On the Statistical Analysis of the Channel Capacity of Double Rayleigh Channels with Equal Gain Combining in V2V Communication Systems

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***Abstract*— In this article, we present a detailed study on the statistical properties of the channel capacity of vehicle-to-vehicle (V2V) fading channels with equal gain combining (EGC). Assuming perfect channel state information (CSI) at the receiver, we have modeled the received signal envelope at the output of the equal gain (EG) combiner as a sum of double Rayleigh processes. These double Rayleigh processes are assumed to be independent but not necessarily identical processes. It is illustrated that the probability density function (PDF) of this sum process can efficiently be approximated using the gamma distribution. Furthermore, exploiting the properties of the gamma distribution, other statistical properties of the sum process are also evaluated. Thus, given the analytical approximations for the statistical properties of the received signal envelope at the output of the EG combiner, the theoretical results associated the statistics of the channel capacity just involves transformation of random variables. Here, simple and closed-form analytical approximations for the PDF, the cumulative distribution function (CDF), the level-crossing rate (LCR), and the average duration of fades (ADF) of the channel capacity are derived. The correctness of the theoretical results is validated by simulations. The presented results can be utilized to optimize the performance of spatial diversity receivers employed in the forthcoming V2V multiple-input multiple-output (MIMO) wireless communication systems.**

I. Introduction

V2V wireless communications has recently gained a fair share of attention by researchers, standardization bodies, and industrial companies, since it offers many applications. These applications are targeted to reduce traffic accidents and to facilitate the flow of traffic [1], [2]. The development of V2V communication systems requires the knowledge of the propagation channel characteristics. It is well-known that the multipath propagation channel in any mobile and wireless communication system can efficiently be described with the help of proper statistical models. For example, the Rayleigh distribution is considered to be a suitable distribution to model the fading channel under non-line-of-sight (NLOS) propagation conditions in classical cellular networks [3]–[5], a Suzuki process represents a reasonable model for land mobile terrestrial channels [6], [7], and the generalized-K distribution is widely accepted in radar systems [8], [9]. To model fading channels under NLOS propagation conditions in V2V communication systems, the double Rayleigh distribution is the appropriate choice (see, e.g., [10], [11] and the references therein). Motivated by the applications of the double Rayleigh channel model, a generalized channel model referred to as the $N * Nakagami$ channel model has been proposed in [12].

Inter-vehicle communication can be considered as a kind of mobile-to-mobile (M2M) communication. In M2M communication systems both the source (transmitter) and the destination (receiver) are mobile stations. If the destination mobile station is equipped with K receive antennas, then the signals reaching the destination mobile station through K diversity branches can be combined in order to mitigate the adverse multipath fading effects. Among other diversity combining techniques [13] aiming to combat the undesirable fading effects, the spatial diversity combining is a well-studied topic in the field of wireless communications. Selection combining (SC) [14], maximal ratio combining (MRC) [14], and EGC [14] are to name a few such combining schemes that provide a spatial diversity gain. MRC has been proved to be the optimum scheme, while the suboptimal EGC scheme is more popular for its simplicity in implementation [14]. Studies pertaining to the statistical properties as well as the performance analysis of both EGC and MRC over Rayleigh, Rice, and Nakagami channels can be found in the literature, see, e.g., [15]–[20]. In a recent work [21], the performance of digital modulation over double Nakagami-m fading channels with MRC diversity is investigated.

In addition to the knowledge about the propagation channel characteristics, a sound understanding of the channel capacity is also indispensable to meet the data rate requirements of future mobile communication systems. For this reason, researchers are currently devoting their time and efforts in investigating the various aspects of the channel capacity. A study on the capacity of Rayleigh and Rice channels with MRC diversity is presented in [22]–[24]. The authors of [25] have analyzed the statistical properties of the channel capacity of Rice fading channels with EGC and MRC. However, to the best of the authors' knowledge, the analysis of the statistical properties of the channel capacity of double Rayleigh fading channels with EGC in V2V communication systems is still an open problem that calls for further work.

This article analyzes the statistical properties of the channel capacity of double Rayleigh fading channels with EGC. The statistical quantities included in our study are the PDF, the CDF, the LCR, and the ADF of the channel capacity. The derived expression for the PDF of the channel capacity allows us to deduce the mean channel capacity and the capacity variance. To obtain an insight into the temporal variations of the channel capacity, an analysis of the LCR and the ADF of the channel capacity is inevitable [26]. We have derived simple and closed-form approximations for all these statistical quantities. The obtained approximate analytical results are compared with the exact simulation results that are considered to be the true results. This allows us to confirm the correctness and to study the accuracy of our approximate solution. Lastly, the influence of the number of diversity branches K on the statistical properties of the channel capacity of double Rayleigh fading channels with EGC has been investigated in detail.

The rest of the article is comprised of five parts. First, a brief review of EGC over double Rayleigh fading channels is given in Section II. In Section III, we discuss the statistical properties of EGC over double Rayleigh fading channels. Section IV deals with the derivation and analysis of the PDF, the CDF, the LCR, and the ADF of the channel capacity of double Rayleigh fading channels with EGC. In Section V, we compare the obtained analytical results with simulation results to validate the correctness of the theory. Finally, we conclude the article in Section VI.

