[9] L. Lin, R. D. Yates, and P. Spasojević, "Adaptive transmission with discrete code rates and power levels," *IEEE Trans. Commun.*, vol. 51, no. 12, pp. 2115–2125, Dec. 2003.
[10] L. Lin, R. D. Yates, and P. Spasojević, "Adaptive transmission with finite code rates," *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 1847–1860, May 2006.
[11] T. Kim and M. Skoglund, "On the expected rate of slowly fading channels with quantized side information," *IEEE Trans. Commun.*, vol. 55, no. 4, pp. 820–829, Apr. 2007.
[12] V. Lau and T. Wu, "On the encoding rate and discrete modulation adaptation design for MIMO links," *IEEE Trans. Wireless Commun.*, vol. 6, no. 1, pp. 50–54, Jan. 2007.
[13] Z. Zhou, B. Vucetic, M. Dohler, and Y. Li, "MIMO systems with adaptive modulation," *IEEE Trans. Veh. Technol.*, vol. 54, no. 5, pp. 1828–1842, Sep. 2005.
[14] A. Forenza, M. McKay, A. Pandharipande, R. Heath, and I. Collings, "Adaptive MIMO transmission for exploiting the capacity of spatially correlated channels," *IEEE Trans. Veh. Technol.*, vol. 56, no. 2, pp. 619–630, Mar. 2007.
[15] M. Kiessling, I. Viering, M. Reinhardt, and J. Speidel, "Short-term and long-term diagonalization of correlated MIMO channels with adaptive modulation," in *Proc. IEEE Int. Symp. Pers., Indoor Mobile Radio Commun.*, 2002, pp. 593–597.
[16] S. de la Kethulle de Ryhove, G. E. Øien, and F. Bøhagen, "Subchannel SNR distributions in dual-branch MIMO systems," in *Proc. IEEE/ITG Int. Workshop Smart Antennas*, Duisburg, Germany, Apr. 2005.
[17] A. J. Goldsmith and P. P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Inf. Theory*, vol. 43, no. 6, pp. 1986–1992, Nov. 1997.
[18] S. de la Kethulle de Ryhove, "Rate-adaptive schemes and capacity issues in wireless systems," Ph.D. dissertation, Norwegian Univ. Sci. Technol., Trondheim, Norway, 2007.
[19] M. Chiani, M. Win, and A. Zanella, "On the capacity of spatially correlated MIMO Rayleigh-fading channels," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2263–2371, Oct. 2003.
[20] F. Bøhagen, P. Orten, and G. E. Øien, "On the Shannon capacity of dual MIMO systems in Ricean fading," in *Proc. 6th IEEE Workshop SPAWC*, New York, Jun. 2005, pp. 114–118.
[21] J. Nocedal and S. J. Wright, "Numerical optimization," in *Springer Series in Operations Research and Financial Engineering*. Berlin, Germany: Springer-Verlag, 1999.
[22] D. Goldberg, *Genetic Algorithms in Search, Optimization, and Machine Learning*. Reading, MA: Addison-Wesley, 1989.
[23] K. J. Hole, H. Holm, and G. E. Øien, "Adaptive multidimensional coded modulation over flat fading channels," *IEEE J. Sel. Areas Commun.*, vol. 18, no. 7, pp. 1153–1158, Jul. 2000.
[24] H. Holm, G. E. Øien, M. S. Alouini, D. Gesbert, and K. J. Hole, "Optimal design of adaptive coded modulation schemes for maximum average spectral efficiency," in *Proc. 4th IEEE Workshop SPAWC*, Rome, Italy, Jun. 2003, pp. 403–407.
[25] C. Lawrence, J. Zhou, and A. Tits, *User's Guide for CFSQP Version 2.5d*. Tucker, GA: AEM Design, 1997. [Online]. Available: http://www.aemdesign.com/downloadfsqp.htm

Exact Closed-Form Expressions for the Distribution, the Level-Crossing Rate, and the Average Duration of Fades of the Capacity of OSTBC-MIMO Channels

Bjørn Olav Hogstad, Matthias Pätzold, *Senior Member, IEEE*, Neji Youssef, *Member, IEEE*, and Valeri Kontorovitch, *Fellow, IEEE*

***Abstract*—This paper deals with some important statistical properties of the channel capacity of multiple-input–multiple-output (MIMO) systems with orthogonal space-time block code (OSTBC) transmission. We assume that all the subchannels are uncorrelated. For OSTBC-MIMO systems, exact closed-form expressions are derived for the probability density function (PDF), the cumulative distribution function (CDF), the level-crossing rate (LCR), and the average duration of fades (ADF) of the channel capacity. Furthermore, it will be shown that these exact closed-form expressions can be used to characterize the channel capacity of single-input–multiple-output (SIMO) and multiple-input–single-output (MISO) systems. In addition, a Gaussian approximation to the exact LCR of the capacity of OSTBC-MIMO systems is derived. The correctness of the derived closed-form expressions and the approximation is confirmed by simulations.**

***Index Terms*—Average duration of fades (ADF), channel capacity, cumulative distribution function (CDF), level-crossing rate (LCR), orthogonal space-time block code (OSTBC)-multiple-input–multiple-output (MIMO) systems, probability density function (PDF), Rayleigh fading channels.**

I. Introduction

Multiple-antenna technology is of growing interest in the field of mobile communications as it allows a significant increase in the transmission capacity of wireless channels [1], [2]. Since wireless channels are time varying, due to the random nature of the propagation environments and the mobility of the terminals, the underlying capacity thus randomly varies with time. Hence, the channel propagation conditions are manifested in the corresponding channel capacities. The resulting time-varying capacity therefore suffers from the random occurrence of capacity fades [3], during which the channel is unable to support a specific data rate. For the purpose of efficient system design and performance evaluation, the statistical characteristics regarding this fundamental limitation should be investigated and understood. In a manner similar to that for mobile fading channels, the autocorrelation function (ACF), level-crossing rate (LCR), and average duration of fades (ADF) are commonly used to describe the temporal variations of the capacity. The investigation of these statistical quantities for the channel capacity of multiple-antenna systems has recently gained

Manuscript received November 13, 2007; revised March 1, 2008. First published June 6, 2008; current version published February 17, 2009. This paper was presented in part at the 65th IEEE Vehicular Technology Conference (VTC2007-Spring), Dublin, Ireland, April 2007. The review of this paper was coordinated by Dr. K. T. Wong.

