A Continuous-Space Analytical Approach for Relay Node Placement in Hybrid Cellular and Ad Hoc Networks

Ziaul Haq Abbas and Frank Y. Li

Dept. of Information and Communication Technology, University of Agder (UiA), N-4898, Grimstad, Norway

{ziaul.h.abbas, frank.li}@uia.no

Abstract—**A hybrid network is composed of a cellular component and an ad hoc component connected by a relay node, for the purpose of coverage extension and/or capacity improvement. In this paper, we analyze the capacity of such a hybrid network by employing a continuous-space analytical methodology based on circular geometry for uniformly distributed nodes. To achieve maximal overall capacity, the relay node needs to be placed in an optimum location between the base station and the mobile station located at the boundary of the hybrid network. Numerical results show that for obtaining the optimum overall capacity for the hybrid network, the placement of the relay node should be in a range which is neither too close nor too far away from the base station. For a given node density and path-loss coefficient, a precise location for relay node placement to achieve maximum overall capacity can be found using the presented method.**

I. INTRODUCTION

Tremendous amount of interest from the research community has been raised in recent years to enhance the coverage of the mobile cellular networks while maintaining better link quality and low cost. The improvement in the capacity of such networks has also been an active research topic. While there have been many attempts for increasing the cellular range (similarly for the capacity), there are practical and/or natural fundamental limits [1] on these quantities. For example, transmission power of a Base Station (BS) can not be increased beyond the limit defined by regulations. Similarly, the transmission power of a Mobile Station (MS) has to be kept within an optimum level due to, for example, interference and battery life considerations. When the coverage area of a BS becomes larger, interference increases due to higher number of interfering nodes, leading to reduced capacity and lower supported user data rate. By reducing the coverage area of the BS, the benefit of spatial reuse of spectrum is achieved. However, this benefit has to be traded off with the installation cost of a BS. As a consequence, the limits on the number of BSs in an area may create uncovered geographical spots. Moreover, the quality of signal in a circular region near a BS is better than in the far region (assuming omnidirectional antennas). Thus, the question of how to improve and extend the coverage (as well as capacity) of the network in the far region from the BS remains valid.

On the other hand, a good amount of research work has been carried out by the scientific community in the field of mobile ad hoc networks [2]. In an ad hoc network, two MSs can communicate with each other, either directly or via multi-hop relay, but the range of communication between two neighboring nodes is relatively short compared with the range between an MS and the BS in cellular networks. Some other advantages of ad hoc networks are that they can be easily deployed in an infrastructure-less manner and are of economically low cost.

A natural question is what happens if we integrate both the pure cellular and pure ad hoc network to create a Hybrid Network (HN) [3]. The low coverage but high capacity of ad hoc networks and the high coverage but low capacity of cellular networks can be traded off with each other to reach an optimum solution. This may lead to overall improvement in the cellular coverage as well as optimum capacity of the HN. Many such attempts have been made within this topic and relevant directions, for example [4].

In [5], the authors evaluated a hybrid network by reducing the coverage of a BS and relaying the traffic outside this coverage area. They intended to determine whether this scheme could improve the total performance in terms of capacity. Hexagonal geometry was used for this purpose and the analysis was carried out with *regular* placement of the BS and users. Fixed spectral bandwidth was divided between the two components of the hybrid network using a weight coefficient. However, *how to find the optimum capacity of the HN has not been addressed in their work*. In [6], the authors studied the impact of mobile relays on throughput and delay. Hexagonal grid was used around the BS and the mobile stations were distributed randomly in a certain region. However, *in their scenario, the relay nodes are preselected closest to the BS.*

In this paper, we analyze the overall capacity of an HN in which a one-hop ad hoc network uses an MS from the cellular part of the HN as a relay node. The goal of this work is to find a position for an *optimal placement* of the relay node. By optimal placement, it is meant that an optimum location for the relay node is determined such that the maximum overall capacity is achieved in the HN. We also study how to find this overall capacity with respect to the change in node density and path-loss coefficient. A salient feature of our method which distinguishes this work from other existing approaches is that it is based on a continuous-space analytical approach with circular geometry while most other related work is based on discrete-space approach.