II. EGC over Double Rayleigh Fading Channels

The instantaneous signal-to-noise ratio (SNR) per symbol $\gamma_{\text{EGC}}(t)$ at the output of the EG combiner can be defined as [27], [28]

$$\gamma_{\text{EGC}}(t) = \frac{\Xi^2(t)}{E\{N^2(t)\}} E_s = \frac{\Xi^2(t)}{K\,N_0} E_s = \gamma_s \Xi^2(t) \qquad (1)$$

where $E\{\cdot\}$ is the expectation operator. The quantity $\gamma_s = E_s/(K\,N_0)$ denotes the average received SNR where E_s gives the energy (in joules) per symbol, K is the total number of diversity branches, and N_0 is the total noise power. The process $\Xi(t)$ corresponds to the total received signal envelope at the output of the EG combiner and $N(t)$ represents the total received noise, i.e., $N(t) = \sum_{k=1}^{K} n^{(k)}(t)$. Here, $n^{(k)}(t)$ $(k = 1, 2, \ldots, K)$ refers to the additive white Gaussian noise (AWGN) in the kth diversity branch. The AWGN is a zero-mean Gaussian process with variance $N_0/2$.

Assuming perfect CSI at the destination mobile station, the received signal envelope $\Xi(t)$ at the output of an EG combiner can be written as [14]

$$\Xi(t) = \sum_{k=1}^{K} \left|\varsigma^{(k)}(t)\right| = \sum_{k=1}^{K} \chi^{(k)}(t) \qquad (2)$$

where $\varsigma^{(k)}(t)$ describes the fading process in the kth diversity branch between the source mobile station and the destination mobile station. We model the fading process $\varsigma^{(k)}(t)$ as a zero-mean complex double Gaussian process, i.e.,

$$\varsigma^{(k)}(t) = \varsigma_1^{(k)}(t) + j\varsigma_2^{(k)}(t) = \mu^{(2k-1)}(t)\,\mu^{(2k)}(t) \qquad (3)$$

for $k = 1, 2, \ldots, K$. In (3), $\mu^{(i)}(t)$ $(i = 1, 2, \ldots, 2K)$ represents a zero-mean complex circular Gaussian process having variance $2\sigma^2_{\mu^{(i)}}$. These Gaussian processes $\mu^{(i)}(t)$ are mutually independent, where each one is characterized by the classical Jakes Doppler power spectral density. The absolute value of $\varsigma^{(k)}(t)$ is denoted by $\chi^{(k)}(t)$ in (2), where each $\chi^{(k)}(t)$ is a double Rayleigh process.

III. Statistical Analysis of EGC over Double Rayleigh Fading Channels

In order to conduct a statistical analysis of the channel capacity, the knowledge of the statistics of the channel itself is imperative. Thus, this section is devoted to briefly discuss the statistical properties of EGC over double Rayleigh fading channels.

In the previous section, we modeled the received signal envelope $\Xi(t)$ at the output of the EG combiner as a sum of K independent but not necessarily identical double Rayleigh processes. Thus, the derivation of the PDF $p_\Xi(x)$ of $\Xi(t)$ involves the computation of a K-dimensional convolution integral. This computation is however quite tedious but still simple as well as closed-form result is not obtained. Here, we follow an approximation approach based on the Laguerre series expansion [29]. The Laguerre series is known to provide a reasonably good approximation for PDFs that have single maximum and fast decaying tails. In addition, the Laguerre series is often used when the first term in the expansion gives a simple approximation of high accuracy [29]. We can thus start by expressing the PDF $p_\Xi(x)$ of $\Xi(t)$ using the Laguerre series expansion as [29]

$$p_\Xi(x) = \sum_{n=0}^{\infty} b_n\, e^{-x} x^{\alpha_L} L_n^{(\alpha_L)}(x) \qquad (4)$$

where

$$L_n^{(\alpha_L)}(x) = e^x \frac{x^{(-\alpha_L)} d^n}{x! dx^n} \left[e^{(-x)} x^{n+\alpha_L}\right], \alpha_L > -1 \qquad (5)$$

represent the Laguerre polynomials. The coefficients b_n can be computed as

$$b_n = \frac{n!}{\Gamma(n+\alpha_L+1)} \int_0^\infty L_n^{(\alpha_L)}(x)\; p_\Xi(x)\, dx \qquad (6)$$

where $x = y/\beta_L$ and $\Gamma(\cdot)$ is the gamma function [30].

By solving the system of equations in [29, p. 21] for $b_1 = 0$ and $b_2 = 0$, the parameters α_L and β_L can easily be obtained. The solution of the said system of equations results in

$$\alpha_L = \frac{(\kappa_1^\Xi)^2}{\kappa_2^\Xi} - 1, \qquad \beta_L = \frac{\kappa_2^\Xi}{\kappa_1^\Xi} \qquad (7a,b)$$

where κ_1^{Ξ} denotes the first cumulant (i.e., the mean value) of the stochastic process $\Xi(t)$ and κ_2^{Ξ} is the second cumulant (i.e., the variance) of $\Xi(t)$. Mathematically, κ_1^{Ξ} and κ_2^{Ξ} can be expressed as

$$\kappa_1^{\Xi} = \sum_{k=1}^{K} \kappa_1^{\chi^{(k)}}, \qquad \kappa_2^{\Xi} = \sum_{k=1}^{K} \kappa_2^{\chi^{(k)}} \tag{8a,b}$$

where $\kappa_n^{\chi^{(k)}}$ $(n = 1, 2)$ represents the nth cumulant associated with the double Rayleigh process $\chi^{(k)}(t)$. The first two cumulants of $\chi^{(k)}(t)$ can be given as [31]

$$\kappa_1^{\chi^{(k)}} = \frac{\sigma_{\mu(2k-1)}\sigma_{\mu(2k)}\pi}{2}, \quad \kappa_2^{\chi^{(k)}} = \frac{1}{4}\sigma_{\mu(2k-1)}^2\sigma_{\mu(2k)}^2\left(16 - \pi^2\right). \tag{9a,b}$$