B. O. Hogstad was with the Faculty of Engineering and Science, University of Agder, 4898 Grimstad, Norway. He is now with the Centro de Estudios e Investigaciones Técnicas de Gipuzkoa (CEIT) and Tecnun, University of Navarra, 20018 San Sebastián, Spain (e-mail: bohogstad@ceit.es).
M. Pätzold is with the Faculty of Engineering and Science, University of Agder, 4898 Grimstad, Norway (e-mail: Matthias.Paetzold@uia.no).
N. Youssef is with the Ecole Supérieure des Communications de Tunis, Route de Raoued, 2083 Tunis, Tunisia (e-mail: neji.youssef@supcom.rnu.tn).
V. Kontorovitch is with the Department of Electrical Engineering, Telecommunications Section, Center for Advanced Study and Research of the National Polytechnic Institute (CINVESTAV), 07360 Mexico City, Mexico (e-mail: valeri@cinvestav.mx).

Digital Object Identifier 10.1109/TVT.2008.927038

much interest. The LCR and ADF of the capacity fade of general multiple-input–multiple-output (MIMO) systems were first considered in [4] and later in [5], by assuming that the capacity can be approximated by a discrete Gaussian model. The analysis of the correlation of the instantaneous capacity of 2×2 MIMO systems was addressed in [6]. For single-input–single-output (SISO) channels, the ACF, LCR, and ADF were presented in [18]. Furthermore, the difference between the MIMO channel capacity and the capacity of OSTBC-MIMO systems has been analyzed in [8].

Although the statistical properties of channel capacity have intensively been studied in the literature, there are still some unsolved problems. In this paper, we provide closed-form expressions for the probability density function (PDF), cumulative distribution function (CDF), LCR, and ADF for the continuous capacity of OSTBC-MIMO channels. Our derivations are performed using the characteristic function (CF) approach. Exact closed-form expressions for the PDFs of the SIMO and MISO channel capacities are derived in [9]. For SIMO and MISO channels, where the powers of all branches are different, an expression for the CDF of the channel capacity can be found in [10]. In this paper, we present the CDF when all the branch powers are equal. It should be mentioned that the obtained CDF in [10] is not valid in this case, since the denominator in [10, eq. (14)] will be equal to zero. In our derivation of the LCR and ADF, we assume the general case of a nonsymmetrical Doppler power spectral density (PSD). We should mention that the statistics of the LCR and ADF of the χ^2 processes [11] cannot be applied here, because they are reported for a symmetrical Doppler PSD. Hence, to the best of our knowledge, the closed-form expressions of the LCR and ADF of the capacity of OSTBC-MIMO systems are completely new.

The remainder of this paper is organized as follows. In Section II, we will describe the Rayleigh process with cross-correlated inphase and quadrature components. A detailed study of the statistical properties of the capacity of OSTBC-MIMO systems is the topic of Section III. Furthermore, a Gaussian approximation of the exact LCR of OSTBC-MIMO systems is derived in Section IV. The analytical results obtained in Sections III and IV will be compared with the simulation results in Section V. Finally, the conclusion is drawn in Section VI.

II. Rayleigh Process

Throughout this paper, we deal with frequency-nonselective mobile channel models, which will be described by making use of the complex baseband representation of the passband signals. The time-varying complex channel gain between a single transmit and a single receive antenna will be denoted by $h(t)$. The complex process $h(t)$ can be expressed as

$$h(t) = h^I(t) + jh^Q(t) \tag{1}$$

where the inphase and quadrature components of $h(t)$ are denoted by $h^I(t)$ and $h^Q(t)$, respectively. In general, it is usually assumed that $h^I(t)$ and $h^Q(t)$ are uncorrelated zero-mean real Gaussian noise processes with identical variances $\sigma^2_{h^I} = \sigma^2_{h^Q} = \sigma^2$ [12]. In this paper, the inphase component $h^I(t)$ and quadrature component $h^Q(t)$ are allowed to be correlated. The correlation properties are described by the correlation matrix in [13, eq. (17)]. Since we will assume that $h(t)$ is a zero-mean complex Gaussian process with unit variance, the absolute value of $h(t)$ (which is also denoted as envelope)

$$\zeta(t) = |h(t)| = \sqrt{[h^I(t)]^2 + [h^Q(t)]^2} \tag{2}$$

follows a Rayleigh distribution. The time derivative of $\zeta(t)$ will be denoted by $\dot{\zeta}(t)$. Throughout this paper, we let the overdote of a process denote the time derivative. From [13], the joint PDF of $\zeta(t)$ and $\dot{\zeta}(t)$ at the same time t can be expressed as

$$p_{\zeta\dot{\zeta}}(z, \dot{z}) = \sqrt{\frac{2}{\pi\beta}} z e^{-z^2 - \dot{z}^2/(2\beta)}, \qquad z \geq 0, \quad |\dot{z}| < \infty \tag{3}$$

where

$$\beta = -\frac{d^2}{d\tau} r_{h^I h^I}(\tau)\bigg|_{\tau=0} - \frac{\frac{d}{d\tau} r_{h^I h^Q}(\tau)\Big|_{\tau=0}}{r_{h^I h^Q(0)}}. \tag{4}$$

In (4), $r_{h^I h^I}(\tau)$ denotes the ACF of $h^I(t)$, and $r_{h^I h^Q}(\tau)$ is the cross-correlation function of $h^I(t)$ and $h^Q(t)$. In case of isotropic scattering, the inphase and quadrature components $h^I(t)$ and $h^Q(t)$ are uncorrelated, and we obtain $\beta = 2(\pi\sigma f_{\max})^2$, where $f_{\max}$ is the maximum Doppler frequency.

