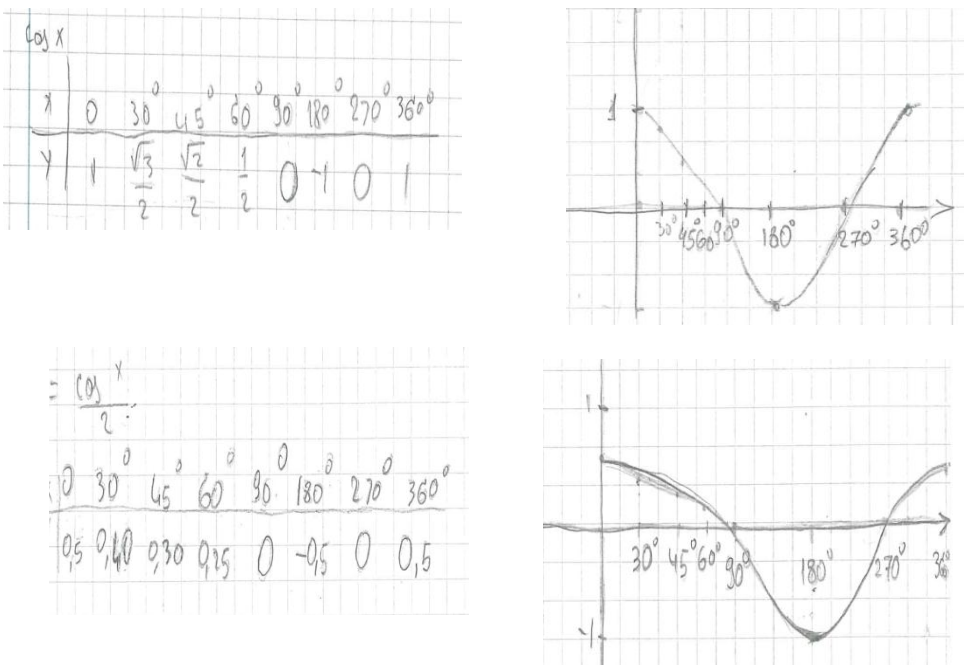
Table 8 presents the categories of solutions, in the same way as for task 1. There are four students from the control group and five students from the experimental who solved correctly task 2.

In the category 'partially correct', there are two students in the experimental group who had a case of absence of values and numbers in their presentation of graphical representation (we show the solution in figure 16).

Figure 16. Partially correct solution of the task 2 from a student in the experimental group (student 3)

Next we present a partially correct solution of task 2 from one of the students in the control group:

Figure 17. Partially correct solution of the task 2 from a student in the control group (student 12)



The student correctly calculated the tables of values and presented the graph of cos(x). But the graph of cos(x)/2 is built partially correct, because the interval from 90° to 270° is the same as in the graph of cos(x).

In the category where nothing is done correctly, there are two students from the control group, and one student from the experimental group. In general, referring to the tables of categories for both tasks, we can say that the students from the experimental group worked better than the students in the control group.

5.1.3 Pre and post test results

After evaluating the pre- and the post-test results for both groups, we organised the scores in tables and using the program SPSS, we made a descriptive analysis of the results, showing the means, the standard deviations, the standard error means and the graphical representations.

In tables 9 and 10, we show the results of the pre- and post-test for the two groups and the mean of the results. To compare the performance of students in the post-test with the one in the pre-test, we calculated the difference of the results, substracting from the post-test scores the pre-test scores.

Table 9. Pre and post test scores in the experimental group

	Exper.Gr	Pre t	Pos t	Diff.
1	Student 1	72	87.5	15.5
2	Student 2	90	97	7
3	Student 3	67	75	8
4	Student 4	68	82	14
5	Student 5	82	85	3
6	Student 6	90	92	2
7	Student 7	77	77	0
8	Student 8	60	71.5	9.5
9	Student 9	79	87	8
10	Student 10	50	44	-6
11	Student 11	81	73	-8
	Mean	**74.2**	**79.2**	**5**

Table 10. Pre and post test scores in the control group

	Cont.Gr	Pre t	Pos t	Diff.
1	Student 12	66	44	-22
2	Student 13	60	68	8
3	Student 14	58	69.5	13.5
4	Student 15	88	86	-2
5	Student 16	90	92	2
6	Student 17	75	62	-13
7	Student 18	73	72	-1
8	Student 19	78	84	6
9	Student 20	91	96	5
10	Student 21	68	81	3
11	Student 22	63	70	7
	Mean	**73.6**	**74.9**	**1.3**

In general, both groups had higher performance in the post-test, compared to the pre-test results. But in the experimental group, we can note that the average of the difference between the pre- and post-test is 5, which is considerably higher than the one in the control group, which is 1.3. This indicates that the students in the experimental group improved more in the post-test than students in the control group.

There are two students in the experimental group, student 10 and 11 and four students in the control group which had lower performance in the post test.

In table 11 are presented the descriptive statistics of the two groups (means, standard deviations and standard error means), calculated with SPSS:

Table 11. Descriptive statistics of pre test scores in the experimental and cont

	Group	N	Mean	Std. Deviation	Std. Error Mean
PreTest	1.Experimental group	11	74.2	12.3	3.7
	2.Control group	11	73.6	11.9	3.6

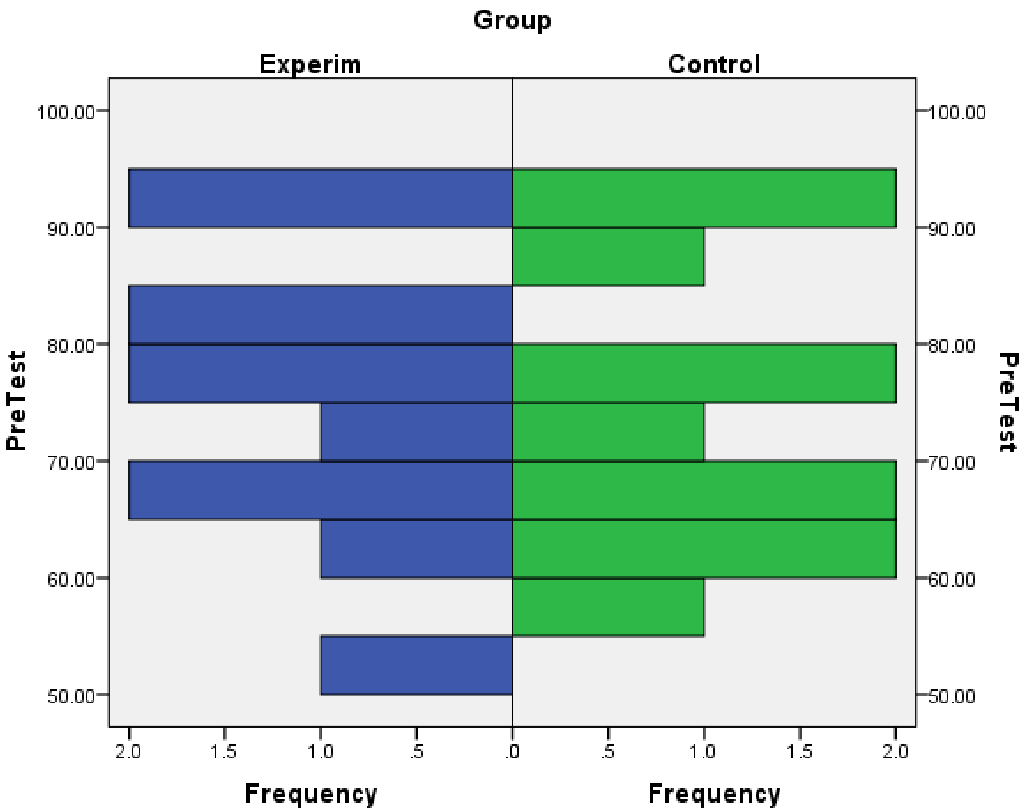
From table 11, we can see that the groups have no significant difference in pre-test results, with a difference 0.6, so we can say that the level of students in general is the same for the two groups. This fact is important for the analysis of the results in the post-test, as we compare the control group which had traditional teaching with the experimental group which participated in experimental lessons using SimReal. It is important as well that the two groups have no significant difference in the mathematical knowledge level.

Given that the maximum of scores in the pre-test was 100 points, the knowledge level of students in both groups regarding the mathematical content tested can be defined as a good level, as their average scores are 73 – 74.

As mentioned in the design of instruments, one of the aims of the pre-test was to relate the students learning to prior knowledge, which would be upgraded during the lessons with the new content. For this reason it is an important fact that both groups had good performance in this test.

In figure 18, we show the graphical representation, according to the frequency of points gained for each group (taken from the SPSS):

Figure 18. Graphical representation of pre test results according to the frequency of points in both groups



We can note that the frequency of points is approximately the same for the two groups, expecially in the intervals of points 65 – 80 and 90 – 95. The other intervals are compensated, for example in the intervals 55 – 60 and 85 – 90 there are no students from the experimental group, but in the interval 80 – 85, there are many students (max frequency) from the experimental group and no students from the control group.

Next we present the descriptive statistics of the scores in the post test (table 12):

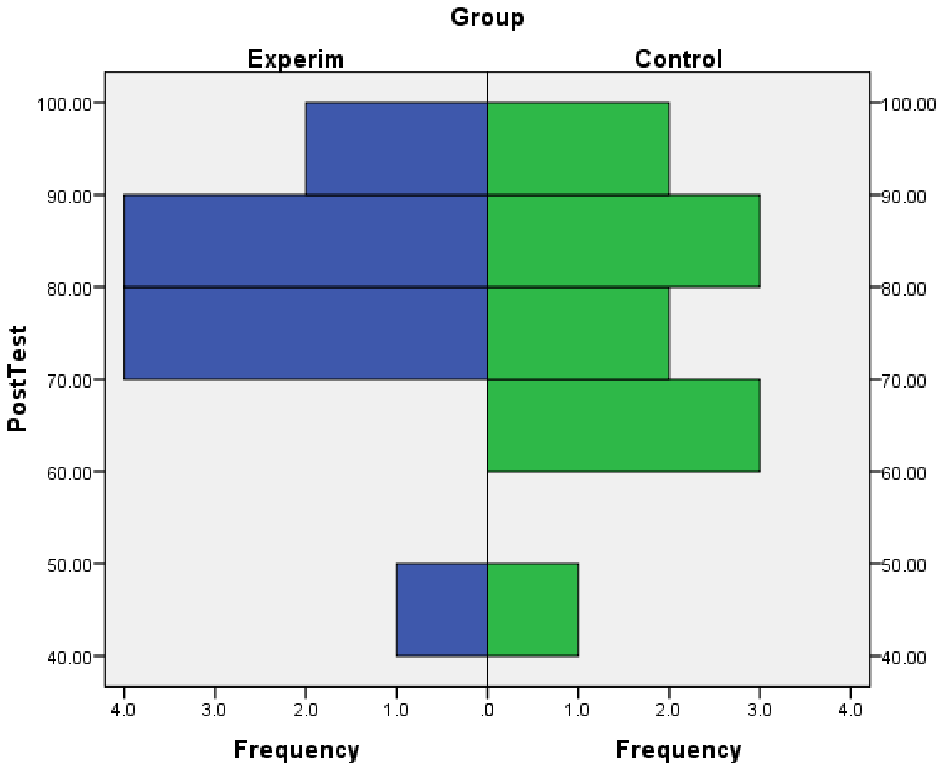
Table 12. Descriptive statistics of post test results in the experimental and control group

Group		N	Mean	Std. Deviation	Std. Error Mean
PostTest	1.Experimental group	11	79.2	14.2	4.3
	2.Control group	11	74.9	14.9	4.5

In table 12 we note that the experimental group has higher mean compared to the control group, with a difference of 4.1. This shows an important result for the study, as we can say that the experimental group had better performance than the control group in the post-test.