Once the cumulants $\kappa_n^{\chi^{(k)}}$ $(n = 1, 2)$ are obtained for all $\chi^{(k)}(t)$ using (9a,b), it is not difficult to compute κ_n^{Ξ} in (8a,b). In addition, given κ_n^{Ξ}, the required quantities α_L and β_L can be found with the help of (7a,b). After substituting α_L and β_L in the Laguerre series expansion, the first term of the series turns out to be the gamma distribution $p_{\Gamma}(x)$ [29]. This allows us to approximate the PDF $p_{\Xi}(x)$ of $\Xi(t)$ as

$$p_{\Xi}(x) \approx p_{\Gamma}(x) = \frac{x^{\alpha_L}}{\beta_L^{(\alpha_L+1)}\Gamma(\alpha_L+1)} e^{-\frac{x}{\beta_L}}, \ x \geq 0. \tag{10}$$

The PDF $p_{\Xi^2}(x)$ of the squared received signal envelope $\Xi^2(t)$ at the output of the EG combiner can be obtained by a simple transformation of the random variables [32, p. 244] as follows:

$$\begin{aligned} p_{\Xi^2}(x) &= \frac{1}{2\sqrt{x}} p_{\Xi}\left(\sqrt{x}\right) \\ &\approx \frac{1}{2\beta_L^{(\alpha_L+1)}\Gamma(\alpha_L+1)} x^{\left(\frac{\alpha_L-1}{2}\right)} e^{-\frac{\sqrt{x}}{\beta_L}} \end{aligned} \tag{11}$$

for $x \geq 0$. This PDF $p_{\Xi^2}(x)$ of $\Xi^2(t)$ will be used in the following section for computing the PDF of the channel capacity.

The derivation of the analytical expression for the LCR of a stochastic process involves the evaluation of the joint PDF of that process and its time derivative at the same time t. Thus, similar to the approximation of the PDF $p_{\Xi}(x)$ of $\Xi(t)$, here the joint PDF $p_{\Xi\dot{\Xi}}(x,\dot{x})$ of $\Xi(t)$ and $\dot{\Xi}(t)$[1] is approximated by the joint PDF $p_{\Gamma\dot{\Gamma}}(x,\dot{x})$ of a gamma process and its corresponding time derivative at the same time t. Numerical investigations have shown that $p_{\Xi\dot{\Xi}}(r,\dot{x}) \approx \frac{1}{\sqrt{3}} p_{\Gamma\dot{\Gamma}}(x,\dot{x})$, where

$$p_{\Gamma\dot{\Gamma}}(x,\dot{x}) = \frac{1}{2\sqrt{2\pi\beta x}} \frac{x^{\alpha_L}}{\beta_L^{(\alpha_L+1)}\Gamma(\alpha_L+1)} e^{-\frac{x}{\beta_L} - \frac{\dot{x}^2}{8\beta x}} \tag{12}$$

for $x \geq 0$ and $|\dot{x}| < \infty$. Furthermore, for the isotropic scattering conditions, the quantity β equals $2\pi^2\beta_L\left(f_{\mathrm{s}_{\max}}^2 + f_{\mathrm{D}_{\max}}^2\right)$ [33], [34]. Here, $f_{\mathrm{s}_{\max}}$ and $f_{\mathrm{D}_{\max}}$ correspond to the maximum Doppler frequencies caused by the motion of the source mobile station and the destination mobile station, respectively.

[1]Throughout this paper, the overdot represents the time derivative.

Finally, making use of the joint PDF $p_{\Xi\dot{\Xi}}(x,\dot{x})$ and applying the concept of transformation of random variables [32, p. 244] allows us to express the joint PDF $p_{\Xi^2\dot{\Xi}^2}(x,\dot{x})$ as

$$\begin{aligned} p_{\Xi^2\dot{\Xi}^2}(x,\dot{x}) &= \frac{1}{4x} p_{\Xi\dot{\Xi}}\left(\sqrt{x}, \frac{\dot{x}}{2\sqrt{x}}\right) \approx \frac{1}{4\sqrt{3}x} p_{\Gamma\dot{\Gamma}}\left(\sqrt{x}, \frac{\dot{x}}{2\sqrt{x}}\right) \\ &= \frac{1}{8\sqrt{6\pi\beta}} \frac{x^{\left(\frac{2\alpha_L-5}{4}\right)}}{\beta_L^{(\alpha_L+1)}\Gamma(\alpha_L+1)} e^{-\frac{\sqrt{x}}{\beta_L} - \frac{\dot{x}^2}{32\beta x^{3/2}}} \end{aligned} \tag{13}$$

for $x \geq 0$ and $|\dot{x}| < \infty$. In (13), $p_{\Xi^2\dot{\Xi}^2}(x,\dot{x})$ represents the joint PDF of the squared received envelope $\Xi^2(t)$ and its corresponding time derivative $\dot{\Xi}^2(t)$ at the same time t. The joint PDF $p_{\Xi^2\dot{\Xi}^2}(x,\dot{x})$ will be utilized in the derivation of the LCR of the channel capacity in the next section.