For the calculations of the statistical properties of the capacity of OSTBC-MIMO systems (see Section III), we need the joint PDF of $\zeta^2(t)$ and $\dot{\zeta}^2(t)$, which is denoted by $p_{\zeta^2\dot{\zeta}^2}(z, \dot{z})$. To find the joint PDF, we apply the concept of transformation of random variables [14]. For fixed values of $t = t_0$, the stochastic processes $\zeta(t)$ and $\dot{\zeta}(t)$ become random variables $\zeta(t_0)$ and $\dot{\zeta}(t_0)$, respectively. The joint PDF $p_{\zeta^2\dot{\zeta}^2}(z, \dot{z})$ of $\zeta(t_0)$ and $\dot{\zeta}(t_0)$ can be expressed as

$$\begin{aligned} p_{\zeta^2\dot{\zeta}^2}(z, \dot{z}) &= \frac{1}{4z} p_{\zeta\dot{\zeta}}\left(\sqrt{z}, \dot{z}/\left(2\sqrt{z}\right)\right) \\ &= \frac{1}{2\sqrt{2\pi\beta z}} e^{-z - \dot{z}^2/(8\beta z)}, \quad z \geq 0, \quad |\dot{z}| < \infty. \end{aligned} \tag{5}$$

From (5), we can easily see that we cannot express the joint PDF $p_{\zeta^2\dot{\zeta}^2}(z, \dot{z})$ as a product of two marginal PDFs, which are denoted by $p_{\zeta^2}(z)$ and $p_{\dot{\zeta}^2}(\dot{z})$. Hence, the stochastic processes $\zeta^2(t)$ and $\dot{\zeta}^2(t)$ are not statistically independent. Nevertheless, we can obtain the PDF $p_{\zeta^2}(z)$ as follows:

$$\begin{aligned} p_{\zeta^2}(z) &= \int_{-\infty}^{\infty} p_{\zeta^2\dot{\zeta}^2}(z, \dot{z}) d\dot{z} \\ &= e^{-z}, \qquad z \geq 0. \end{aligned} \tag{6}$$

Equation (6), together with the CF of $p_{\zeta^2}(z)$, will be used in the next section to derive the PDF of the capacity of OSTBC-MIMO systems. The CF of $p_{\zeta^2}(z)$ is denoted by $\Phi_{\zeta^2}(\omega)$. This function is defined as the Fourier transform of $p_{\zeta^2}(z)$, i.e.,

$$\begin{aligned} \Phi_{\zeta^2}(\omega) &= \int_{-\infty}^{\infty} p_{\zeta^2}(z) e^{j\omega z} dz \\ &= \frac{1}{1 - j\omega}. \end{aligned} \tag{7}$$

For the calculation of the LCR of the capacity of OSTBC-MIMO systems, we need the joint CF of $\zeta^2(t)$ and $\dot{\zeta}^2(t)$, which is denoted by $\Phi_{\zeta^2\dot{\zeta}^2}(\omega,\dot{\omega})$. This function can be expressed as

$$\Phi_{\zeta^2\dot{\zeta}^2}(\omega,\dot{\omega}) = \int_{-\infty}^{\infty}\int_{0}^{\infty} p_{\zeta^2\dot{\zeta}^2}(z,\dot{z})e^{j(\omega z+\dot{\omega}\dot{z})}dzd\dot{z} = \frac{1+2\beta\omega^2+j\dot{\omega}}{(1+2\beta\omega^2)^2+\dot{\omega}^2}. \quad (8)$$

In the next section, we will present the capacity of OSTBC-MIMO systems. In addition, the SIMO and MISO channels will be considered by using the preceding Rayleigh fading process.

III. Statistical Properties of the Channel Capacity

In the following, we consider a MIMO channel with M_T and M_R antennas at the transmitter and receiver, respectively. The complex channel gains will be denoted by $h_m(t)$ for $m = 1,\ldots,M_TM_R$. It is assumed that stochastic processes $h_m(t)$ for $m = 1,\ldots,M_TM_R$ are uncorrelated complex Gaussian processes.

For OSTBC-MIMO systems, the channel capacity can be written as [15]

$$C_{\mathrm{MIMO}}(t) = \log_2\left(1+\frac{\gamma}{M_T}\mathbf{h}^H(t)\mathbf{h}(t)\right) \quad (9)$$

where $\mathbf{h}(t) = [h_1(t),\ldots,h_{M_TM_R}(t)]^T$ is the $M_TM_R \times 1$ complex channel gain vector. The transpose and the complex conjugate transpose operators are denoted by $(\cdot)^T$ and $(\cdot)^H$, respectively. Finally, the quantity γ is the average signal-to-noise ratio (SNR). Alternatively, the capacity $C_{\mathrm{MIMO}}(t)$ in (9) can be expressed in component form as

$$C_{\mathrm{MIMO}}(t) = \log_2\left(1+\frac{\gamma}{M_T}\sum_{m=1}^{M_TM_R}\zeta_m^2(t)\right) \quad (10)$$