The rest of the paper is organized as follows. In Section II we describe the scenario of the studied network and present basic assumptions for the analysis which is given in Section

III. Section IV presents and discusses the numerical results obtained through our method. Section V concludes the paper.

II. HYBRID NETWORK SCENARIO AND ASSUMPTIONS

In this section, we describe the network architecture we are considering in an HN. Fig. 1 shows the pictorial representation of the scenario studied in this paper. The overall region is divided into three sub-regions, namely, region-A, region-AB, and region-B. The values of R_1, R_2 and R_3 are not fixed so that the areas of all the three mentioned sub-regions also vary accordingly; and the distance from the BS to the MS located at the boundary of the region-B is $D=R_1+R_2+R_3$.

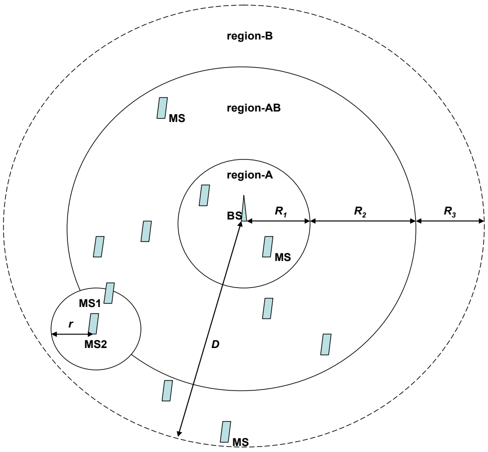


Fig. 1. Hybrid Network Architecture.

In our HN architecture, the MSs in region-A access the BS directly, and are referred to as *cellular nodes*. The mobile stations in region-B can also be reached by the BS, but with lower capacity. However, if a relay node within region-AB is used to forward packets then an MS in region-B may reach the BS with higher capacity. In other words, an MS in region-B, referred to as *ad hoc node*, can use MSs in region-AB as relay nodes to reach the BS. Hence, MSs in region-AB are referred to as *relay nodes*, and may operate either in cellular or ad hoc mode.

We assume that each MS is equipped with dual radio interface, i.e., a cellular interface and an ad hoc interface. The operating frequency for an ad hoc node is different from the operating frequency of a cellular node. At any time instant, an MS can use one interface only. Each MS has a buffer of sufficient size to store data coming on its one interface and it may then relay the data through the same or the other interface (depending on which mode it is working on). A relay node may send its own data as well. We further assume that there is negligible processing delay at a node and no incoming packets are dropped.

In the considered network, all MSs are distributed uniformly. By using a kind of pilot signal, the BS can inform the MSs in the whole network about their working mode at any given instant either directly or through the relay nodes. The algorithm for updating the network topology information is assumed to be working on the backend.

For the HN, identical transmission power is assumed for all MSs so that the Signal-to-Interference Ratio (SIR) is mainly dependent on the distance and path-loss coefficient. All other types of noise are assumed to be negligible for the whole network. We assume omnidirectional antennas for both the BS and MS, and further assume that the effects of the uncovered regions due to circular geometry are negligible.

The analysis is based on the assumption that there are negligible errors when we approximate from hexagonal cell to a circular cell. Also, there may be nodes existing outside region-B (in order to make the interference calculations, for nodes close to the boundary, more complete). Hence, the formulas for interference power are consistent.

III. ANALYSIS OF THE HYBRID NETWORK CAPACITY

The purpose of the following analytical work is to find the optimum overall capacity for the HN, and then to find the optimum range of distance from the BS to place relay nodes. To achieve this purpose, we start with the calculation of interference level for both the ad hoc and the cellular networks. Circular geometry is used for calculations. Initially, the total interference for a receiving MS in ad hoc mode is determined and then SIR is computed for this link. This also enables us to calculate the capacity for this ad hoc link, C_{adhoc}. For the capacity of a cellular link, $C_{cellular}$, a straightforward methodology is adopted but with a constraint (to be described in Section III-B). When both C_{adhoc} and $C_{cellular}$ are obtained, the overall capacity of HN can be found. This also helps towards the optimal placement of the relay node.