IV. Statistical Analysis of the Channel Capacity of Double Rayleigh Fading Channels with EGC

In this section, we present a statistical analysis of the channel capacity of double Rayleigh fading channels with EGC. The statistical quantities studied here include the PDF, the CDF, the LCR, and the ADF of the channel capacity.

The instantaneous channel capacity $C(t)$ of double Rayleigh fading channels with EGC is defined as

$$C(t) = \log_2\left(1 + \gamma_{\mathrm{EGC}}(t)\right) \ \text{(bits/s/Hz)} \tag{14}$$

where $\gamma_{\mathrm{EGC}}(t)$ introduced in (1) is the instantaneous SNR per symbol. Equation (14) illustrates the mapping of the instantaneous SNR $\gamma_{\mathrm{EGC}}(t)$ on the channel capacity $C(t)$. This mapping makes it possible to derive the analytical expressions for the statistical properties of the channel capacity $C(t)$ with the help of those associated with $\gamma_{\mathrm{EGC}}(t)$.

We start the statistical analysis of the channel capacity $C(t)$ by deriving an expression for the PDF $p_C(r)$ of $C(t)$. However, in order to proceed with this derivation, we require the PDF $p_{\gamma_{\mathrm{EGC}}}(z)$ of $\gamma_{\mathrm{EGC}}(t)$. The PDF $p_{\gamma_{\mathrm{EGC}}}(z)$ can be obtained using the relation $p_{\gamma_{\mathrm{EGC}}}(z) = (1/\gamma_s)\, p_{\Xi^2}(z/\gamma_s)$, where $p_{\Xi^2}(z)$ is presented in (11). Using the concept of transformation of random variables [32, p. 244], the PDF $p_C(r)$ of $C(t)$ can be expressed in closed form as

$$\begin{aligned} p_C(r) &= 2^r \ln(2)\, p_{\gamma_{\mathrm{EGC}}}(2^r - 1) \\ &\approx \frac{2^{r-1}\ln(2)}{\gamma_s^{\left(\frac{\alpha_L+1}{2}\right)}} \frac{(2^r-1)^{\left(\frac{\alpha_L-1}{2}\right)} e^{-\frac{\sqrt{(2^r-1)/\gamma_s}}{\beta_L}}}{\beta_L^{(\alpha_L+1)}\Gamma(\alpha_L+1)}, \ r \geq 0. \end{aligned} \tag{15}$$

The probability that the channel capacity $C(t)$ remains below a certain threshold level r defines the CDF $F_C(r)$ of $C(t)$ [32]. Substituting (15) in $F_C(r) = 1 - \int_r^{\infty} p_C(z)\,dz$ results in the following approximation

$$\begin{aligned} F_C(r) &\approx 1 - \frac{\ln(2)}{2\,\gamma_s^{\left(\frac{\alpha_L+1}{2}\right)}} \frac{1}{\beta_L^{(\alpha_L+1)}\Gamma(\alpha_L+1)} \\ &\quad \times \int_r^{\infty} 2^z (2^z-1)^{\left(\frac{\alpha_L-1}{2}\right)} e^{-\frac{\sqrt{(2^z-1)/\gamma_s}}{\beta_L}} dz, \ r \geq 0. \end{aligned} \tag{16}$$

The LCR $N_C(r)$ of $C(t)$ describes the average number of times the channel capacity $C(t)$ crosses a certain threshold level r from up to down (or from down to up) per second. Mathematically, the LCR $N_C(r)$ of $C(t)$ can be expressed as [35]

$$N_C(r) = \int_0^{\infty} \dot{z}\, p_{C\dot{C}}(r, \dot{z})\, d\dot{z}, \ r \geq 0 \tag{17}$$

where $p_{C\dot{C}}(r, \dot{z})$ is the joint PDF of the stochastic process $C(t)$ and its corresponding time derivative $\dot{C}(t)$ at the same time t. Here, we first need to find the joint PDF $p_{\gamma_{\text{EGC}}\dot{\gamma}_{\text{EGC}}}(z, \dot{z})$ of $\gamma_{\text{EGC}}(t)$ and $\dot{\gamma}_{\text{EGC}}(t)$ at the same time t. The joint PDF $p_{\gamma_{\text{EGC}}\dot{\gamma}_{\text{EGC}}}(z, \dot{z})$ can be found using the relation $p_{\gamma_{\text{EGC}}\dot{\gamma}_{\text{EGC}}}(z, \dot{z}) = (1/\gamma_s)^2\, p_{\Xi^2\dot{\Xi}^2}(z/\gamma_s, \dot{z}/\gamma_s)$ along with (13). Thus, given the joint PDF $p_{\gamma_{\text{EGC}}\dot{\gamma}_{\text{EGC}}}(z, \dot{z})$ and using the concept of transformation of random variables [32, p. 244], the joint PDF $p_{C\dot{C}}(r, \dot{z})$ can be expressed as

$$\begin{aligned} p_{C\dot{C}}(z, \dot{z}) &= \left(2^z \ln(2)\right)^2 p_{\gamma_{\text{EGC}}\dot{\gamma}_{\text{EGC}}}\left(2^z - 1, 2^z \dot{z} \ln(2)\right) \\ &\approx \frac{\left(2^z \ln(2)\right)^2}{8\sqrt{6\pi\beta}\,\gamma_s^{\left(\frac{2\alpha_L+3}{4}\right)}} \frac{\left(2^z - 1\right)^{\left(\frac{2\alpha_L-5}{4}\right)}}{\beta_L^{(\alpha_L+1)}\Gamma(\alpha_L + 1)} \\ &\quad \times e^{-\frac{\sqrt{(2^z-1)/\gamma_s}}{\beta_L} - \frac{\left(2^z \dot{z} \ln(2)\right)^2/\sqrt{\gamma_s}}{32\beta(2^z-1)^{3/2}}}, \quad z \geq 0, \ |\dot{z}| < \infty \,. \end{aligned} \tag{18}$$