where $\zeta_m^2(t) = |h_m(t)|^2$. For simplicity, we denote

$$\Lambda(t) = \sum_{m=1}^{M_TM_R}\zeta_m^2(t). \quad (11)$$

By using [16, eq. (2.32)], the PDF of $\Lambda(t)$ is obtained as

$$p_\Lambda(z) = \frac{1}{\Gamma(M_TM_R)}z^{M_TM_R-1}e^{-z}, \qquad z \geq 0 \quad (12)$$

where $\Gamma(\cdot)$ is the Gamma function [17, eq. (8.310)]. Applying the concept of transformation of random variables [14] allows us to find the PDF of $C_{\mathrm{MIMO}}(t)$, which is denoted by $p_{C,\mathrm{MIMO}}(r)$, as a function of the PDF $p_\Lambda(z)$ of the stochastic process $\Lambda(t)$ in the following form:

$$p_{C,\mathrm{MIMO}}(r) = \frac{2^rM_T\ln 2}{\gamma}p_\Lambda\left(M_T(2^r-1)/\gamma\right) = \frac{(M_T)^{M_TM_R}\ln 2}{\Gamma(M_TM_R)\gamma^{M_TM_R}}2^r(2^r-1)^{M_TM_R-1} \times e^{-M_T(2^r-1)/\gamma}, \qquad r \geq 0. \quad (13)$$

The CDF $F_{C,\mathrm{MIMO}}(r)$ of $C_{\mathrm{MIMO}}(t)$ can be expressed as

$$F_{C,\mathrm{MIMO}}(r) = 1-\left(\frac{\gamma}{M_T}\right)^{1-M_TM_R} \times e^{-M_T(2^r-1)/\gamma}(2^r-1)^{M_TM_R-1} \times \sum_{k=0}^{M_TM_R-1}\frac{\gamma^k}{\Gamma(M_TM_R-k)\left[M_T(2^r-1)\right]^k}. \quad (14)$$

To find the LCR and ADF of the capacity $C_{\mathrm{MIMO}}(t)$, we continue as follows. In the Appendix, it is shown that the joint PDF of $\Lambda(t)$ and $\dot{\Lambda}(t)$, which is denoted by $p_{\Lambda\dot{\Lambda}}(z,\dot{z})$, is given by

$$p_{\Lambda\dot{\Lambda}}(z,\dot{z}) = \frac{z^{M_TM_R-1}e^{-z-\dot{z}^2/(8\beta z)}}{2\Gamma(M_TM_R)\sqrt{2\pi\beta z}}, \qquad z \geq 0, |\dot{z}| < \infty. \quad (15)$$

From (9) and (10), it follows that $C_{\mathrm{MIMO}}(t)$ is a function of $\Lambda(t)$. Thus, by applying the concept of transformation of random variables, we obtain

$$p_{C\dot{C},\mathrm{MIMO}}(z,\dot{z}) = \left(\frac{2^zM_T\ln 2}{\gamma}\right)^2 \times p_{\Lambda\dot{\Lambda}}\left(M_T(2^z-1)/\gamma, 2^z\dot{z}M_T\ln 2/\gamma\right) = \frac{2^{2z-1}(M_T)^{M_R}\ln^2 2}{\Gamma(M_TM_R)\gamma^{M_R}\sqrt{2\pi\beta(2^z-1)}} \times e^{-M_T(2^z-1)/\gamma-M_T(2^z\dot{z}\ln 2)^2/(8\beta\gamma(2^z-1))}. \quad (16)$$

The LCR $N_{C,\mathrm{MIMO}}(r)$ of the channel capacity $C_{\mathrm{MIMO}}(t)$ is defined as

$$N_{C,\mathrm{MIMO}}(r) = \int_0^\infty \dot{z}p_{C\dot{C},\mathrm{MIMO}}(r,\dot{z})d\dot{z}, \qquad r \geq 0. \quad (17)$$

After substituting (16) in (17) and carrying out some lengthy algebraic computations, we finally find the result

$$N_{C,\mathrm{MIMO}}(r) = \frac{(M_T)^{M_TM_R}\sqrt{2\gamma\beta(2^r-1)}}{\Gamma(M_TM_R)\gamma^{M_TM_R}\sqrt{\pi M_T}} \times (2^r-1)^{M_TM_R-1}e^{-M_T(2^r-1)/\gamma}. \quad (18)$$

It should be noted that $N_{C,\mathrm{MIMO}}(r)$ is proportional to $f_{\max}$, as can easily be shown by substituting β with [13, eq. (21b)]. Thus, the normalization of $N_{C,\mathrm{MIMO}}(r)$ onto $f_{\max}$ removes the influence of the vehicle speed.

Let us consider the LCR of the SISO channel capacity. When $M_T = M_R = 1$, then (18) reduces to

$$N_{C,\mathrm{SISO}}(r) = \sqrt{\frac{2\beta(2^r-1)}{\pi\gamma}}e^{-(2^r-1)/\gamma}. \quad (19)$$

It should be mentioned that an expression similar to (19) can also be found in [18] if we use the von Mises density [19] for the distribution of the angle of arrival seen at the receiver. Finally, by means of [20], the ADF $T_{C,\mathrm{MIMO}}(r)$ of the MIMO channel capacity is obtained as

$$T_{C,\mathrm{MIMO}}(r) = \frac{F_{C,\mathrm{MIMO}}(r)}{N_{C,\mathrm{MIMO}}(r)}. \quad (20)$$

A closed-form solution can directly be obtained for $T_{C,\mathrm{MIMO}}(r)$ by using the results in (14) and (18).