In figure 19 we shown the graphical representation of the post test results, according to the frequency of points:

Figure 19. Graphical representation of post test results according to the frequency of points in both groups



From the graph in figure 19 we can note that the frequency of points 80 - 90 is greater in experimental group, and the frequency of points 60 – 70 is greater in the control group, which clearly indicates that the performance of students in the experimental group was better than in the control group.

We have already mentioned that the test results are not a perfect measure for students' learning and understanding of the mathematical content, because there are many other factors which should be considered. However, they are an indicator which allows us to create an idea related to the students' achievement. We decided to go a little more in detail in the students' solutions of the tasks in the post-test and to look for evidence where the perfomance of the students from the experimental group is better than the one of the students in the control group.

5.1.4 Analysing individual tasks from the mathematical post-test

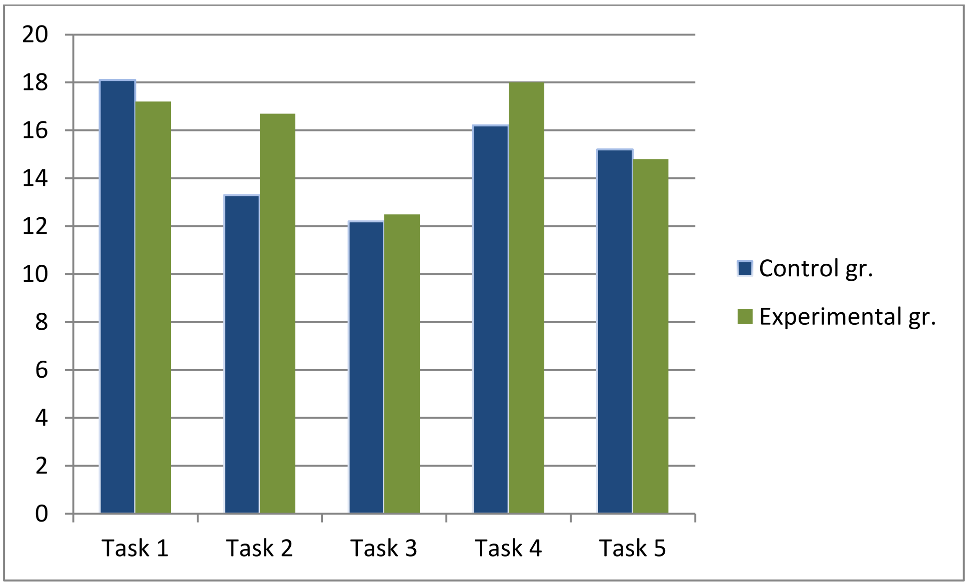
We will take the five exercises of the post-test individually and analyse the performance of the two groups in each of them. In table 13 we show the points for the control and the experimental group.

Table 13. Points of tasks in the post test for both groups

	Control gr.	Experimental gr.
Task 1	18.1	17.2
Task 2	13.3	16.7
Task 3	12.2	12.5
Task 4	16.2	18
Task 5	15.2	14.8

In the figure 20 it is translated the table 13 in the graphical representation, which shows clearly that the experimental group had better performance in the tasks 2, 3 and 4:

Figure 20. Graphical representation of post-test points for each task.



Referring to the table where the tasks of the post-test are described table 4, we can observe how students performed in these tasks:

Task 1. What is an angle 1 radian? Find the values in grade of the angles: 3π, $3\pi/2$, $4\pi/3$, $7\pi/6$.

In the first task, which is a theoretical question and it needs some calculation related to the unit of the angle, the two groups had approximately the same results.

Task 2. Draw a unit circle, and take an angle x. Show the sine, cosine and the tangent of the angle x.Which of these functions are limited? Find the period of each function and draw the graph of sinx.

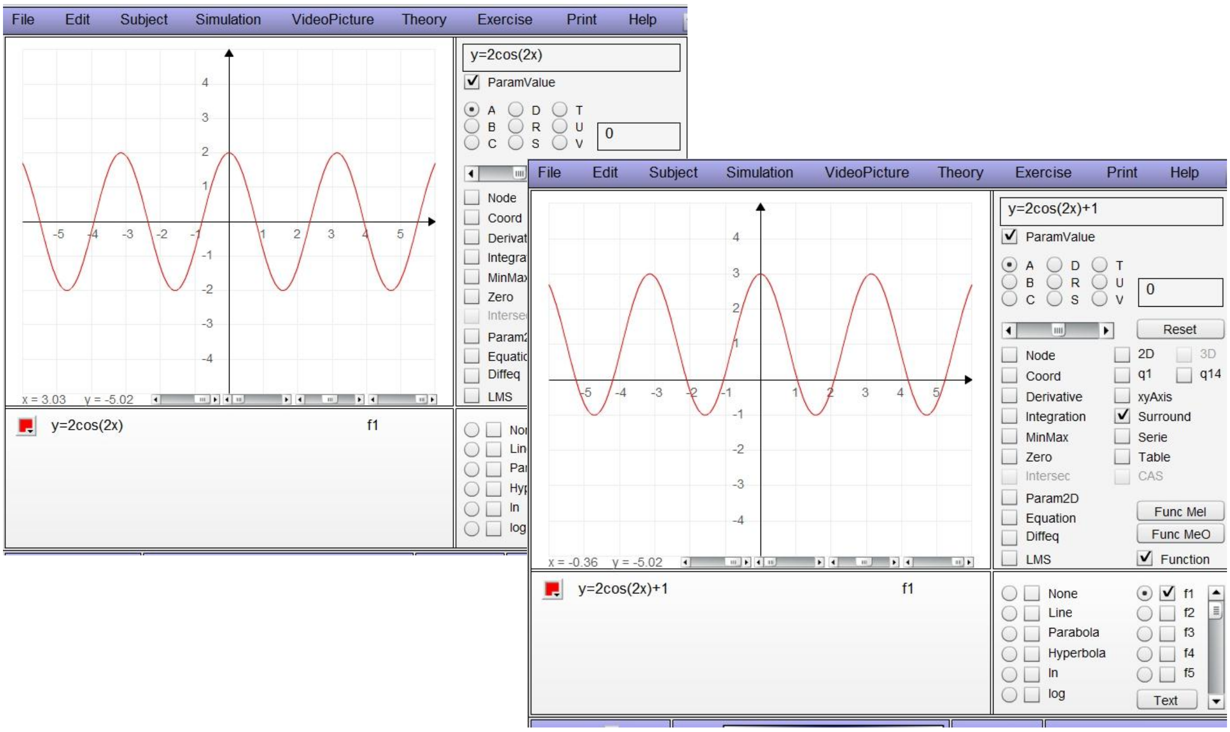
In this second task, students from the experimental group had better results. This is important in relation to the students' understanding of the mathematical content, given that this task includes some essential concepts regarding the variation of these functions, their limits and periods.

Task 3. Given the equation of motion of a point with the function y = 2cos(2x) + 1, find the amplitude, period and the frequency of the motion. Draw the graph of this function.

According to the points, it seems that the students accumulated less points in the third task. Actually, it is one of the exercises which is related to the application of the trigonometric functions in physics, and it differs a little from the standard exercises.

In figure 20, we show how this task has been explained during the lessons, with the help of SimReal:

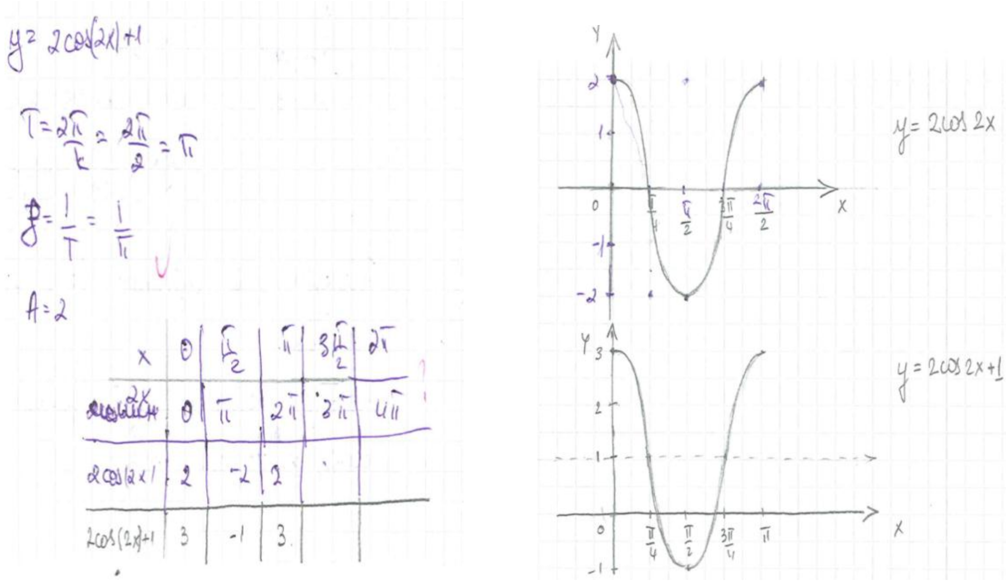
Figure 21. Task 3 of the post test developed with SimReal



It can be noticed that the graph of the function in the second picture is the same as the one in the first picture, but it is moved with one unit up in the Oy axis. Or we can say that the Ox axis has been moved with a unit down. This means that going from the graphical representation of the function 2cos(2x) to the one of the function 2cos(2x) +1, we have just to move the Ox axis with a unit down, in a way for the graphs' values to go one unit up (as they increased by a unit).