A. Capacity Experienced by a Receiving Mobile Station (in Ad Hoc Mode) from a Relay Node

Fig. 2 is used as a basic model for the calculation of total interference experienced by a receiving MS from all MSs within the interference range from the MS to the relay. In this

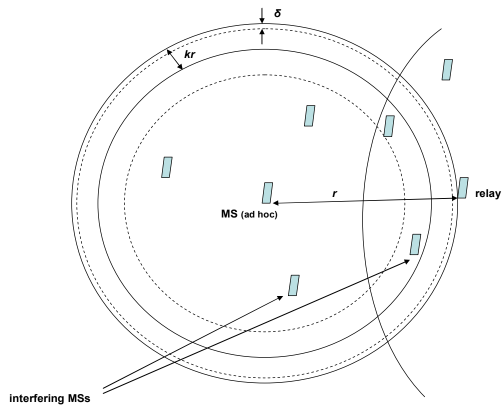


Fig. 2. Basic Model for Total Interference Calculation (Ad Hoc to Relay).

figure, r is the radius of the interference region of the MS (not BS, since here we are calculating only for the ad hoc part). [This r can be varied to see the effects on the capacity (to be described in Section IV).] This region is subdivided into $1/k$ sub-regions, where $0<k<<1$. To perform continuous analysis, we consider, within a sub-region, a concentric annular ring with width δ between its outer and inner boundaries. The area of such a ring is given as

$$A_\delta = \pi r^2 - \pi (r-\delta)^2, \quad (1)$$

which is the elemental area for the whole HN.

Thus, any two MSs in the above mentioned ring may be considered to be at the same distance from the receiving MS at the center of the circle. The interference generated in this ring towards the receiving MS becomes

$$I_\delta = \frac{P_t \rho A_\delta}{x^\alpha}, \quad (2)$$

where x is the distance of an MS inside this ring from the receiving MS, ρ is the normalized node density, P_t is the transmit power of each MS, and α is the path-loss coefficient.

In each sub-region with width kr, there are sufficiently large number of rings each with width δ. Therefore, the integration of (2) with respect to δ from 0 *to* kr gives the interference contribution by a sub-region, I_{kr}, as

$$I_{kr} = \int_0^{kr} I_\delta d\delta \cdot \quad (3)$$

At any instant, for the receiving MS, there is only one transmitting relay and all other transmitting MSs are simply contributing as interferers. The contribution of each of these interferers is also a function of their individual distance from the receiving MS. Hence the total interference, I_{total}, towards the receiving MS is obtained by integrating all the individual interference contributions over all the interference region of the receiving MS, (here, 0 to r). Thus,

$$I_{total} = \int_0^{r} I_{kr} dx \; = P_t \rho \left(\pi k^2 - \frac{\pi}{3} k^3 \right) \left(\frac{r^{4-\alpha}}{4-\alpha} \right) \quad (4)$$

where P_t is the same for all MSs, and is fixed for a certain communication instant.

Now we consider the signal received at the MS. Given the distance between the relay node and the MS as r, the signal strength received at the MS, S_a, is obtained by

$$S_a = \frac{P_t}{r^\alpha} \cdot \quad (5)$$

Since I_{total} is already found for a link between the sender and the receiver MS, the ad hoc signal-to-interference ratio, SIR_{adhoc}, can be calculated as

$$SIR_{adhoc} = \frac{S_a}{I_{total}} = K_1 \frac{4-\alpha}{r^4} \;, \quad (6)$$

where $K_1 = \dfrac{1}{\rho \left(\pi k^2 - \frac{\pi}{3} k^3 \right)}$. (7)

Given SIR_{adhoc}, the ad hoc capacity can be found using the famous Shannon formula [1],

$$C_{adhoc} = B \log_2 \left(1 + SIR_{adhoc} \right), \quad (8)$$

where B is the bandwidth of the channel.

B. Capacity Between the Relay Station and the Base Station

Considering Fig. 3, the maximum possible distance for an MS from the BS is D. Given that a relay node is placed somewhere between the MS and the BS on a straight line, D can be simply divided into two parts, i.e., r, which is the distance between MS and relay node, and D-r, which is the distance between relay node and BS. Increasing r means placing a relay node closer to the BS, and vice-versa.