Finally, substituting (18) in (17) and solving the integral over $\dot{z}$ using [30, Eq. (3.326-2)] leads to a closed-form approximation for the LCR $N_C(r)$ of $C(t)$ in the form

$$N_C(r) \approx \sqrt{\frac{2}{3\pi}\beta}\, \frac{\left(2^r - 1\right)^{\left(\frac{2\alpha_L+1}{4}\right)} e^{-\frac{\sqrt{(2^r-1)/\gamma_s}}{\beta_L}}}{\gamma_s^{\left(\frac{2\alpha_L+1}{4}\right)} \beta_L^{(\alpha_L+1)}\Gamma(\alpha_L + 1)}, \ r \geq 0 \,. \tag{19}$$

The ADF $T_C(r)$ of the channel capacity $C(t)$ is the expected value of the time intervals over which $C(t)$ remains below a certain threshold level r. The ADF $T_C(r)$ of $C(t)$ can be computed by evaluating the ratio of the CDF $F_C(r)$ and the LCR $N_C(r)$ of $C(t)$ [14], i.e.,

$$T_C(r) = \frac{F_C(r)}{N_C(r)} \,. \tag{20}$$

Thus, the substitution of (16) and (19) in (20) results in an approximation for the ADF $T_C(r)$ of $C(t)$.

V. Numerical Results

This section illustrates the important theoretical results by evaluating the expressions in (10), (15), (16), (19), and (20). The correctness of the approximations is then validated by simulations. Here, the simulation results are considered as the true results. A sum-of-sinusoids (SOS) channel simulator [36] has been employed to obtain the simulation results. Meaning thereby, the SOS concept has been exploited to simulate the uncorrelated Gaussian noise processes making up the received signal envelope at the output of the EG combiner. The model parameters of the channel simulator are computed by utilizing the generalized method of exact Doppler spread (GMEDS_1) [37]. Each Gaussian process $\mu^{(i)}(t)$ was simulated using $\text{N}_l^{(i)} = 14$ for $i = 1, 2, \ldots, 2K$ and $l = 1, 2$, where $\text{N}_l^{(i)}$ is the number of sinusoids required to simulate the inphase ($l = 1$) and quadrature components ($l = 2$) of $\mu^{(i)}(t)$. It is required that the simulated distribution of $\left|\hat{\mu}^{(i)}(t)\right|^2$ closely approximates the Rayleigh distribution for all $i = 1, 2, \ldots, 2K$. Furthermore, it has been shown in [36] that any value of $\text{N}_l^{(i)} \geq 7$ ($l = 1, 2$) serves this purpose. Thus, our selection of $\text{N}_l^{(i)} = 14$ for $i = 1, 2, \ldots, 2K$ and $l = 1, 2$ provides us with the waveforms having the desired distribution. The maximum Doppler frequencies caused by the motion of the source mobile station and the destination mobile station, denoted by $f_{\text{S}_{\max}}$ and $f_{\text{D}_{\max}}$, respectively, were selected as 154 Hz and 273 Hz. The variances $\sigma^2_{\mu^{(i)}}$ were set to unity $\forall\, i = 1, 2, \ldots, 2K$ unless stated otherwise. In addition, the analysis of the statistical properties of the channel capacity $C(t)$ is carried out for an SNR E_s/N_0 of 15 dB.

The results presented in Figs. 1-5 show a good fitting of the approximated analytical and the exact simulation results considered as the true results. Figure 1 illustrates the theoretical results of the PDF $p_{\Xi}(x)$ of $\Xi(t)$ described by the approximation in (10) as well as the simulation results obtained by evaluating the statistics of the waveforms generated by using the SOS-based channel simulator. This figure shows the influence of the number of diversity branches K on the behavior of the PDF $p_{\Xi}(x)$. It can be observed that as the number of diversity branches K increases, the mean value and the variance of the stochastic process $\Xi(t)$ increases. Furthermore, for $K = 1$, the PDF $p_{\Xi}(x)$ of $\Xi(t)$ reduces to the double Rayleigh distribution. This result confirms the validity of the approximation in (10).

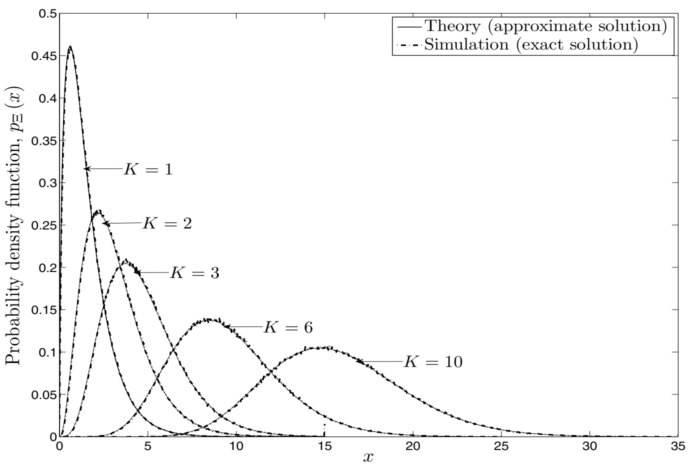

Fig. 1. The PDF $p_{\Xi}(x)$ of the received signal envelope at the output of the EG combiner $\Xi(t)$ for a different number K of diversity branches.