In figure 22 we can observe how one of the students in the experimental group solved it:

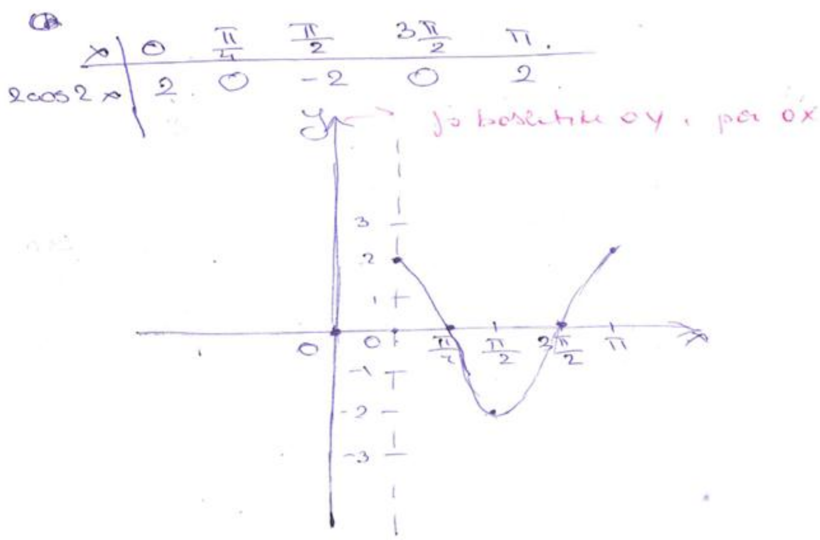
Figure 22. Task 3 of the post test solved by one of the students in the experimental group



We note that she solved it as it was presented in SimReal, she moved the Ox axis from 1 (which is drawn with dotted line) to 0. Also she correctly calculated the period (π) and the amplitude (2) of the function and correctly presented the values on the table.

Next we present a solution for task 3 from one student in the control group (figure 23):

Figure 23. Task 3 of the post test solved by a student in the control group



The values in the table are correctly calculated and presented, also the graph of the function 2 cos(2x) is represented accordingly. But the movement of the axis is wrongly presented, as the student moved the Oy axis to the left with a unit, instead of moving Ox down with a unit.

This case indicates how students can get confused if they try to remember the rule without knowing the meaning of the procedure. Also, it shows how the program influenced the solutions of the students' work in the experimental group, linking representations of the two graphics and making it easier for students to see how it changes after we add 1 in the algebraic expression of the function.

Task 4. *Transform the expressions: cos (90°-α) – sin (180°-α) + tan (90°+ α) + cos (-α), 1/(1-cosx)+1/(1+cosx).*

In this task, where students have to use the reduction formulas to make the neccesary transformations and the main formula of trigonometry, students in the experimental group had an average of 18 points, and the students in the control group had 16.2 points.

Task 5. *Solve the equations: sinx = - cosx , 2cos2x – 5cosx + 3 = 0*

And in the last task, in solving equations, the two groups had approximately the same results, with a difference of 0.4 points.

5.2. Students' attitudes toward using SimReal during mathematics lessons

5.2.1. The responses to the questionnaire of attitudes towars using SimReal in learning mathematics

As already described in the chapter on methods, one of the instruments was aquestionnaire about the attitudes toward using SimReal during the mathematics lessons. There are 11 items in this questionnaire, which we are going to show and analyse. The positive statement related to the use of the program are noted in green, and the negative expressions are noted in red.

1. Computing power makes it easier to explore mathematical ideas
2. I know computers are important but I don't feel I need to use them to learn mathematics
3. Computers and graphics calculators are good tools for calculation, but not for my learning of mathematics
4. I think using technology is too new and strange to make it worthwhile for learning mathematics
5. I think using technology wastes too much time in the learning of mathematics
6. I prefer to do all the calculations and graphing myself , without using a computer or graphics calculator
7. Using technology for the calculations makes it easier for me to do more realistic applications
8. I like the idea of exploring mathematical methods and ideas using technology
9. I want to get better at using computers to help me with mathematics
10. The symbols and language of mathematics are bad enough already without the addition of technology
11. Having technology to do routine work makes me more likely to try different methods and approaches

In the table 14 we shown the points for each student in the experimental group.The mean of points gathered is 41.6 from a total of 55 points, and most students reacted positively and had positive attitudes (table 15).

Table 14. Points from the evaluation of items of the questionnaire

Code of student	Points
Student 1	43
Student 2	49
Student 3	32
Student 4	40
Student 5	41
Student 6	42
Student 7	44
Student 8	44
Student 9	41
Student 10	38
Student 11	44
Mean	**41.6**

Table 15. Categories of attitudes according to the responses from the questionnaire

Category	Number of students
Excellent	1
Very good	8
Acceptable	1
Barely acceptable	1
Not acceptable	0

From table 15 we can observe that students mostly had “very good” attitudes related to the use of the program in learning mathematics. Student 3 had lower points, compared to the others, and is in the category “barely acceptable” attitude. She did not react in a very good way, and looking at the questionnaire filled out by her, we noticed that she totally agreed that students lose much time using the program during the lesson, and that she prefers to do the calculation and to construct the graphs by herself. Another negative response from the students was that mathematics is complex and difficult enough without adding the computer, and this comment came from a student who had difficulties both in mathematics and in using the computer (student 10).

5.2.2 Analysis of the interviews

The interviews had the same content for the five interviewees, specific questions related to the use of SimReal in teaching and learning trigonometric functions.

According to the students' response, we point out these three advantages of using SimReal:

1) Useful to learn mathematics – students responded to the question whether they found the program useful, and where exactly it helped them:

> *"it helped me drawing graphs and to make calculations" (Student 4)*

> *"it was interesting to solve equations in a different way, using the program"(Student 2)*

> *"…drawing graphs, it is easier than doing it with paper and pen" (Student 5)*

2) Easy to use :*"The program is easy to use, and it can be learned very quickly* (Student 4)

3) Helps in the concretisation of the material explained by the teacher -

"The program is more helpful to make clearer the teacher explanation in the blackboard" (Student 4)

"The program helped me to understand better some concepts and problems that I found difficult from the explanation in the blackboard. So, I think it is better when the lesson is explained first in the blackboard, than in the program"(Student11)

Also some disavantages were mentioned by students:

Time constraints - they mentioned that a little more time was needed in order to practice it, and to learn how to use it better.

"It is not a difficult program, but I think we need more time to learn it, and to explore it"(Student 5)

Difficulties in using the computer – some of them found it difficult to learn how to use the program, mainly because they were not very much confident in using the computer.:

> *"I found it very difficult to learn, …more useful was the part when teacher used the program during the explanation* " (Student 7)

5.2.3 Field notes analysis

These data we tried to organise according to some thematic analysis related to the use of the program. To show the results from students' opinions and their feedback which have important implications for the investigation of the role of the program during the teaching and learning process, we will use some thematic analysis of representations of successful use of computer programs, made by Ruthven (2002). This emerged from analysis of what practioners conceive successful use of computers to support mathematics teaching and learning. We divide these thematic analysis into five components:

1. **Participation of students and the ambience change**

Related to this theme, we can say that students seemed to enjoy the lessons with SimReal, expressing that "the time goes quickly, and is less boring", "I like to do mathematics always in this way". Concerning the ambience, there was a change in the general form and feelings of classroom activity, given that the experimental lessons were conducted in the computer science classroom, not in their usual classroom. This gave them a break from the usual teaching routines. And given that they like computer science, they were enthusiastic that they were to work with computers.

2. **Productivity of lessons and routine facitilated**

Specifically in relation to the mathematical content which was the focus of the lessons, drawing a lot of graphs can take too much time during the lessons. This happened because we had to draw first the graph of one trigonometric function, y = sinx, and then to change parameters in the algebraic expression, for example y = 2sinx, and to find out how the graph changes, compared to the first. During the classwork, there were many exercises of this kind, and drawing the graphs with SimReal was a very quick task and at the same time a convincing way.

Another important use of the program which can be defined as productive is the calculations part. For example, before drawing the graphs, students have to find some values and write the table which can be used as a reference for drawing the graph. It was easier to find these values with the help of the program, because it is not like a simple calculator. The students could see the value at the same time in the unit circle and then they could convert it to radian and also look at the point in the graph.

Some students said:

"it is easier to draw the graphs, and you can finish the work faster",

"it helps me for the calculations, I don't need to write down all the calculations when I want to get values for the graphs"

3. **Progression of students and features accentuated**

It was very important to hear from the students about specific areas where they found the program helpful: *"it helps me to understand better the applications of trigonometric functions", "you can see how the graphs change".* Features accentuated - in providing vivid images and striking effects which highlight properties and relations, for example to find out the meaning of the period and the amplitude.

4. **Restraints alleviated and engagement intensified**

This is related to the possibility that students have to explore and experiment with the computer. Even if they do something wrong, they can look at a classmates' computer, or at the projector where the teacher is working, and they can correct what they have done wrong. Regarding the restraints, the program helps in minimazing the effects of factors that indicate the students' participation in the lessons such as difficult tasks, which usually are done only by the good students, on the blackboard, and the others are not given the opportunity to discuss; when students make mistakes on the blackboard, they become insecure and it infuences their attitudes toward doing mathematics, but when they make mistakes in the program, they can just go back and correct it.

Regarding the engagement, students can be more active and more engaged, compared to the traditional lessons, where the teachers explain, students take notes, and when they work in class, not all the students have the opportunity to be active or to discuss topics which they don't understand, thereby causing gaps in their understanding of the subject.

5. **Attention raised and activity effected**

We observed that even the low-knowledge level students worked with the program and were interested in learning how to use it. All the students had the opportunity to be active, as they explore the task on their computers, and can formulate questions for the issues that they don't understand. So the program offers the opportunity for individual work, but also collaborative, as the students can help each other and can discuss issues together.

5.3. Limitations on the use of computer program in teaching and learning mathematics

From the data analysis emerged some limitations in relation to the use of the program during the mathematics lessons. Given that these constraints came directly from the practical work using the tool, it is important to highlight some of them:

1. The students capability of using computers

As mentioned before, students filled out a questionnaire related to attitutes toward the computer, motivation and confidence in using it. We found out that in the experimental group, 10 students had very good attitudes, only one student had "barely acceptable" attitudes (Student 10)

"Mathematics scares me, having to use the computer it becomes a nightmare"
Another student stated (interview):
"I preferred the explanation part, when the teacher uses the program"(student 7)

However this concern is related also in the ability to use the program, SimReal.When students do not know how to use the program, it is difficult to follow the activities taking place in the class, and they can become confused, focusing more on the use of the computer than on the mathematical content. For example, while working with the applications of the trigonometric functions, many students were more focused on the simulation in the program, how to make it work, than in what we wanted to show with it.