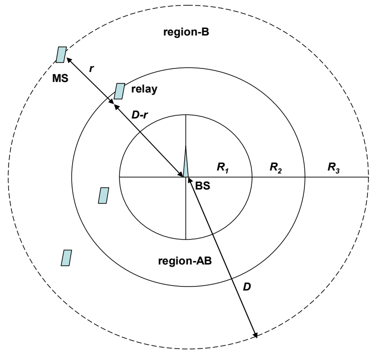


Fig. 3. Variation of r and Placement of Relay Node.

According to [7], the SIR for a receiver in a cellular network (e.g., a CDMA system) can be approximated as

$$SIR = \frac{1}{N-1}, \quad (9)$$

where N is the number of MSs in the interference region of a cellular node. As we are using the continuous-analysis approach, the signal-to-interference ratio would also depend on the node density, ρ, and the distance, D-r. Since all interfering MSs are at most D-r distance away from the BS, the number of MSs inside this range becomes $N = \rho \pi (D-r)^2$. Thus, the signal-to-interference ratio for the cellular link, $SIR_{cellular}$, is given by

$$SIR_{cellular} = \frac{1}{\pi \rho (D-r)^2 - 1} \;, \quad (10)$$

with a constraint

$$\pi \rho (D-r)^2 \geq 2, \text{ that is, } r \leq D - \sqrt{\frac{2}{\pi \rho}}. \quad (11)$$

The above mentioned inequality (11) is imposed by considering that there should be at least two stations in this region in order to have the communication to take place. That is, one station is the transmitter (the relay node in this case) and the other station is acting as a receiver (the BS in this case). Other stations are regarded as interferers. The capacity for the cellular link, $C_{cellular}$, can now be calculated as

$$C_{cellular} = B \log_2 \left(1 + SIR_{cellular} \right). \quad (12)$$

C. Total Capacity of the Hybrid Network

For the ad hoc network part, the capacity C_{adhoc} will decrease as r is increased (clear from Eqs. (6) and (8)). For the cellular network part, the capacity $C_{cellular}$ will increase as an MS moves closer to the BS (clear from Eqs. (10) and (12). Thus, there would be a point in the HN where the overall total capacity of the HN is optimum. This means that there exists a cross point where the curves for C_{adhoc} and $C_{cellular}$ intersect, and this would be the total optimum capacity, C_{total}^{opt}, for the whole HN. Hence, the task of placing the relay node in an optimal location can be summarized in the following expression:

$$\textbf{maximize} \quad C_{total}^{opt}$$
$$\textbf{subject to} \quad d_{MS\text{-}relay} + d_{relay\text{-}BS} = D, \qquad (13)$$

where $d_{MS\text{-}relay}$ is the distance between the MS and its relay node and $d_{relay\text{-}BS}$ is the distance between the relay node and the BS. In other words, $C_{total}^{opt} = \max\{\min\{C_{adhoc}, C_{cellular}\}\}$. That is, we need to find an optimal value of r which gives maximum value for C_{total}^{opt}. This means to find an appropriate location for a relay node between an MS and the BS such that the maximum overall capacity for the whole HN is achieved.

Since the BS knows the number of MSs operating in a specific mode, it can dictate a node to work as a relay if it is within a certain range where optimum capacity is achieved. This is done by the BS by measuring the values of node density and path-loss constant.

IV. Numerical Results and Discussions

In this section we present the numerical results achieved from the analytical model of our scheme. MATLAB is used to analyze the performance of the system. An ideal medium access protocol is assumed. As mentioned earlier, the MSs are placed uniformly in the whole region of interest. The maximum interference region centered from the BS is a circle of radius $R_1+R_2+R_3=D$. The maximum distance between the BS and the boundary of the network, D, is set as one kilometer. For illustration convenience, we assign this maximum radius at a normalized scale of D=10 in Figs. 4 and 5. For example, an MS placed at distance r=7 means that it is placed at a distance of 300 meters from the BS, of the whole possible distance of 1000 meters. Point 10 on the horizontal axis in all the graphs represents the location of the BS and point 0 represents the location of an MS at the boundary of the HN. It is again mentioned that there are some MSs assumed to be existing outside the boundary of the network in order to make the interference calculations complete. The vertical axis in all graphs illustrates the calculated capacity, C, in Mbps. The path-loss coefficient, α, is kept as 2 for Fig. 4 and as 3.5 for Fig. 5. The whole network region is divided into 100 concentric annular sub regions, i.e., k=0.01.