The closed-form approximate expression of the PDF $p_C(r)$ of the channel capacity $C(t)$ given in (15) is presented in Fig. 2 along with the simulation results. A close agreement can be seen between the approximate solution and the exact (simulation) results. Figure 2 also compares the PDF $p_C(r)$ of $C(t)$ for a different number of diversity branches K. It

[2]The process $\hat{\mu}^{(i)}(t)$ denotes a simulated Gaussian process.

is obvious that increasing K, increases the mean channel capacity, whereas the variance of $C(t)$ decreases. Similarly, in Fig. 3, the theoretical results of the CDF $F_C(r)$ of $C(t)$ described by the approximation in (16) are illustrated.

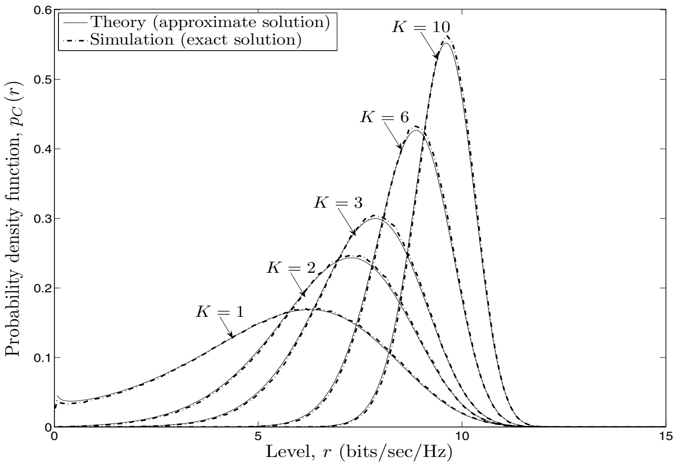

Fig. 2. The PDF $p_C(r)$ of the channel capacity $C(t)$ of the double Rayleigh fading channels with EGC for a different number K of diversity branches.

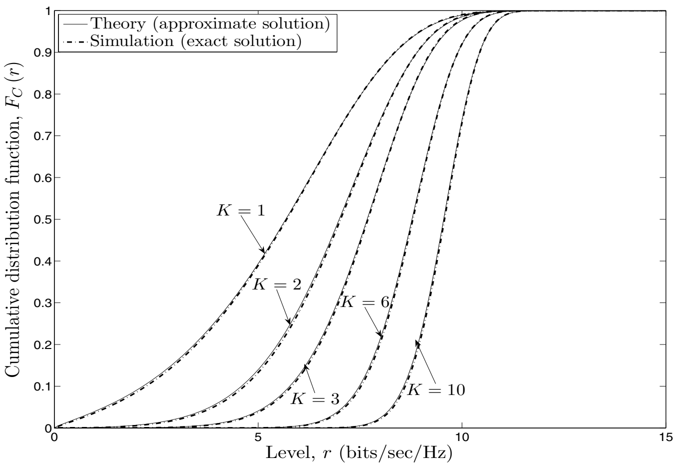

Fig. 3. The CDF $F_C(r)$ of the channel capacity $C(t)$ of the double Rayleigh fading channels with EGC for a different number K of diversity branches.

The LCR $N_C(r)$ of the channel capacity $C(t)$ described by the closed-form approximate expression in (19) is evaluated together with the exact (simulation) results in Fig. 4. This figure depicts the LCR $N_C(r)$ of $C(t)$ for a different number of diversity branches K. Studying the graphs shows that increasing the value of K results in a drastic decrease in the LCR $N_C(r)$ at low signal levels r. The LCR $N_C(r)$ at higher signal levels r is however the same for all values of K. Furthermore, a very good fitting of the approximate solution and the exact simulation results can be witnessed for any number of diversity branches $K > 1$.

Figure 5 displays the ADF $T_C(r)$ of the channel capacity $C(t)$ given in (20) for a different number of diversity branches K. For the purpose of validation of the obtained approximate solution, the exact (simulation) results are also plotted in Fig. 5. The presented results reveal the fact that the ADF $T_C(r)$ of $C(t)$ decreases with the increase in the number of diversity branches K.

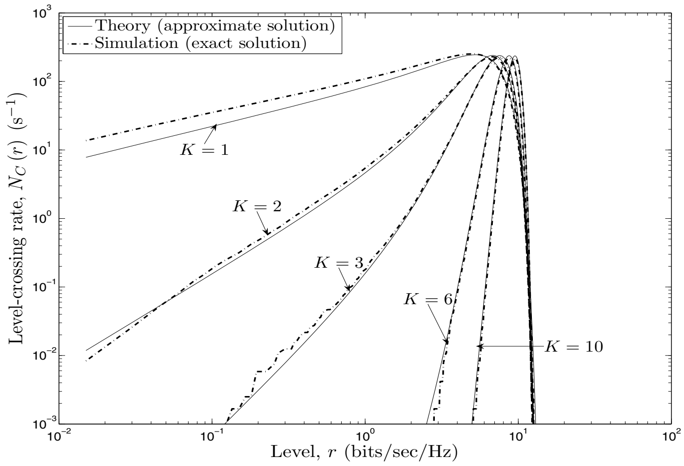

Fig. 4. The LCR $N_C(r)$ of the channel capacity $C(t)$ of the double Rayleigh fading channels with EGC for a different number K of diversity branches.