2. The short time to learn how to use the program

From the interviews, we can say that students found the program easy to use, and that they learned it quickly, but also they mentioned that it would have been better if there was a little more time to practice it and to learn how to use it better.

Next we show some quotations from students' responses during the interviews and their feedback about the use of the program:

"I wish I could have more time to learn how to use it and to explore it better"

"I had difficulties to learn how to use the program,

2. How the technology is used

The first concern about the way that technology is used by students is related to the point discussed above, i.e. the students' capability of using computers. They will not have any benefit if they don't know how to use it properly.

The second is related to the use of the computer which can give students the confidence that they know how to develop the exercises represented by the program, without even trying by themselves with paper and pen. This was a particular issue regarding the homework assignments, which were often neglected with the justification that they saw what was required of them in the program, and this could be done easily.

A third concern is about the possibility for students to shift the attention from the mathematical content to the tool. They may be distracted by visual elements, focusing on superficial features of practical tasks, rather than on trigonometric concepts. For example, when the program was used in relation to how the sine function is used in physics, by showing the harmonic motion of an object, students were focused on the animation and some of them did not relate it to the mathematical content. And also, many students liked to explore the program, but this distracted them from the explanation of the teacher or the discussions related to the mathematical content.

3. Difficulty of looking at the screen and taking notes at the same time

Related to the requirement of working with pencil and paper, we found it difficult for students to stare at the screen and to write on their notebooks at the same time.

4. The change of ambience and activities:

"it doesn't seem to me that we are doing mathematics" (Student 4)

According to this study, even if the changing of the general form of classroom activities had positive effects on the students' attitudes toward the learning process, we cannot say that they were affected positively in the learning of mathematics for all. Since the experimental lessons were conducted in the computer science classroom, students were enthusiastic that they were to work with computers. But they pointed out that they did not feel they were doing mathematics. This can make them not take the subject seriously.

6 Discussion and Conclusions

After analyzing the data and presenting the findings in relation to the participants' performances and their attitudes toward using SimReal during trigonometry lessons, we present a discussion of these results (6.1) based on review of literature and the theoretical framework described in the chapter 2.

In section 6.2, we refer to the three research questions and give answers according to the findings and the discussion about them. In addition, we make some suggestions for further research and reflections in relation to the conclusions and their validity.

6.1 Discussion

Given that the computer program used in this study, SimReal, has been recently developed, there are few evidences from other studies about its use in teaching mathematics. We can mention the studies of Hogstad and Brekke (2010), and Brekke (2009), who used the program in this field, and concluded that it has positive effects in students learning.

Also, there are no studies which tested technology in teaching and learning mathematics in Albanian schools (as far as we can tell). As mentioned before, it is a new practice in the education system in this country.

The discussion of the results is based on the two components of this study: the integration of SimReal in teaching and learning trigonometry and the investigation of this practice. We are going to relate the evidence from the findings for these two components to the theoretical approach.

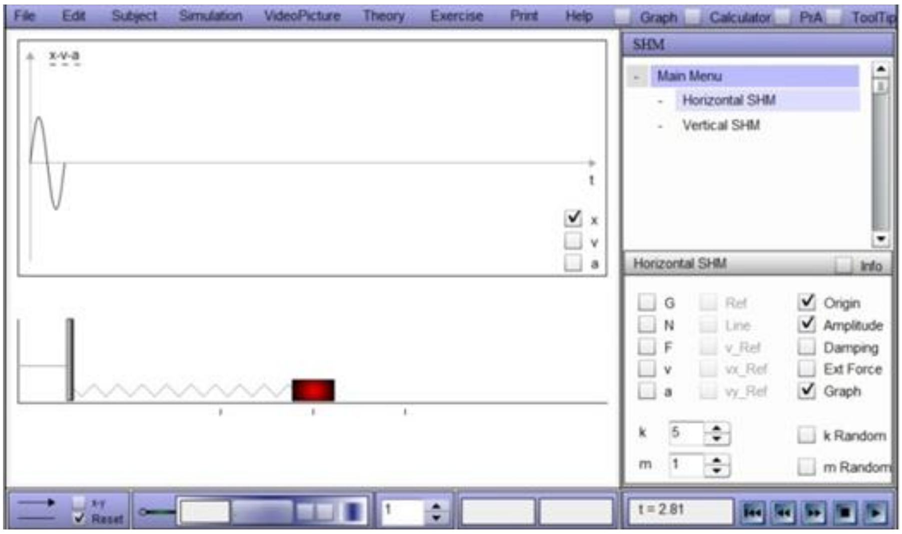
6.1.1 SimReal as a cognitive tool

Following the definition presented by Pea (1987) about cognitive tools, we are going to present the functionalities of SimReal, as a cognitive tool:

1. *The environment of the tool*

SimReal provided an environment where students had the possibility to observe the applications of the mathematical content immediately. One example is the application of the trigonometric functions in physics, where the program showed the harmonic motion of a given object (figure 24):

Figure 24. Presentation of the horizontal harmonic motion of an object in SimReal

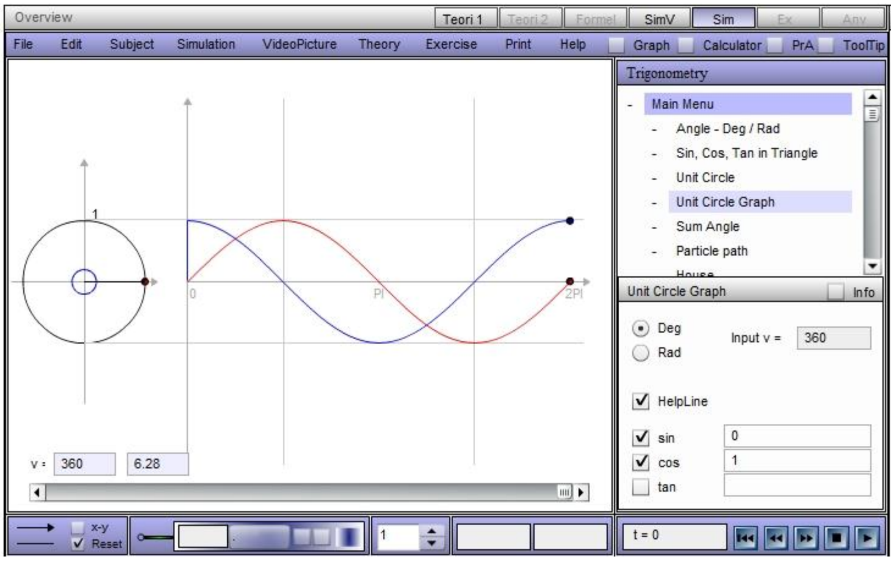


In figure 24, at the lower part, there is an object making a harmonic motion. In the upper section, we observe the curve moving according to the graphical representation of the sine function. In this animation offered by SimReal, students had the possibilty to make a relation between the graphical representation of trigonometric functions and their application in presenting harmotic motion in physics.

2. *Development of conceptual fluency*

By using it for routine computations, SimReal helped students to become more fluent in performing activities, such as convertion of measures of the angle, calculations of trigonometric functions for different values of angles and drawing graphs (figure 25).

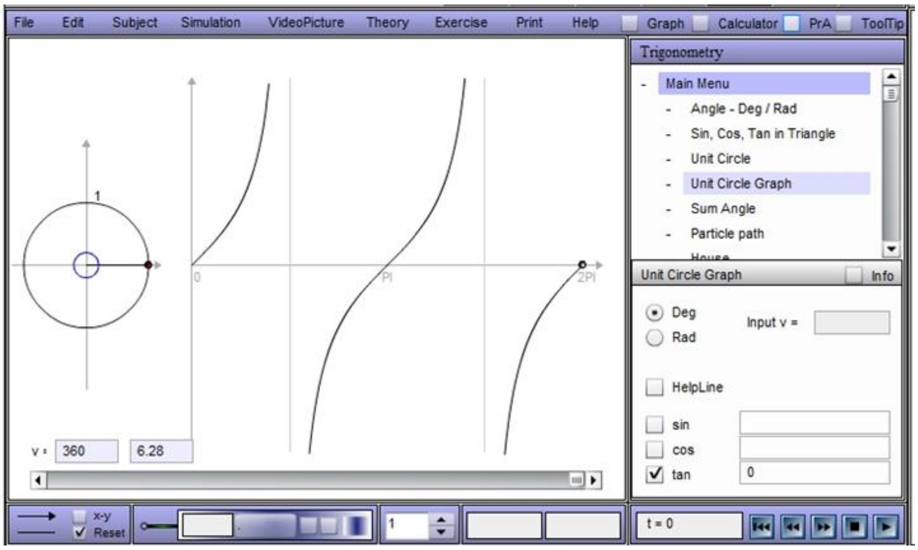
Figure 25. Presentation the angle 360^o in the unit circle and in the sine and cosine graph



3. Mathematical exploration

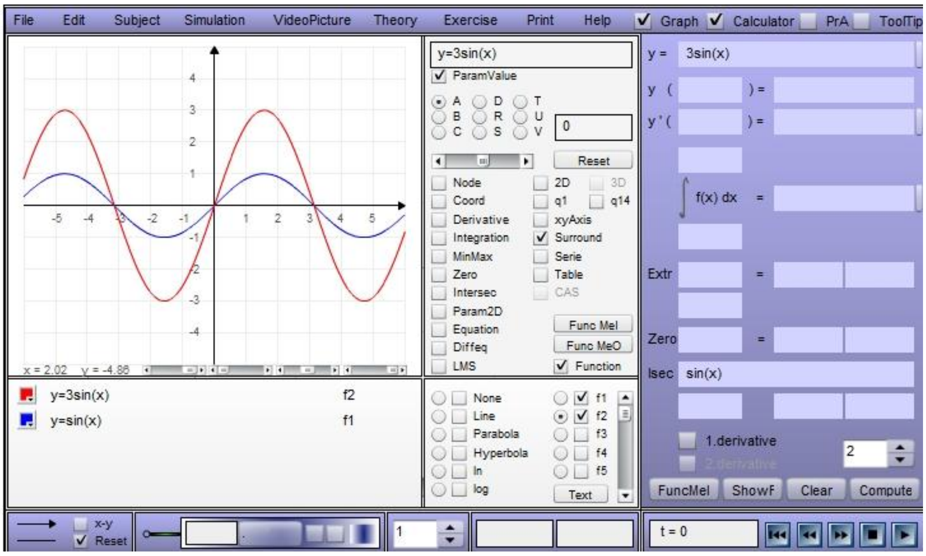
SimReal helped students to explore mathematical features and to discover the meaning of important properties of trigonometric functions, as period and amplitude. For example, it can be observed in figure 26 that the tangent function presented in the graph is periodic, which means that after the angle 180^o , the graphical representation is the same.