All the preceding results can easily be applied to characterize the SIMO and MISO channel capacity. First, we consider a SIMO channel with M_R transmit antennas. The SIMO channel capacity $C_{C,\mathrm{SIMO}}(t)$ is defined as

$$C_{\mathrm{SIMO}}(t) = \log_2\left(1+\gamma\mathbf{h}^H(t)\mathbf{h}(t)\right) \quad (21)$$

where $\mathbf{h}(t) = [h_1(t),\ldots,h_{M_R}(t)]^T$ is the $M_R \times 1$ complex channel gain vector. By setting $M_T = 1$ in (13), (14), (17), and (20), we obtain the PDF, CDF, LCR, and ADF, respectively, of the SIMO channel capacity. Second, we consider a MISO channel with M_T transmit antennas. The MISO channel capacity $C_{C,\mathrm{MISO}}(t)$ is defined as

$$C_{\mathrm{MISO}}(t) = \log_2\left(1 + \frac{\gamma}{M_T}\mathbf{h}^H(t)\mathbf{h}(t)\right) \tag{22}$$

where $\mathbf{h}(t) = [h_1(t), \ldots, h_{M_T}(t)]^T$ is the $M_T \times 1$ complex channel gain vector. Again, by setting $M_R = 1$ in (13), (14), (17), and (20), we obtain the PDF, CDF, LCR, and ADF, respectively, of the MIMO channel capacity. Note that the SIMO and MISO systems in (21) and (22) are known as the maximal ratio combiner and maximal ratio transmitter, respectively [15]. In Section V, we confirm the correctness of our analytical expressions by simulations.

Another approach in solving the level-crossing problem is to use a Gaussian approximation. This is the topic of the next section.

IV. Gaussian Approximation of the LCR for OSTBC-MIMO Systems

If the capacity $C_{\mathrm{MIMO}}(t)$ is a standardized continuous-time real Gaussian process with ACF $r_C(\tau)$, then the LCR of $C_{\mathrm{MIMO}}(t)$ has a Gaussian shape (see [4] and [21]). If $C_{\mathrm{MIMO}}(t)$ is a real-valued Gaussian process with mean m_C and variance σ_C^2, then the LCR $N_{C,\mathrm{MIMO}}(r)$ of $C_{\mathrm{MIMO}}(t)$ can be obtained as

$$N_{C,\mathrm{MIMO}}(r) = \frac{\sqrt{-\ddot{\tilde{r}}_C(0)}}{2\pi} e^{-(r-m_C)^2/(2\sigma_C^2)} \tag{23}$$

where the quantity $\ddot{\tilde{r}}_C(0)$ denotes the double derivative of the normalized ACF $\tilde{r}_C(\tau)$ at $\tau = 0$. The ACF $r_C(\tau)$, which is not normalized, is defined by

$$\begin{aligned} r_C(\tau) &= E\left\{C(t)C(t+\tau)\right\} \\ &= E\left\{\log_2\left(1 + \frac{\gamma}{M_T}\mathbf{h}^H(t)\mathbf{h}(t)\right)\right. \\ &\qquad \left. \times \log_2\left(1 + \frac{\gamma}{M_T}\mathbf{h}^H(t+\tau)\mathbf{h}(t+\tau)\right)\right\} \end{aligned} \tag{24}$$

where $E\{\cdot\}$ denotes the expectation operator. Furthermore, the normalized ACF $\tilde{r}_C(\tau)$ is defined by

$$\tilde{r}_C(\tau) = \frac{r_C(\tau) - m_C}{\sigma_C^2}. \tag{25}$$

In general, it seems hard to calculate (24) and, hence, (25) for an arbitrary SNR [18]. However, some simple approximations of (24) can be obtained in the low- and high-SNR regimes. For the low-SNR regime and, hence, $\gamma \to 0$, the function $\log_2(1 + \gamma x)$ can be approximated by γx for fixed x. On the other hand, when $\gamma \to \infty$, the function $\log_2(1 + \gamma x)$ can be approximated by $\log_2(\gamma x)$. For high values of γ, an approximation of the LCR of the SIMO/MISO channel capacity can be found in [4]. It is straightforward to extend this approximation to OSTBC-MIMO channels. By using [17, eq. (9.122)], the formula in [4, eq. (34)] can be simplified as follows:

$$\ddot{\tilde{r}}_C(0) = \frac{2}{(M_T M_R - 1)\dot{\psi}(M_T M_R)} \ddot{r}_h(0) \tag{26}$$

where the function $\dot{\psi}(\cdot)$ is the first derivative of the *Euler's digamma function* [17, eq. (8.360)], and $\ddot{r}_h(\cdot)$ is the second derivative of the ACF of the underlying complex Gaussian subchannels. It should be mentioned that (26) and [4, eq. (34)] are not valid for SISO channels.

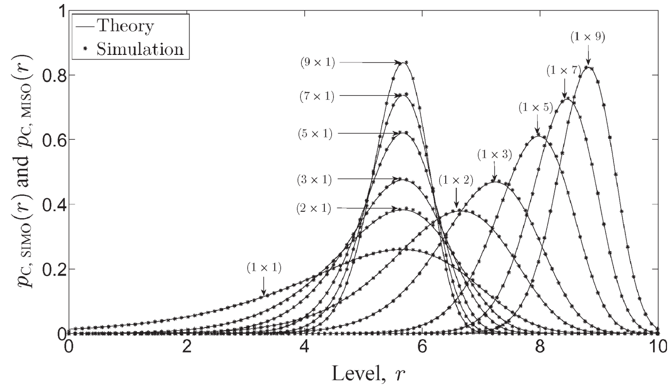

Fig. 1. PDF of the $(1 \times M_R)$ SIMO and $(M_T \times 1)$ MISO channel capacities.