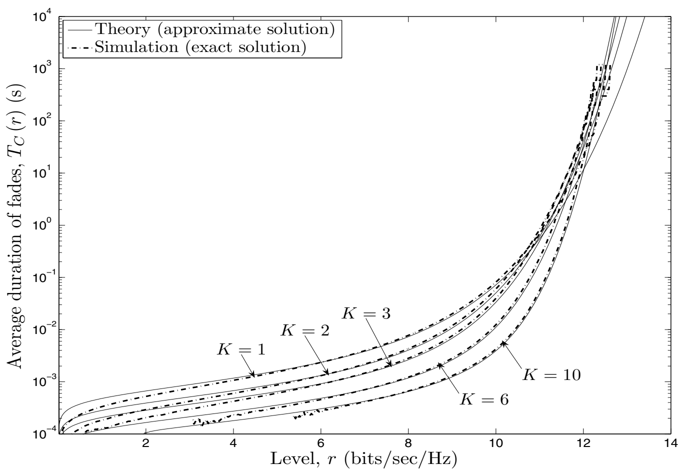

Fig. 5. The ADF $T_C(r)$ of the channel capacity $C(t)$ of the double Rayleigh fading channels with EGC for a different number K of diversity branches.

VI. Conclusion

In this article, we present a thorough analysis of the statistical properties of the channel capacity of double Rayleigh fading channels with EGC. A V2V communication system is considered here, where the destination mobile station is equipped with K receive antennas. The signal reaching the destination mobile station through K diversity branches is then combined using EGC. Simple and closed-form analytical approximations for the PDF, the CDF, the LCR, and the ADF of the channel capacity of double Rayleigh fading channels with EGC have been derived. We have modeled the received signal envelope at the output of the EG combiner as a sum of

K independent but not necessarily identical double Rayleigh processes. With the help of the Laguerre series expansion, the PDF of this sum process can be approximated by the gamma distribution. Furthermore, making use of the properties of the gamma distribution, other statistical properties of the sum process can also be evaluated. Once the analytical expressions for the statistical properties of the received signal envelope at the output of the EG combiner are obtained, the computation of the theoretical results associated with the statistics of the channel capacity can be performed using the concept of transformation of random variables. The correctness of the approximate theoretical results are then validated by simulations. We have presented the results demonstrating the influence of the number of diversity branches K on the PDF, the CDF, the LCR, and the ADF of the channel capacity of double Rayleigh fading channels with EGC. It has been shown that the mean channel capacity increases and the capacity variance decreases with an increase in the number of diversity branches K. In addition, at low signal levels r, the LCR of the channel capacity decreases as the value of K increases. The LCR however remains the same at high signal levels r for all values of K. A decrease in the ADF of the channel capacity can be observed if the number of diversity branches K increases. The results presented in this article can be utilized to optimize the spatial diversity receivers employed in the future V2V MIMO communication systems.

REFERENCES

[1] J. Zhu and S. Roy, "MAC for dedicated short range communications in intelligent transport system," *IEEE Commun. Mag.*, vol. 41, no. 12, pp. 60–67, Dec. 2003.

[2] V. Gradinescu *et al.*, "Adaptive traffic lights using car-to-car communication," in *Proc. IEEE 65th Veh. Technol. Conf., VTC'07-Spring*. Dublin, Ireland, Apr. 2007, pp. 21–25, DOI 10.1109/VETECS.2007.17.

[3] W. R. Young, "Comparison of mobile radio transmission at 150, 450, 900, and 3700 MHz," vol. 31, pp. 1068–1085, Nov. 1952.

[4] H. W. Nylund, "Characteristics of small-area signal fading on mobile circuits in the 150 MHz band," *IEEE Trans. Veh. Technol.*, vol. 17, pp. 24–30, Oct. 1968.

[5] Y. Okumura, E. Ohmori, T. Kawano, and K. Fukuda, "Field strength and its variability in VHF and UHF land mobile radio services," *Rev. Elec. Commun. Lab.*, vol. 16, pp. 825–873, Sep./Oct. 1968.

[6] M. Pätzold, U. Killat, and F. Laue, "An extended Suzuki model for land mobile satellite channels and its statistical properties," *IEEE Trans. Veh. Technol.*, vol. 47, no. 2, pp. 617–630, May 1998.

[7] H. Suzuki, "A statistical model for urban radio propagation," *IEEE Trans. Commun.*, vol. 25, no. 7, pp. 673–680, Jul. 1977.

[8] S. Chitroub, A. Houacine, and B. Sansal, "Statistical characterisation and modelling of SAR images," *Elsevier, Signal Processing*, vol. 82, no. 1, pp. 69–92, Jan. 2002, DOI http://dx.doi.org/10.1016/S0165-1684(01)00158-X.

[9] P. M. Shankar, "Error rates in generalized shadowed fading channels," *Wireless Personal Communications*, vol. 28, no. 3, pp. 233–238, Feb. 2004, DOI http://dx.doi.org/10.1023/B:wire.0000032253.68423.86.

[10] I. Z. Kovacs, P. C. F. Eggers, K. Olesen, and L. G. Petersen, "Investigations of outdoor-to-indoor mobile-to-mobile radio communication channels," in *Proc. IEEE 56th Veh. Technol. Conf., VTC'02-Fall*, vol. 1. Vancouver BC, Canada, Sep. 2002, pp. 430–434.