Figure 26. Graphical representation of tangent function



Also in figure 27, we see the interface of SimReal which made it possible for students to observe the change of the amplitude in a trigonometric function.

Figure 27. Graphical representation of the functions sin(x) and 3sin(x)



It can be observed in figure 27 that the blue line represents the graph of sin(x), where the amplitude is 1 (the greatest value in the Oy axes), and the red line repesents the graph of 3sin(x) where the amplitude is 3.

4. Different mathematical representations

This functionality of SimReal is related to the use of multiple representations or, as Duval (1999) defines them, registers of representations. What is important for the students' understanding is the change of these registers, especially the second type of change, named as conversions. The connection of registers of representations and conversions were performed by students with the help of SimReal. Concretely, they had the possibility to construct the graphical representations of trigonometric functions, to change the values in the algebraic expressions of these functions and to observe the immediate changes in the graphical representation.

SimReal offered visualisations and interactivity, which helped students to visualise the graphs of the trigonometric functions, and also to engage them in exploring and discovering important features in relation to the content in focus.

6.1.2 Use of SimReal

Regarding the use of SimReal during mathematics lessons, we present three areas which we found important to discuss and which can have implications for the role of the program in the teaching process.

1. Difficulties around visualisation

The first area is related to difficulties around visulazation. As we presented in the review of the literature, Guzman (2002) mentioned two difficulties in relation to the use of visualization to represent mathematical content: it can lead to errors, if the interpretation of the figure is incorrect, and it is difficult when the students do not have the neccesary knowledge and preparation to interpret it.

From the analysis of the students' solutions of tasks in the experimental group, it has been observed that some of them represented the graphics of functions, but they were not able to construct it refering to the values of the functions. In the graphical representation, there was absence of numbers, which indicates that the students only drew the graph as it was visualized in the program, without understanding the procedure.

2. Students' characteristics

Another important area, which has implications especially for students' attitudes toward the use of the program during the lessons, is related to three aspects of students' characteristics:

- The non-visual nature of some students (according to Presmeg, 1986), who do not find visualization very useful, or need to draw the graphs by themselves in a way to understand it better.
  As presented in the findings, one of the students in the experimental group (Student 3) pointed out that it was easier for her to learn mathematics without the program.

- Some students were not confident in using computers, which contributes to their insecurity to use the program in the learning of mathematics. As Fogarty et. al (2001) concluded in their study, computer attitudes are influential in facilitating the active engagement of computer-related activities in mathematical learning. As we have shown in the section on the results, one of the students was not confident in using the computer, and also he had difficulties in learning mathematics, and said, "*Mathematics scares me, having to use the computer it becomes even worse*" (Student 10)

3. *Sequence of teaching*

In the study of Ross et al. (2011), was found that students who experienced the tool after the teaching of core concepts, learned more than students who began with technology-supported simulations. A similar reaction had also the students in the experimental group. As presented in the finding from the analysis of the tasks in classwork, the student 10 solved the task in two variants. First he solved it refering to the textbook, than he tried to use the program. Also from the interviews, students expressed that it was easier for them when the teacher first explain the content in blackboard, and after look at the concretisation in the program:

> *"It is easier to look at the demonstration in the program after the explanation in the blackboard"* (Student 7)

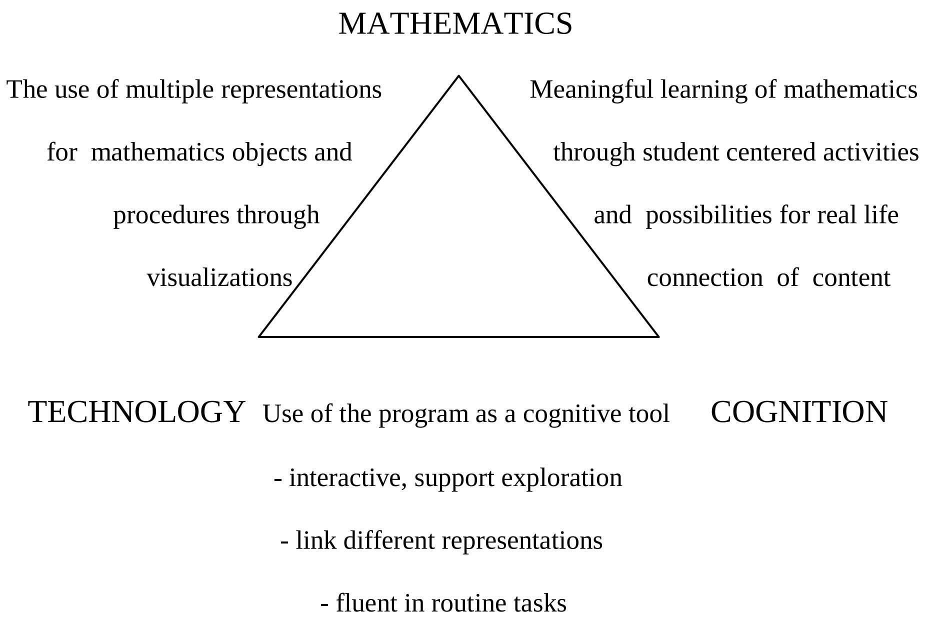
6.2 Conclusions

The main aim of the research is to investigate the roles of the computer program, SimReal, in teaching and learning trigonometry in upper secondary school. We present the answers of the research questions, based on the results from data analysis:

1. *What are the roles of the program in the learning and understanding of trigonometry?*

We can say that this research, together with previous studies on the use of computer programs for education purposes, shows the importance of using these tools in enhancing students' learning. Interactivity and visualizations are two important components offered by SimReal. They help students to understand better the mathematical content related to trigonometry, as shown by the results from the experimental group of students in the post-test compared to the results of the control group students.

Also from the qualitative analysis of students' work we noticed conceptual understanding of the mathematical content, mainly supported by the use of the program. This qualitative analysis is based on the model below, inspired by the constructivist theory, and has also been introduced in the theoreical framework.



This model consists in the relation of three elements: mathematics, technology and cognition.

Starting from the use of technology as a cognitive tool, we can say that SimReal supports the interaction between students and the program. In fact, the students had the possibility to explore the properties of trigonometric functions and to link different representations, changing from algebraic to the graphical register. The program also offered fluency in routine tasks, as calculations of trigonometric functions for different angles, and changing measures of angles in radian and degrees.

Second is the relation between mathematics and the use of technology, where the main benefitial part is the visualization, especially of the graphics of trigonometric functions, which helped students to focus and to visualize better how they are presented and how the graphs are transformed when the parameters are changed in the algebraic expression.

The third and the most important component of the triangle is the connection of mathematics and cognition, which is related to the meaningful learning of mathematics and student-centered activities during the lessons. Meaningful learning is supported by technology offering real life context, for example, the application of trigonometric functions in physics. Interesting to observe is that this way of learning also offers an active engagement of students during the lessons.

2. *What are the students' attitutes toward using the program in the classroom?*

This study reports in general positive attitudes toward the use of the program. Students enjoyed the experience and they expressed the desire to implement the program in every mathematics lesson.

As indicated by students' responses during the inteviews and in the questionnaire, as well as their general comments in relation to the use of the program, they appear to be receptive to use the program to support their exploration of mathematics and also they were more engaged to do mathematics with SimReal during the lessons.

3. *What are the limitations on the implementation of the program?*

Making these tools part of the educational curriculum is not simple. There are many issues and limitations to be considered, which are mostly related to the way how technology is used in the classroom. The students' possibilities to reflect over the content are reduced by the elements offered by the computer. Technology offers a variety of different distractions, which results in students doing other things than mathematics in the classroom. Also they can shift the attention from the mathematical content to the use of the program. As stated by the National Council of Teachers of Mathematics (2000), "students can learn mathematics more deeply with the appropriate use of technology" (p.25).

The results of this study are related to previous ones by extending them, supporting their conclusions, proposing new directions or challenging the generality of previous results.

This study is original and it has not been carried out before, as it consists in the implementation of a recently programmed tool, in an upper secondary school in Albania, where this way of teaching and learning mathematics is a completely new experience.

6.3 Limitations of the study

This study and its results may have some limitations, namely:

1. Sample constraint : Participants came from one particular sample place, a private high school;

2. Time constraint : Students needed more time to be familiar with the program and to explore it in order to be able to use it appropriately during the teaching and learning process;

We should mentioned that the use of technology for learning purposes is a new practice for Albanian schools and education. The first steps in the process of implementing a new didactical tool are always difficult, and this also can influence the results of the research study, because it is a complete new experience for students to use computers as tools for learning and exploration.

Despite these limitations, we hope that this study will have implications for teachers to use the technology, and particularly SimReal, in their teaching. With access to technology, the curriculum might be extended to provide students with the opportunity to adress a wider range of practical situations and to make connections between the study of mathematics and other sciences.

6.4 Further studies

To have a full understanding of the impact of SimReal, further research should broaden the outcome measures in both cognitive (e.g., different, specific mathematical areas) and affective (e.g., attitude, anxiety, self-confidence) domains as a way to provide comprehensive information on how this tool may affect mathematics learning.

Also future studies are necessary in relation to the teacher role in the environments where technology is used for educational purposes, and to teacher education, as an essential point for the process of promoting technology integration into meaningful learning of mathematics.

7 Pedagogical Implications

7.1 Implication for me as teacher and researcher

1) From idealist to realist: facing the difficulties prepares you for future obstacles

At the beginning I was enthusiastic looking to the fuctionalities of SimReal. The first impression was that it would be wonderful to concretize mathematic topics. But I did not take into consideration the fact that it would not be easy to implement it in teaching mathematics. It was a facilitating tool as I already have the mathematical knowledge. However, it is another thing to use the program to construct this knowledge. Also I did not take into consideration the fact that many students are already used to learning mathematics in the traditional way, especially those called "non-visual" students, who prefer to do everything by paper and pencil, in order to understand it better.

2) First experience as a researcher and as teacher in upper secondary school

This research process enabled me to travel in a path that led to my professional growth and development. Each phase of doing the research had an important implication for me and my future as a teacher. Structuring, analysing and developing such a work was not easy, but at the end I found myself more able to analyse practices which take place during lessons and also more analytical to the content which should be transmitted to the students. During the practice, as I was focused all the time on students' understanding of the content, it helped me to realise how students learn and how we can promote meaningful learning.

Going from the theoretical rationale to their practical use had strong implications, i.e. finding out that you can substantially learn from practical work and new difficulties and issues to be discussed emerge from there.

7.2 Implication for mathematics educators

This study intends to have two important implications for mathematics teachers, expecially in Albania:

The first one is related to the demonstration of the role of technology, specifically of SimReal, in teaching and learning mathematics, so they can have an example and results from such practice, and can consider the integration of these programs, especially SimReal, in their teaching process.