In the following, we assume low SNR. Under this assumption, the ACF $r_C(\tau)$ can be expressed as

$$r_C(\tau) = \frac{M_R \gamma^2 \log_2^2 e \left(M_T M_R + r_h^2(\tau)\right)}{M_T} \tag{27}$$

where $e = 2.71828\ldots$. Since the product $\mathbf{h}^H(t)\mathbf{h}(t)$ is a random process, following the chi-square distribution with $2M_T M_R$ degrees of freedom, the mean and variance of $C_{\mathrm{MIMO}}(t)$ are given by [16, eq. (2.35)]

$$m_C = M_R \gamma \log_2 e \tag{28}$$

$$\sigma_C^2 = \frac{M_R \gamma^2 \log_2^2 e}{M_T} \tag{29}$$

respectively. By assuming $r_h(0) = 1$ and $\dot{r}_h(0) = 0$, we obtain

$$\ddot{\tilde{r}}_C(0) = 2\ddot{r}_h(0). \tag{30}$$

For example, if we consider isotropic scattering and, hence, $r_h(\tau) = J_0(2\pi f_{\max}\tau)$, where $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind, we obtain $\ddot{r}_h(0) = -2\pi^2 f_{\max}^2$, and the LCR $N_{C,\mathrm{MIMO}}(r)$ becomes

$$N_{C,\mathrm{MIMO}}(r) = 2 f_{\max} e^{-M_T(r - M_R\gamma \log_2 e)^2 / \left(2 M_R \gamma^2 \log_2^2 e\right)}. \tag{31}$$

In the next section, the Gaussian approximation presented in (31) is evaluated for its accuracy.

V. Simulation Results

In the following, we present analytical and simulation results of the statistical properties of the channel capacity for various OSTBC-MIMO systems. To generate mutually uncorrelated Rayleigh fading waveforms, we have used the sum-of-sinusoids principle. For the computation of the model parameters, we have used the generalized method of exact Doppler spread (GMEDS_1) [22]. In the applied Rayleigh fading channel simulator, the following parameters have been used: The numbers of sinusoids were $N_1 = 35$ and $N_2 = 36$. The maximum Doppler frequency was 91 Hz. For the simulations of the channel capacity of OSTBC-MIMO systems, the SNR was set to 17 dB. First, we consider the PDF of the capacity for various numbers of receive and transmit antennas in Fig. 1. In all cases, there is an excellent fit between the analytical and simulation results. Fig. 1 shows that the expected value of $C_{\mathrm{MISO}}(t)$ is nearly independent of the number of transmit antennas. In addition, from Fig. 1, we observe that to obtain high capacity, it is more important to have a high number

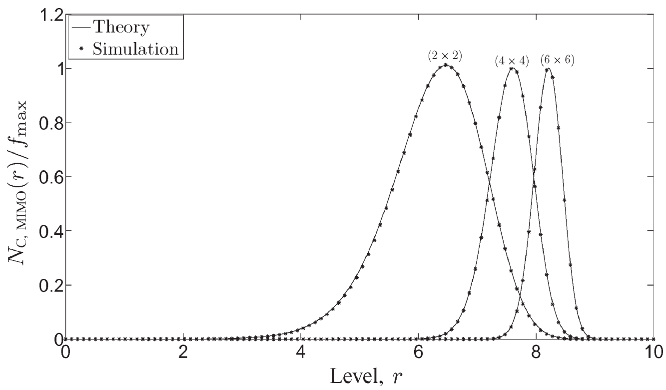

Fig. 2. Normalized LCR of the $(M_T \times M_R)$ OSTBC-MIMO channel capacity.

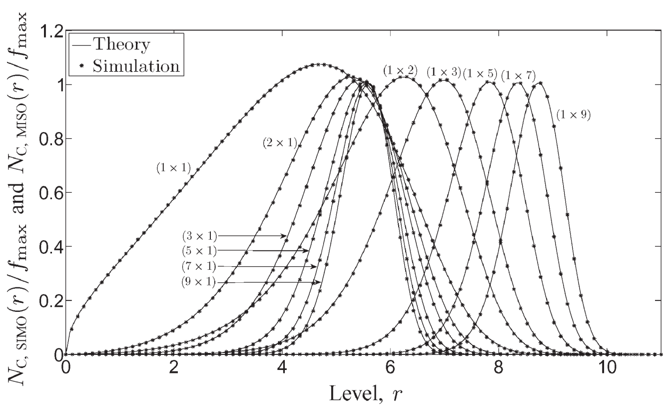

Fig. 3. Normalized LCR of the $(1 \times M_R)$ SIMO and $(M_T \times 1)$ MISO channel capacities.

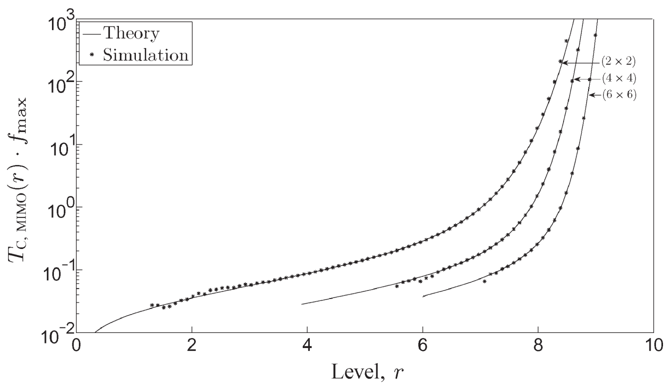

Fig. 4. Normalized ADF of the $(M_T \times M_R)$ OSTBC-MIMO channel capacity.