[11] M. Pätzold, B. O. Hogstad, and N. Youssef, "Modeling, analysis, and simulation of MIMO mobile-to-mobile fading channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 2, pp. 510–520, Feb. 2008.

[12] G. K. Karagiannidis, N. C. Sagias, and P. T. Mathiopoulos, "N*Nakagami: A novel stochastic model for cascaded fading channels," *IEEE Trans. Commun.*, vol. 55, no. 8, pp. 1453–1458, Aug. 2007.

[13] T. S. Rappaport, *Wireless Communications: Principles and Practice*, 2nd ed. Upper Saddle River, New Jersey: Prentice-Hall, 1996.

[14] W. C. Jakes, Ed., *Microwave Mobile Communications*. Piscataway, NJ: IEEE Press, 1994.

[15] M. D. Yacoub, C. R. C. M. da Silva, and J. E. V. Bautista, "Second-order statistics for diversity-combining techniques in Nakagami-fading channels," *IEEE Trans. Veh. Technol.*, vol. 50, no. 6, pp. 1464–1470, Nov. 2001.

[16] N. C. Beaulieu and X. Dong, "Level crossing rate and average fade duration of MRC and EGC diversity in Ricean fading," *IEEE Trans. Commun.*, vol. 51, no. 5, pp. 722–726, May 2003.

[17] P. Ivanis, D. Drajic, and B. Vucetic, "The second order statistics of maximal ratio combining with unbalanced branches," *IEEE Communications Letters*, vol. 12, no. 7, pp. 508–510, Jul. 2008.

[18] Q. T. Zhang, "Probability of error for equal-gain combiners over Rayleigh channels: some closed form solutions," *IEEE Trans. Commun.*, vol. 45, no. 3, pp. 270–273, Mar. 1997.

[19] D. Zogas, G. Karagiannidis, and S. A. Kotsopoulos, "Equal gain combining over Nakagami-n (Rice) and Nakagami-q (Hoyt) generalized fading channels," *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 374–379, mar 2005.

[20] H. Samimi and P. Azmi, "An approximate analytical framework for performance analysis of equal gain combining technique over independent Nakagami, Rician and Weibull fading channels," *Wireless Personal Communications*, vol. 43, no. 4, pp. 1399–1408, dec 2007, DOI 10.1007/s11277-007-9314-z.

[21] W. Wongtrairat and P. Supnithi, "Performance of digital modulation in double Nakagami-m fading channels with MRC diversity," *IEICE Trans. Commun.*, vol. E92-B, no. 2, pp. 559–566, Feb. 2009.

[22] M. S. Alouini and A. J. Goldsmith, "Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques," *IEEE Trans. Veh. Technol.*, vol. 48, no. 4, pp. 1165–1181, Jul. 1999.

[23] S. Khatalin and J. P. Fonseka, "On the channel capacity in Rician and Hoyt fading environments with MRC diversity," *IEEE Trans. Veh. Technol.*, vol. 55, no. 1, pp. 137–141, Jan. 2006.

[24] K. A. Hamdi, "Capacity of MRC on correlated Rician fading channels," *IEEE Trans. Commun.*, vol. 56, no. 5, pp. 708–711, May 2008.

[25] G. Rafiq and M. Pätzold, "Statistical properties of the capacity of Rice channels with MRC and EGC," in *IEEE Int. Conf. on Wireless Communications & Signal Processing, WCSP 2009*. Nanjing, China, Nov. 2009, DOI 10.1109/WCSP.2009.5371578.

[26] A. Giorgetti, P. J. Smith, M. Shafi, and M. Chiani, "MIMO capacity, level crossing rates and fades: The impact of spatial/temporal channel correlation," *J. Commun. Net.*, vol. 5, no. 2, pp. 104–115, Jun. 2003.

[27] M. K. Simon and M. S. Alouini, *Digital Communications over Fading Channels*, 2nd ed. New Jersey: John Wiley & Sons, 2005.

[28] M. Schwartz, W. R. Bennett, and S. Stein, *Communication Systems and Techniques*. New York: McGram Hill, 1966, vol. 4.

[29] S. Primak, V. Kontorovich, and V. Lyandres, Eds., *Stochastic Methods and their Applications to Communications: Stochastic Differential Equations Approach*. Chichester: John Wiley & Sons, 2004.

[30] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 6th ed. New York: Academic Press, 2000.

[31] M. K. Simon, *Probability Distributions Involving Gaussian Random Variables: A Handbook for Engineers and Scientists*. Dordrecht: Kluwer Academic Publishers, 2002.

[32] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th ed. New York: McGraw-Hill, 2002.

[33] A. S. Akki and F. Haber, "A statistical model of mobile-to-mobile land communication channel," *IEEE Trans. Veh. Technol.*, vol. 35, no. 1, pp. 2–7, Feb. 1986.

[34] A. S. Akki, "Statistical properties of mobile-to-mobile land communication channels," *IEEE Trans. Veh. Technol.*, vol. 43, no. 4, pp. 826–831, Nov. 1994.

[35] S. O. Rice, "Mathematical analysis of random noise," *Bell Syst. Tech. J.*, vol. 24, pp. 46–156, Jan. 1945.

[36] M. Pätzold, *Mobile Fading Channels*. Chichester: John Wiley & Sons, 2002.

[37] M. Pätzold, C. X. Wang, and B. O. Hogstad, "Two new methods for the efficient generation of multiple uncorrelated Rayleigh fading waveforms," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 3122–3131, Jun. 2009.