The second implication is related to the different issues which should be taken into consideration before planning to implement such tools in mathematics teaching and learning.

8 References

Aas, O.T. (2005). ParAbel is a Norwegian success – it delivers mathematics and physics at high school level on the Internet and illustrates how animations can bring quality to e-learning. *Higher Colleges of Technology Abu Dhabi*, UAE, November 19-21. http://www.admc.hct.ac.ae/emel2005/presenters.aspx?Ola%20T.%20Aas

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics*, *52*(3), 215-241.

Ashburn, E. A., & Floden, R. E. (2006). *Meaningful learning using technology: What educators need to know and do.* New York: Teachers College Press.

Blackett, N., & Tall, D. O. (1991). Gender and the versatile learning of trigonometry using computer software. In F. Furinghetti (Ed.), *Proceedings of the 15th conference* of the *International Group for the Psychology of Mathematics Education*, *1*, 144-151. Assisi, Italy.

Boaler, J. (2000). Mathematics from Another World: Traditional Communities and the Alienation of Learners. *The Journal of Mathematical Behavior, 18*(4), 379-397.

Bransford, J. D., Brown, A. L., Cocking, R. R., Donovan, M. S., & Pellegrino, J. W. (Eds.). (2000). *How people learn: Brain, mind, experience, and school* (Expanded ed.). Washington, DC: National Academy Press. Retrieved from: http://books.nap.edu/openbook.php?record_id=9853&page

Brekke, M. (2009). Effects of bringing computer technology in physics and mathematics into the classroom. *London International Conference on Education* (LICE), 9-12, London.

Bryman, A. (2008). *Social research methods*, Oxford University Press.

Byers, P. (2010). Investigating trigonometric representations in the transition to college mathematics. *College Quarterly*, *13*, 2.

Chiappini G., Bottino R.M. (*1999).* Visualisation in teaching-learning mathematics: the role of the computer. *Proceedings of Computer Graphics and Visualization Education, 147-152.* Coimbra, Portugal: Eurographics Workshop.

Choi-Koh, S. S. (2003). Effect of a graphing calculator on a 10th-grade student's study of trigonometry. *The Journal of Educational Research, 96*(6), 359-369.

Christy, D. T. (1993). *Essentials of precalculus algebra and trigonometry* (5^{th}ed.). Dubuque, IA: Wm. C. Brown.

Churchill, D. (2005). Learning objects: an interactive representation and a mediating tool in a learning activity. *Educational Media International*, *42*(4), 333-349.

Davis, J. (2005). Connecting procedural and conceptual knowledge of functions. *Mathematics Teacher*, *99*, 36-39.

Dreyfus T. (1995). Immagery for diagrams. In R. Sutherland & J. Mason, (eds), *Exploiting mental imagery with computer in mathematics education*, NATO ASI Series, Springer - Verlag, Berlin, Series F, *138*, 3-19.

Drijvers, P., Kieran, C., Mariotti, M-A., Ainley, J., Andresen, M., Chan, Y., Dana-Picard, T-D., Gueudet,G., Kidron, I., Leun, A., Meagher, M., & Leung, A. (2010). Integrating technology into mathematics education: Theoretical perspectives. In C. Hoyles & J. B. Lagrange (Eds.), *Mathematics Education and Technology-Rethinking the Terrain*, 89-132: Springer US.

Duffy, T. & Cunningham, D. (1996). Constructivism: Implications for the design and delivery of instruction. In D. Jonassen (Ed.), *Handbook of research on educational communications and technology*. New York: Simon & Schuster.

Duval, R. (1999). Representation, vision and visualization: Cognitive functions in mathematical thinking. Basic Issues for Learning. In: *Proceeding of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (21st),* Cuernavaca, Morelos, Mexico.

Faugli, B. (2003). Evolution of education, theories of learning and computer supported learning. Research report, 20, Elverum: Høgskolen.

Fogarty, G., Cretchley, P., Harman, C., Ellerton, N., & Konki, N. (2001). Validation of a questionnaire to measure mathematics confidence, computer confidence and attitudes towards technology for learning mathematics. *Mathematics Education Research Journal*, *13(2)*, 154-159.

Galbraith, P., & Haines, C. (1998). Disentangling the nexus: Attitudes to mathematics and technology in a computer learning environment. *Educational Studies in Mathematics, 36*(3), 275-290.

González, G., & Herbst, P. (2009). Students' conceptions of congruency through the use of dynamic geometry software. *International Journal of Computers for Mathematical Learning*, *14*(2), 153-182.

Grabingar, R. S. & Dunlap, J. C. (1995). Can multimedia support constructivist learning? *Journal of Teaching Review, 4*(2).

Greeno, J.G., & Hall, R.B. (1997). Practicing representation: Learning with and about representational forms. *Phi Delta Kappan, 78*, 361-367.

Guzman, M. (2002). The role of visualization in the teaching and learning of mathematical analysis. In: Proceedings of the International Conference on the Teaching of Mathematics (at the Undergraduate Level), 2nd, Hersonnisos, Crete, Greece.

Heid, M. K. (1997). The technological revolution and the reform of school mathematics. *American Journal of Education 106*(1), 5-61.

Henning, P. (1998). Everyday cognition and situated learning. In D. Jonassen (Ed.), *Handbook of Research on Educational Communications and Technology*. (2nd. Ed.). New York: Simon & Schuster.

Hogstad, P. H. & Brekke, M. (2010). New teaching methods - Using computer technology in physics, mathematics and computer science. *International Journal of Digital Society* (IJDS), *1*(1), 17-24.

Hohenwarter, M. & Lavicza, Z. (2007). Mathematics teacher development with ICT: Towards an International GeoGebra Institute. *Proceedings of the British Society for Research into Learning Mathematics, 27*(3) 49-54.

Johnson, R. B. & Onwuegbuzie, A. J. (2004). Mixed methods research: A research paradigm whose time has come. *Educational Researcher 33*(7), 14-26.

Jonassen, D. H. (2000). *Computers as mindtools for schools: Engaging critical thinking*. Upper Saddle River, NJ: Prentice Hall, Inc.

Jonassen, D.H. (1994). Thinking technology: Toward a constructivist design model. *Educational Technology*, *34*(4), 34-37.

Jonassen, D.H. (1994). Technology as cognitive tools: Learners as designers [Electronic Version]. ITForum from http://itech1.coe.uga.edu/itforum/paper1/paper1.html

Jonassen, D.H., & Reeves, T. C. (1996). Learning with technology: Using computers as cognitive tools. In D.H. Jonassen (Ed.), *Handbook of research for educational communications and technology*, 693-719. New York: Macmillan.

Lagrange, J.B. & Chiappini, G. (2007). Integrating the learning of algebra with technology at the European level: Two examples in the ReMath project, *Proceedings of 5th Congress of the European Society for Research in Mathematics Education*, Larnaka, Cyprus. Available at http://www.erme-soc.eu.

Lagrange, J.B. (2010). Teaching and learning about functions at upper secondary level: designing and experimenting the software environment Casyopée. *International Journal of Mathematical Education in Science and Technology*, *41*(2), 243-255.

Lawless, K. A., & Pellegrino, J. W. (2007). Professional development in integrating technology into teaching and learning: Knowns, unknowns, and ways to pursue better questions and answers. *Review of Educational Research*, *77*(4), 575-614

Lesser, L. M., & Tchshanov, M. A. (2005). The effect of representation and representational sequence on students' understanding. (G. Lloyd, M. Wilson, J. Wilkins, & S. Behm eds). *Proceedings of the 27the annual meeting of Psychology of Mathematics Education*, 281–296. North America Roanoke VA, PME-NA.

Liang, H.-N., & Sedig, K. (2006). Interactivity of visual mathematical representations: Factors affecting learning and cognitive processes. *Journal of Interactive Learning Research,17*(2), 179–212.

Liang, H.-N., & Sedig, K. (2010). Can interactive visualization tools engage and support pre-university students in exploring non-trivial mathematical concepts? *Computers & Education*, *54*(4), 972-991.

Lotfi, F. H. & Mafi, E. (2012). Efficacy of computer software on trigonometry. *Applied Mathematical Sciences, 6*(5), 229-236.

Lulja, E. & Babamusta, N. (2009). *Matematika 10*, Tiranë, Albania: Pegi.

Lulja, E., Babamusta, N. & Bozdo, Sh. (2010). *Matematika 11*, Tiranë, Albania: Pegi.

Mayer, R. (2001). *Multimedia Learning*, New York: Cambridge University Press.

Mayer, R. E. & Anderson, R. B. (1991). Animations need narrations: An experimental test of a dual-coding hypothesis. *Journal of Educational Psychology*, *83*, 484-490.

McCormick, B.H., DeFantim, T.A. & Brown, M.D. (1987). Visualization in scientific computing: Definition, domain, and recommendations, *Computer Graphics*, *21*, 3–13.

Milovanović, M., Takači, D., & Milajić, A. (2011). Multimedia approach in teaching mathematics – example of lesson about the definite integral application for determining an area. *International Journal of Mathematical Education in Science and Technology*, *42*(2), 175-187.

Moos, D. C., & Azevedo, R. (2009). Learning with computer-based learning environments: A literature review of computer self-efficacy. *Review of Educational Research*, 79, 576-600.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). (2008). *Technology in Teaching and Learning Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). (2011). *Technology in Teaching and Learning Mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Ng, B. K., & Hu, C. (2006). Use web-based simulation to learn trigonometric curves. Retrieved from: http://www.cimt.plymouth.ac.uk/journal/chunhu.pdf.

Paivio, A. (2006). Dual coding theory and education. *Pathways to Literacy Achievement for High Poverty Children.* University of Michigan School of Education. http://www.umich.edu/~rdytolrn/pathwaysconference/presentations/paivio.pdf

Pape, S. J., & Tchoshanov, M.A. (2001). The role of representation(s) in developing mathematical undertanding. *Theory into Practice*, *40*, 118-127.

Park, O. (1994). Dynamic visual displays in media-based instruction. *Educational Technology*, *34*(4), 21-25.

Pea, R. D. (1987). Cognitive technologies for mathematics education. In A. Schoenfeld (Ed.), *Cognitive science and mathematics education* (89–122). Hillsdale, NJ: Erlbaum.

Perjési, I. (2003). Application of CAS for teaching of integral-transforming theorems. *ZDM*, *35*(2), 43-47.

Presmeg, N. C. (1986). Visualisation in high school mathematics. *For the Learning of Mathematics*, *6*(3), 42-46.

Pritchard, L. & Simpson, A (1999). The role of pictorial images in trigonometry problems. In O. Zaslavsky (Ed.), *Proceeding of the 23rd PME International Conference*, *4*, 81-88.