of receive antennas rather than a high number of transmit antennas. In Figs. 2 and 3, we have presented the normalized LCR of the capacity. In the SIMO and MISO cases, the spread of the capacity decreases with increasing number of antennas. Fig. 3 shows that the maximum LCR is nearly independent of M_R for $M_R \geq 2$. In addition, Fig. 3 shows that the maximum LCR of $C_{\mathrm{MISO}}(t)$ is nearly independent of M_T for $M_T \geq 2$. Again, there is an excellent correspondence between theory and simulation. Fig. 4 shows the normalized ADF of the OSTBC-MIMO channel capacity and the fact that the mean value for the length of the time intervals in which the capacity $C_{\mathrm{MIMO}}(t)$ is below a given length r is decreasing with the number of transmit and receive antennas. In Fig. 5, we can observe the behavior of the ADF of the SIMO channel capacity. Similar to the MIMO case, the ADF

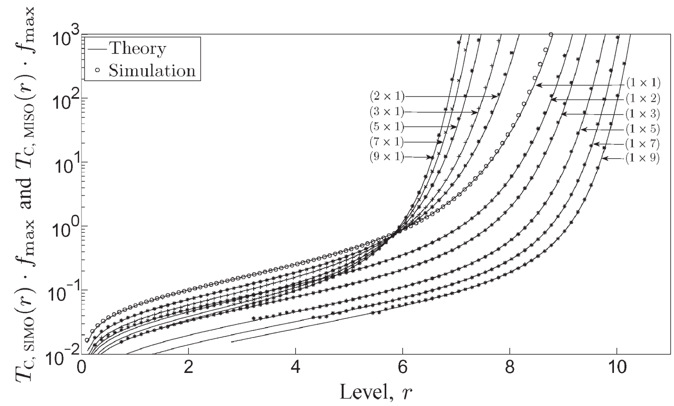

Fig. 5. Normalized ADF of the $(1 \times M_R)$ SIMO and $(M_T \times 1)$ MISO channel capacities.

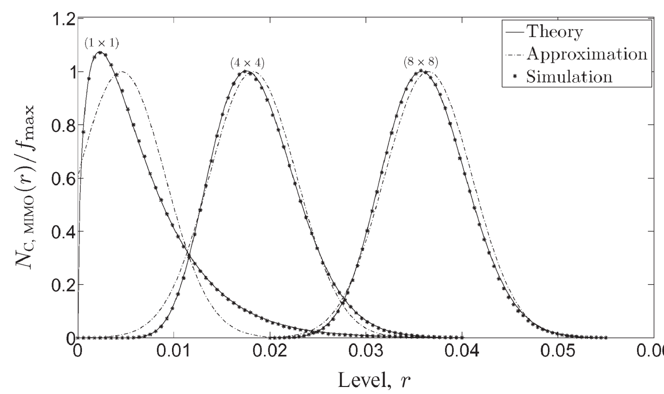

Fig. 6. Normalized LCR of the $(M_T \times M_R)$ OSTBC-MIMO channel capacity.

of $C_{\mathrm{SIMO}}(t)$ decreases with the number of antennas. Furthermore, Fig. 5 shows the ADF of the MISO channel capacity. Around the mean value of $C_{\mathrm{MISO}}(t)$, the ADF of $C_{\mathrm{MISO}}(t)$ is nearly independent of the number of transmit antennas. For levels r less than the mean value of $C_{\mathrm{MISO}}(t)$, the ADF decreases with the number of transmit antennas. The opposite occurs for levels r that are larger than the mean value of $C_{\mathrm{MISO}}(t)$. Finally, in Fig. 6, we present the Gaussian approximation of the LCR for OSTBC-MIMO systems. Here, we have used $\gamma = -25$ dB. Remember that our Gaussian approximation only works for low SNR. Furthermore, the approximation works very well, even for moderate numbers of antennas.

VI. Conclusion

In this paper, we have studied the statistical properties of the capacity for various OSTBC-MIMO channels. Exact closed-form solutions for the PDF, CDF, LCR, and ADF of the capacity have been derived. The analytical expressions are valid for any number of transmit and receive antennas. In addition, an accuracy Gaussian approximation of the LCR for OSTBC-MIMO systems has been derived. Our analytical developments help us understand and predict the capacity gain expected from the MIMO technique in terms of the channel dimension and parameters. Simulation results show an excellent correspondence between theory and simulation.

Appendix

Since we have assumed that $h_m(t)$ and, hence, $\zeta_m^2(t)$ are independent stochastic processes, the CF of $\Lambda(t)$ and $\dot{\Lambda}(t)$, which is denoted by $\Phi_{\Lambda\dot{\Lambda}}(\omega, \dot{\omega})$, can be expressed as

$$\Phi_{\Lambda\dot{\Lambda}}(\omega, \dot{\omega}) = \prod_{m=1}^{M_T M_R} \Phi_{\zeta^2 \dot{\zeta}^2}(\omega, \dot{\omega}) \tag{32}$$

where $\Phi_{\zeta^2 \dot{\zeta}^2}(\omega, \dot{\omega})$ is given by (8). Hence, we obtain

$$\Phi_{\Lambda\dot{\Lambda}}(\omega, \dot{\omega}) = \frac{1}{(1 + 2\beta\omega^2 - j\dot{\omega})^{M_T M_R}}. \tag{33}$$

Using the inversion formula of the 2-D Fourier transforms, it follows that the joint PDF $p_{\Lambda\dot{\Lambda}}(z, \dot{z})$ of $\Lambda(t)$ and $\dot{\Lambda}(t)$ can be expressed as

$$p_{\Lambda\dot{\Lambda}}(z, \dot{z}) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{\Lambda\dot{\Lambda}}(\omega, \dot{\omega}) e^{-j(\omega z + \dot{\omega}\dot{z})} d\omega d\dot{\omega} \tag{34}$$

for $z \geq 0$ and $|\dot{z}| < \infty$. By substituting (33) in (34), we obtain, after some lengthy algebraic computations, the following expression:

$$p_{\Lambda\dot{\Lambda}}(z, \dot{z}) = \frac{z^{M_T M_R - 1} e^{-z - \dot{z}^2/(8\beta z)}}{2\Gamma(M_T M_R)\sqrt{2\pi\beta z}}, \qquad z \geq 0, \quad |\dot{z}| < \infty. \tag{35}$$

ACKNOWLEDGMENT

The authors would like to thank the reviewers for their constructive and valuable comments.