Fig. 4 and 5 illustrate the capacity curves for the cellular link as well as the ad hoc link with respect to the distance from the BS. The dashed curves represent the ad hoc link capacity and solid curves represent the cellular link capacity. It can be easily observed that although the BS can be reached by nodes in region-B in case of pure cellular network, the capacity goes down as the distance from the BS increases. However, if we introduce a hybrid network, higher system capacity can be achieved. That is, when a relay node is placed between the BS and region-B, a better overall capacity may be achieved. By placing the relay node in most appropriate location, optimal network capacity can be reached, as described in the following two subsections.

A. *Case I: Free-space Environment*

Considering Fig. 4, as expected, we observe that as r increases, C_{adhoc} decreases while $C_{cellular}$ increases. Thus, there is an optimal location, r_{opt}, for optimum overall capacity of the HN, which is obtained at the cross point of these two curves. Fig. 4(a) depicts a sparsely populated network (ρ=0.2). Here we achieve an optimum capacity of 3.87 Mbps at r_{opt}=7.4. In Fig. 4(b), which represents a moderately populated network, the optimum capacity decreases to 2.25 Mbps and is achieved at r_{opt}=7.9. This is about 40% decrease with respect to the optimum capacity in Fig. 4(a). Fig. 4(c) represents a densely populated network. Here, the cross point of the capacity curves is achieved at r_{opt}=8, with C_{total}^{opt}=1.52 Mbps. Hence, the optimum capacity has dropped about 33% with respect to Fig. 4(b) and about 60% with respect to Fig. 4(a). This is due to the fact that as ρ increases, interference increases and thus the capacity decreases. Therefore, one needs to put a relay node closer to the BS in order to achieve the optimal capacity because more nodes are now generating interference. From the above observations, we conclude that an optimum range of distance from the BS for a relay node placement is found, which is, in this case, $r_{opt} \approx (7.4 \sim 8)$.

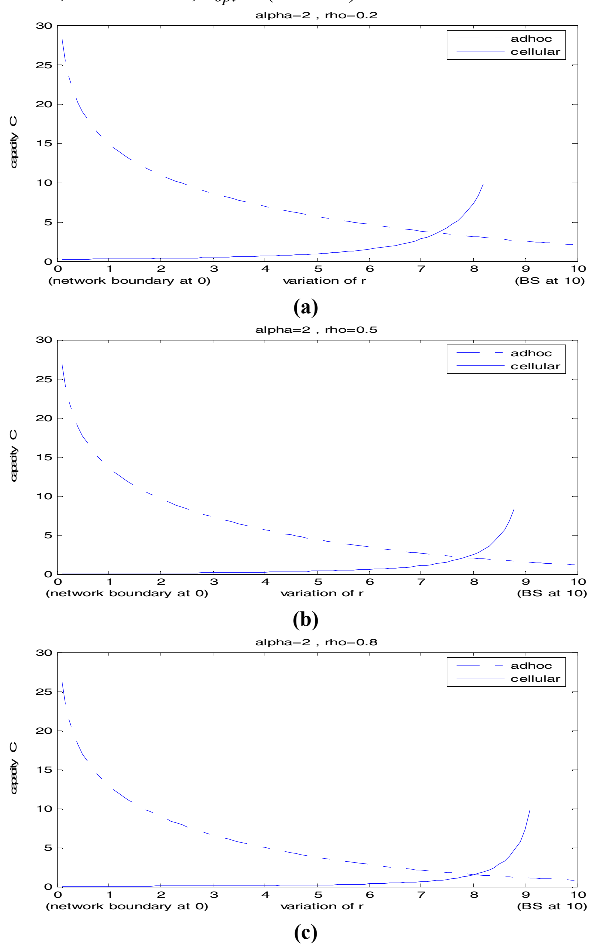
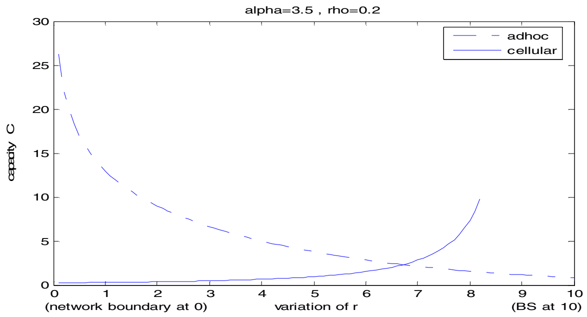