Ross, J. A., Bruce, C. D., & Sibbald, T. M. (2011). Sequencing computer-assisted learning of transformations of trigonometric functions. *Teaching Mathematics and its Applications*, *30*(3), 120-137.

Ruthven, K., & Hennessy, S. (2002). A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning. *Educational Studies in Mathematics*, *49*(1), 47-88.

Shulman, L. (1987). Knowledge and Teaching: Foundations of the New Reform. *Harvard Educational Review*, *57*(1), 1-23.

Skemp, R. R. (1979). Goals of learning and qualities of understanding. *Mathematics Teaching*, *88*, 44-49.

Skemp, R. R. (1987). *The psychology of learning mathematics*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Steckroth, J. J. (2007). Technology enhanced mathematics instruction: Effects of visualization on student understanding of trigonometry. Ph.D. Thesis, University of Virginia, Charlottesville, VA.

Stylianides, A. J. & Stylianides, G. J. (2007). Learning Mathematics with Understanding: A Critical Consideration of the Learning Principle in the Principles and Standards for School Mathematics. *The Montana Mathematics Enthusiast*, *4*(1), 103-114.

Takaci, D, R. Stojkovic, R. & Radovanovic, J. (2008). The influence of computer on examining trigonometric functions, *Teaching Mathematics and Computer Science*, *6*(1), 111-123, Debrecen, Hungary.

Takaci, D., Herceg, D., & Stojkovic, R. (2005). Possibilities and limitations of scientific workplace in studying trigonometric functions. *Teaching Mathematics*, *7*, 61-72.

Van Voorst, C. (1999). Technology in mathematics teacher education. Retrieved from http://www.icte.org/T99_Library/T99_54.PDF

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*, Cambridge: Harvard University Press.

Weber, K. (2005). Students' understanding of trigonometric functions. *Mathematics Education Research Journal*, *17*(3), 91-112.

Wheatley, G. & Brown, D. (1994). The Construction and representation of images in mathematical activity. In J. P. Ponte & J. F. Matos (Eds.), *Proceedings of the 18th PME International Conference*, 1, 81.

Wilson, S. J. (2008). Dynamic web tools for trigonometry, *Innovations in Math Technology*. Paper 2.

Zimmermann, W. & Cunningham, S. (1991). *Visualization in teaching and learning mathematics*, Washington D. C: Mathematical Association of America.

Ministria e Arsimit dhe Shkences , www.mash.gov.al

Instituti i Zhvillimit te Arsimit, www.izha.edu.al

SimReal , http://grimstad.uia.no/perhh/phh/

9 Annexes

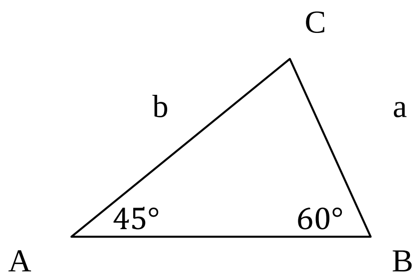
9.1 Pre- and Post- test forms and results

Pre -test

1. What is 1 Radian? Turn into radian the angles:

   20^0, 30^0, 270^0, 3240^0.

2. Give the definitions of the trigonometric functions in the right angle triangle ABC.
3. It is given in a triangle ABC the side a = 10 and the angles α=45°,β=60°. Find the length of the side b.



4. Make the necessary transformations:

$$\sin(90° - \alpha) - \cos(180° - \alpha) + \tan(180° - \alpha) + \sin(-\alpha)$$

5. a) Can you find the area of a triangle if you know the length of two sides b and c and the angle x between them?
   b) If we can change the angle x, for which value of this angle will we have the triangle with the largest area?

Results of the experimental group in the pre-test

Exper. gr.	**Task1**	**Task2**	**Task3**	**Task4**	**Task5**	**Total**	**Mark**
Student 1	2	20	20	20	10	**72**	**8**
Student 2	10	20	20	20	20	**90**	**10**
Student 3	16	20	16	10	5	**67**	**7**
Student 4	18	20	10	10	10	**68**	**7**
Student 5	10	20	20	20	12	**82**	**9**
Student 6	10	20	20	20	20	**90**	**10**
Student 7	7	20	20	20	10	**77**	**8**
Student 8	10	10	15	15	10	**60**	**7**
Student 9	13	20	16	20	10	**79**	**8**
Student 10	2	10	20	10	8	**50**	**6**
Student 11	10	18	18	20	15	**81**	**9**
Mean	**9.8**	**18**	**17.7**	**16.8**	**11.8**	**74.2**	**8**

Results of the control group in the pre-test

Contr. gr.	Task1	Task2	Task3	Task4	Task5	Total	Mark
Student 12	16	15	20	0	15	**66**	**7**
Student 13	8	4	20	20	8	**60**	**7**
Student 14	5	5	20	20	8	**58**	**6**
Student 15	18	20	20	20	10	**88**	**9**
Student 16	20	20	20	15	15	**90**	**10**
Student 17	8	20	19	10	18	**75**	**8**
Student 18	8	20	20	15	10	**73**	**8**
Student 19	8	20	20	20	10	**78**	**8**
Student 20	18	20	18	20	15	**91**	**10**
Student 21	8	20	20	15	5	**68**	**7**
Student 22	17	8	15	15	8	**63**	**7**
Mean	**12.2**	**15.6**	**19.3**	**15.5**	**11.1**	**73.6**	**7.9**

Post -test

1. What is called an angle 1 radian? Find the values in grade of the angles:

   $3\pi,\ \frac{3\pi}{2},\ \frac{4\pi}{3},\ \frac{7\pi}{6}.$

2. Draw an unit cirlce, and take an angle x. Show the sine, cosine and the tangent of the angle x. Which of these functions are limited? Find the period of each function and draw the graph of sinx.

3. It is given the equation of motion of a point i levizjes with the function y = 2cos(2x) + 1. Find the amplitude, period and the frequency of the motion. Draw the graph of this function.

4. Make the neccesary transformations:

   a) $\cos(90° - \alpha) - \sin(180° - \alpha) + \tan(90° + \alpha) + \cos(-\alpha)$

   $b)\ \frac{1}{1 - cosx} + \frac{1}{1 + cosx}$

5. Solve the equations:
   a) sinx = -cosx
   b) $2\cos^2 x - 5\cos x + 3 = 0$

Results of the experimental group in the post-test

Exp. gr.	Task1	Task2	Task3	Task4	Task5	Total	Mark
Student 1	17.5	17	17	19	17	**87.5**	**9**
Student 2	20	20	20	20	17	**97**	**10**
Student 3	15	13	10	20	17	**75**	**8**
Student 4	20	17	10	18	17	**82**	**9**
Student 5	20	17	13	20	15	**85**	**9**
Student 6	20	18	17	20	17	**92**	**10**
Student 7	10	17	13	20	17	**77**	**8**
Student 8	16	17	7	17.5	14	**71.5**	**8**
Student 9	20	17	13	20	17	**87**	**9**
Student 10	15	14	7	8	0	**44**	**5**
Student 11	16	17	10	15	15	**73**	**8**
Mean	**17.2**	**16.7**	**12.5**	**18**	**14.8**	**79.18182**	**8.454545**

Results of the control group in the post-test

Contr. gr.	Task1	Task2	Task3	Task4	Task5	Total	Mark
Student 12	20	14	10	0	0	**44**	**5**
Student 13	18	12	10	15	13	**68**	**7**
Student 14	20	5	10	17.5	17	**69.5**	**8**
Student 15	20	17	13	20	16	**86**	**9**
Student 16	16	17	20	20	19	**92**	**10**
Student 17	18	7	7	17	13	**62**	**7**
Student 18	17	10	10	18	17	**72**	**8**
Student 19	20	17	10	18	19	**84**	**9**
Student 20	20	20	17	20	19	**96**	**10**
Student 21	15	14	17	18	17	**81**	**9**
Student 22	15	13	10	15	17	**70**	**8**
Mean	**18.1**	**13.3**	**12.2**	**16.2**	**15.2**	**74.9**	**8.2**

9.2 Questionnaire forms and scores

Questionnaire 1 - Mathematics confidence attitudes

		Strongly agree	Agree	Not sure	Disagree	Strongly disagree
1	I have less trouble learning how to use a computer than I do learning other things.					
2	When I have difficulties using a computer I know I can handle them.					
3	I am not what I would call a computer person.					
4	It takes me much longer to understand how to use computers than the average person.					
5	I have never felt myself able to learn how to use computers.					
6	I enjoy trying new things on a computer.					
7	I find having to use computers frightening.					
8	I find many aspects of using computers interesting and challenging.					
9	I don't understand how some people can seem to enjoy spending so much time using computers.					
10	I have never been very excited about using computers.					
11	I find using computers confusing.					

Questionnaire 2 - Computer confidence attitudes

		Strongly agree	Agree	Not sure	Disagree	Strongly disagree
1	I have less trouble learning how to use a computer than I do learning other things.					
2	When I have difficulties using a computer I know I can handle them.					
3	I am not what I would call a computer person.					
4	It takes me much longer to understand how to use computers than the average person.					
5	I have never felt myself able to learn how to use computers.					
6	I enjoy trying new things on a computer.					
7	I find having to use computers frightening.					
8	I find many aspects of using computers interesting and challenging.					
9	I don't understand how some people can seem to enjoy spending so much time using computers.					
10	I have never been very excited about using computers.					
11	I find using computers confusing.					
12	I'm nervous that I'm not good enough with computers to be able to use them to learn					

Questionnaire 3 - Attitudes toward using computers in mathematics

		Strongly agree	Agree	Not sure	Disagree	Strongly disagree
1	Computing power makes it easier to explore mathematical ideas.					
2	I know computers are important but I don't feel I need to use them to learn mathematics.					
3	Computers and graphics calculators are good tools for calculation, but not for my learning of mathematics.					
4	I think using technology is too new and strange to make it worthwhile for learning mathematics.					
5	I think using technology wastes too much time in the learning of mathematics.					
6	I prefer to do all the calculations and graphing myself , without using a computer or graphics calculator.					
7	Using technology for the calculations makes it easier for me to do more realistic applications.					
8	I like the idea of exploring mathematical methods and ideas using technology.					
9	I want to get better at using computers to help me with mathematics.					
10	The symbols and language of mathematics are bad enough already without the addition of technology.					
11	Having technology to do routine work makes me more likely to try different methods and approaches					

Scores of students in the experimental group in the questionnaire 1 and 2

Codes of students	Mathematics confidence	Computer confidence
Student 1	42	56
Student 2	38	54
Student 3	45	60
Student 4	43	57
Student 5	42	45
Student 6	37	53
Student 7	41	42
Student 8	41	58
Student 9	38	46
Student 10	22	36
Student 11	46	44
Mean	**39.5**	**50.1**