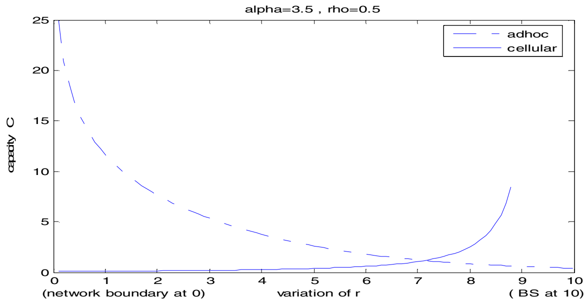
Fig. 4. Capacity curves for the hybrid network with lower path-loss constant: (a) Sparsely populated, (b) Moderately populated, (c) Densely populated.

B. *Case II: Urban Area Semi-open Environment*

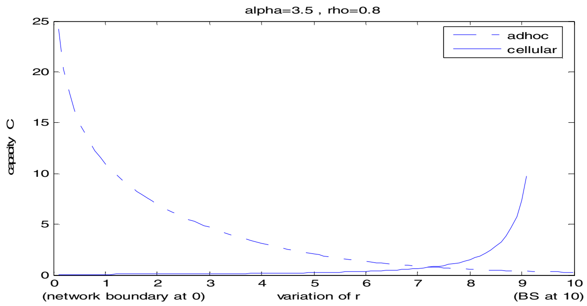
Fig. 5(a, b, c) represents the results for a greater value of α, i.e., α=3.5, for the sparsely, moderately, and densely populated HN, respectively.



(a)



(b)



(c)

Fig. 5. Capacity curves for the hybrid network with higher path-loss constant: (a) Sparsely populated, (b) Moderately populated, (c) Densely populated.

As shown in the figure, when we move the relay node closer to the BS, i.e., as r is increased, a change in optimum capacity is observed. However, the comparison of Fig. 4(a) and Fig. 5(a) shows that the cross point of the capacity curves in Fig. 5(a), (as r_{opt}=6.8), is lower than the cross point in Fig. 4(a), The reason for this is that as α increases, the capacity decreases for both network components (provided all the other relevant parameters are kept the same). This means, in order to achieve optimal overall capacity, one needs to place the relay node closer to the BS. Similarly, Fig. 4(b) can be compared with Fig. 5(b), and Fig. 4(c) with Fig. 5(c), and the same effect can be observed, i.e., a corresponding drop in the capacity with increase in α. More specifically, the achieved C_{total}^{opt} in Fig. 5(a,b,c) are 2.28, 1.14, and 0.8 Mbps, respectively. These values are substantially lower than the corresponding values we observed in the free-space environment. However, again, an optimum range for the relay node placement is found, which is $r_{opt} \approx (6.8 \sim 7.3)$.

From both Fig. 4 and Fig. 5, two important observations can be made. Firstly, the overall optimum capacity of the HN does not improve if a relay node is placed beyond a certain distance from the BS. However, a range for optimal placement of the relay node is found, as $r_{opt} \approx (6.8 \sim 8)$, in general. Secondly, for known values of ρ and α, a precise location for the relay node can be found which ensures maximum overall capacity for the HN.

V. Conclusions and Future Work

In this paper, we have proposed an approach of finding the optimum range for relay node placement in a Hybrid Network consisting of an ad hoc component and a cellular component. The major contributions of this paper are that we have proposed a method which employs continuous-space circular geometry for analyzing the optimum overall capacity of HN and that we have obtained optimal range of positions for relay node placement. From the numerical results, it is also found that, for known values of node density and path-loss coefficient, a *precise location* for a relay node placement can be determined which can give maximum overall capacity of the hybrid network.

As part of the future work, we are investigating the methods for efficient selection of a relay node when multiple relay nodes are available within the optimum range. Another possible future direction is to explore methods to improve the overall optimum capacity for such a scenario.

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