Scores of students in the control group in the questionnaire 1 and 2

Codes of students	Mathematics confidence	Computer confidence
Student 12	29	48
Student 13	40	51
Student 14	38	40
Student 15	38	45
Student 16	47	46
Student 17	38	36
Student 18	37	53
Student 19	55	60
Student 20	45	47
Student 21	46	47
Student 22	53	60
Mean	**42.4**	**48.5**

9.3 Schedule of lessons, activities and field notes

Chapter 3 – Trigonometry (13 hours)

Week	Date	Title of the lesson	Activities during the lessons
46	09.11	Pre-test	
	10.11	Radian. Unit circle. Trigonometric arcs and trigonometric angles	-Definition of an angle (as a rotation) -Units for angles : degree, radian -What is the meaning of 1 radian -SimReal: the radian definition in the circle (demonstration in projector) -Exercise: convert from degree to radian -Definition of the unit circle -Trigonometric arcs of the same angle: show in SimReal
47	14.11	Trigonometric functions (in the unit circle)	-Introduce the program to the student, learn how to open and to use it -Demonstration in SimReal: Trigonometric functions in a triangle (relate with the content of 10th grade) -Blackboard: Trigonometric functions in a unit circle -Demonstration in SimReal: Unit circle, trigonometric functions, negative angles -Classwork: 4,5 page 63 (SimReal)
	16.11	Sine and cosine variations	-Demonstration in SimReal: the graph of sine related to the unit circle -Students using SimReal: the graph of cosine
	17.11	Tangent function, properties and variation	-Blackboard: definition of tangent function, the formula , the sign -Demostration and students using SimReal: drawing the graph
48	23.11	Trigonometric identities	-SimReal: discussion about the homeworks: 7, 8 p. 68 -The main formula of trigonometric funcitons, from Pythagora -Exercises using the formula
	24.11	Exercises	-Exploring with SimReal the meaning of variables in trigonometric functions expressions: amplitude, period -Using SimReal to look at applications in physics
49	30.11	Reduction formulas	-Blackboard: reduction formulas explained by triangles' congruence -Demostration in SimReal: sum of angles
	02.12	Exercises	Using reduction formulas for the tangent function
50	05.12	Elementary trigonometric equations	-Blackboard: algebraic solution of equations -Demonstration and students using SimReal: how can be solved geometrically an equation
	07.12	Exercises	Solving equation in blackboard and in SimReal
51	12.12	Exercises for the chapter	
	14.12	Post-test	

Field notes – Comments of students after the lessons

Week	Date	Title of the lesson	Students comments
46	10.11	Radian. Unit circle. Trigonometric arcs and trigonometric angles	*Student 7 : "I have problem to understand, it is in English"* *Student 2: "It looks interesting"*
47	14.11	Trigonometric functions (in the unit circle)	*Student 4: "it doesn't seem to me that we are doing mathematics"* *Student 10: "Mathematics scares me, having to use the computer it becomes worse"* *Student 6: "it takes me time to understand how to use it, I think I lost what happened in the blackboard"*
	16.11	Sine and cosine variations	*"It helps me for the calculations, I don't need to write down all the calculations when I want to get values for the graphs" student1* *"you can see how the graphs change"*
	17.11	Tangent function, properties and variation	
48	23.11	Trigonometric identities	
	24.11	Exercises	
49	30.11	Reduction formulas	*Student3: "I think I don't need the program for the reduction formulas, it is easier when you draw the unit circle"*
	02.12	Exercises	
50	05.12	Elementary trigonometric equations	*Student 4: "it is easier to solve equations by paper and pencil"*
	07.12	Exercises	*"it is easier to draw the graphs, and you can finish the work faster", student 6*
51	12.12	Exercises for the chapter	*Student 4 :"the time goes quickly, and is less boring"* *Student 2: "I like to do mathematics always in this way"*

9.4 Transcriptions of five interviews

	Interview 1 – Student 4
1	Was it useful for you to use the program SimReal in mathematics lessons?
St.4	Yes, I found the program very useful during the mathematics hours
2	Which part of the program was more useful for you, the simulations, the interactive part, or doing calculations
St.4	The program helped me with the drawing of the graphs and with the calculations
3	Do you think that is easier to understand the lessons explained with SimReal, or it is better to explain before the lesson in the blackboard and then to look at the program
St.4	The program is more helpful to make clearer the teacher explanation in the blackboard, but in some cases I had no problem to understand the lesson directly from the program. Concreatly, when the sine and cosine graphs were explained, or when we compared two trigonometric functions. Everything was very clear from the program
4	Is SimReal easy to use? Do you think it was necessary more practice to learn how to use it better?
St.4	The program is easy to use, and it can be learned very quickly. I think I did not need more time to practise it.
5	Can you say any concrete example were SimReal helped you to understand better the mathematical content?
St.4	The program helped me with the radian concept, and also with the equations, in drawing the graphs and solving them graphically. But I found easier to sovle them with paper and pencil.

	Interview 2 – Student 11
1	Was it useful for you to use the program SimReal in mathematics lessons?
St.11	Yes, the program was very useful during the mathematics hours
2	Which part of the program was more useful for you, the simulations,

	the interactive part, or doing calculations
St.11	I found the program with many advantages, in calculating the radian, sine, cosine, drawing the graphs, and also in solving different trigonometric functions.
3	Do you think that is easier to understand the lessons explained with SimReal, or it is better to explain before the lesson in the blackboard and then to look at the program
St.11	The program helped me to understand better some concepts and problems that I found difficult from the explanation in the blackboard. So, I think it is better when the lesson is explained first in the blackboard, than in the program.
4	Is SimReal easy to use? Do you think it was necessary more practice to learn how to use it better?
St.11	It was easy to learn it, but it needed to be very concretrated, and to learn it step by step.
5	Can you say any concrete example were SimReal helped you to understand better the mathematical content?
St.11	Concrete content…hmmm…in calculation of the radian, drawing sine and cosine graphs, expecially in drawing graphs. But also I found it useful to explain and make more concrete the amplitude and period meanings.

	Interview 3 – Student 5
1	Was it useful for you to use the program SimReal in mathematics lessons?
St.5	Yes, it was very useful as a tool to learn better mathematics.
2	Which part of the program was more useful for you, the simulations, the interactive part, or doing calculations
St.5	More useful I found the interactive part, excpecially in drawing graphs, it is easier than doing it with paper and pen.
3	Do you think that is easier to understand the lessons explained with SimReal, or it is better to explain before the lesson in the blackboard and then to look at the program
St.5	I think it is better to make the two ways of explenations in paralel, so doing it at the blackboard, and explain it with the program. Is much better I think.
4	Is SimReal easy to use? Do you think it was necessary more practice to learn how to use it better?
St.5	It is not a difficult program, but I think we need more time to learn it, and to explore it.
5	Can you say any concrete example were SimReal helped you to understand better the mathematical content?
St.5	Concretly it helped me with the equations, it is not difficult to solve them, but doing it with the program, drawing the graphs, makes everything easier to understand, as it is more concrete, not just a calculations

	Interview 4 – Student 2
1	Was it useful for you to use the program SimReal in mathematics lessons?
St.2	Yes, I think it was useful to use this program during the mathematics hours
2	Which part of the program was more useful for you, the simulations, the interactive part, or doing calculations
St.2	More useful for me it was the explanation with the program, so the simulation, or when teacher used it. This is because it took me some time to practise the program, and it was difficult for me to interact with it. Also I can say that I did not found it useful for calcualtions, it take so much time doing them with the program, considering that they were not very complicated calculations, and that can be done easy by hand.
3	Do you think that is easier to understand the lessons explained with SimReal, or it is better to explain before the lesson in the blackboard and then to look at the program
St.2	Easier and better I think it is when the teacher introduce first the lesson in the blackboard, than makes the concretization with the program, so we know what we are talking about, and understand it better.
4	Is SimReal easy to use? Do you think it was necessary more practice to learn how to use it better?
St.2	It is not very difficult, it can be learned, but as I said, it took me time to learn and to work with it.
5	Can you say any concrete example were SimReal helped you to understand better the mathematical content?
St.2	I think it helped me to understand the chapter in general, and in particular the applications of the trigonometric functions, concretely what is the amplitude, period of a function, and how it is related with physics. (harmonic motion)

	Interview 5 – Student 7
1	Was it useful for you to use the program SimReal in mathematics lessons?
St.7	Yes, it was useful to use it during the lessons
2	Which part of the program was more useful for you, the simulations, the interactive part, or doing calculations
St.7	More useful was the part when teacher used the program during the explanation, because I found it very difficult to use during the lessons, as we are not used with computer programs, excpecially to use them during mathematics hours.
3	Do you think that is easier to understand the lessons explained with SimReal, or it is better to explain before the lesson in the blackboard and then to look at the program
St.7	It is easier to look at the demonstration in the program after the explanation

	in the blackboard.
4	Is SimReal easy to use? Do you think it was necessary more practice to learn how to use it better?
St.7	I found it very difficult to learn, but after some practice I started to be more familiar with it.
5	Can you say any concrete example were SimReal helped you to understand better the mathematical content?
St.7	Concreatly I found it useful in showing with simulation how the pictures and drawings of the problems should be done.…you can see how the pictures are placed.

9.5. The permission for the data collection

The Albanian version

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Deklaratë

Unë, e nënshkruara Violeta Volumi, drejtoreshë e shkollës së mesme "Scutari" në Shkodër, deklaroj nën përgjegjësinë time të plotë se lejoj studenten e masterit Elira Curri që të bëjë çdo veprim që i shërben mbledhjes së të dhënave studimore në vitin e dytë të shkollës sonë, të dhëna që do t'i shërbejnë në punimin e temës së saj të masterit "Përdorimi i teknologjisë kompjuterike në mësimdhënien e matematikës në një shkollë të mesme në Shqipëri", për të cilën po ndjek studimet në Universitetin "Agder" në Kristiansand, Norvegji.

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REPUBLIC OF ALBANIA
MINISTRY OF EDUCATION AND SCIENCE

REGIONAL SCHOOL DISTRICT Shkoder
EDUCATION OFFICE Shkoder

Non public High School "*Scutari*". Shkoder

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Declaration

Me, the undersigned Violeta Volumi, headmaster of Upper Secondary School "Scutari" in Shkoder, declare in all my responsibility that I allow the master student Elira Curri to do all she needs to make the data collection in the second year of our school, for her master thesis "Using computer technology in teaching and learning mathematics in Albanian Upper Secondary school", during her studies in Agder University, Kristiansand, Norway.

Headmaster

VIOLETA VOLUMI

Shkoder, 05.11.2012

REPUBLIKA E SHQIPERISE
GJIMNAZI JOPUBLIK "SCUTARI"
SHKODER - ALBANIA