REFERENCES

[1] G. H. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wirel. Pers. Commun.*, vol. 6, no. 3, pp. 311–335, Mar. 1998.

[2] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, 1999.

[3] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 5, pp. 684–702, Jun. 2003.

[4] A. Giorgetti, P. J. Smith, M. Shafi, and M. Chiani, "MIMO capacity, level crossing rates and fades: The impacts of spatial/temporal channel correlation," *J. Commun. Netw.*, vol. 5, no. 2, pp. 104–115, 2003.

[5] P. H. Kuo and P. J. Smith, "On the probability of adaptation error in MIMO systems," in *Proc. 17th IEEE Int. Symp. PIMRC*, Helsinki, Finland, Sep. 2006, pp. 1–5.

[6] S. Wang and A. Abdi, "Correlation analysis on instantaneous mutual information in 2 × 2 MIMO systems," in *Proc. 40th Annu. Conf. Inf. Sci. Syst.*, Princeton, NJ, Feb. 2006, pp. 542–546.

[7] B. Kedem, *Time Series Analysis by Higher Order Crossings*. Piscataway, NJ: IEEE Press, 1994.

[8] S. Sandhu and A. Paulraj, "Space-time block codes: A capacity perspective," *IEEE Commun. Lett.*, vol. 4, no. 12, pp. 384–386, Dec. 2000.

[9] H. Ge, K. D. Wong, M. Barton, and J. C. Liberti, "Statistical characterization of multiple-input multiple-output (MIMO) channel capacity," in *Proc. IEEE Wireless Commun. Netw. Conf.*, Mar. 2002, vol. 2, pp. 789–793.

[10] P. J. Smith, L. M. Garth, and S. Loyka, "Exact capacity distributions for MIMO systems with small numbers of antennas," *IEEE Commun. Lett.*, vol. 7, no. 10, pp. 481–483, Oct. 2003.

[11] R. Silverman, "The fluctuation rate of the chi process," *IEEE Trans. Inf. Theory*, vol. IT-4, no. 1, pp. 30–34, Mar. 1958.

[12] M. Pätzold, *Mobile Fading Channels*. Chichester, U.K.: Wiley, 2002.

[13] M. Pätzold, U. Killat, and F. Laue, "An extended Suzuki model for land mobile satellite channels and its statistical properties," *IEEE Trans. Veh. Technol.*, vol. 47, no. 2, pp. 617–630, May 1998.

[14] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th ed. New York: McGraw-Hill, 2002.

[15] A. J. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge, U.K.: Cambridge Univ. Press, 2003.

[16] M. K. Simon, *Probability Distributions Involving Gaussian Random Variables: A Handbook for Engineers and Scientists*. Boston, MA: Kluwer, 2002.

[17] I. S. Gradstein and I. M. Ryshik, *Tables of Series, Products, and Integrals*, 5th ed, vol. I/II. Frankfurt, Germany: Harri Deutsch, 1981.

[18] S. Wang and A. Abdi, "On the second-order statistics of the instantaneous mutual information of time-varying fading channels," in *Proc. 6th IEEE Workshop Signal Process. Adv. Wireless Commun.*, New York, 2005, pp. 405–409.

[19] A. Abdi, J. A. Barger, and M. Kaveh, "A parametric model for the distribution of the angle of arrival and the associated correlation function and power spectrum at the mobile station," *IEEE Trans. Veh. Technol.*, vol. 51, no. 3, pp. 425–434, May 2002.

[20] W. C. Jakes, Ed., *Microwave Mobile Communications*. Piscataway, NJ: IEEE Press, 1993.

[21] W. C. Jakes, Ed., *Microwave Mobile Communications*. Piscataway, NJ: IEEE Press, 1995. classic reissue ed.

[22] M. Pätzold and B. O. Hogstad, "Two new methods for the generation of multiple uncorrelated Rayleigh fading waveforms," in *Proc. 63rd IEEE Semiannu. VTC—Spring*, Melbourne, Australia, May 2006, vol. 6, pp. 2782–2786.

SVD-Assisted Multiuser Transmitter and Multiuser Detector Design for MIMO Systems

W. Liu, L. L. Yang, and L. Hanzo

***Abstract*—A novel singular value decomposition (SVD)-based joint multiuser transmitter (MUT) and multiuser detector (MUD) aided multiple-input–multiple-output (MIMO) system is proposed, which takes advantage of the channel state information (CSI) of all users at the base station (BS), but only of the mobile station (MS)'s own CSI, to decompose the multiuser (MU) MIMO channels into parallel single-input–single-output (SISO) channels, where each SISO channel corresponds to the singular values of a particular MS's channel matrix. Based on the proposed scheme, the SVD-based transmission carried out in the context of a single user can readily be extended to the MU case for both the uplink (UL) and downlink (DL). As a beneficial application of the proposed scheme, we improve the system's achievable throughput and highlight its future applications.**

***Index Terms*—Multiple-input multiple-output (MIMO), postprocessing, preprocessing, singular value decomposition (SVD), space-division multiple access (SDMA), zero forcing (ZF).**

I. INTRODUCTION

In multiple-input–multiple-output (MIMO)-aided multiuser systems, both the uplink (UL) and downlink (DL) transmissions experience multiuser interference (MUI), also referred to as multiple access interference (MAI), as well as interantenna interference (IAI). The optimum maximum-likelihood (ML) receiver employed at the mobile station (MS) often imposes excessive computational complexity. To reduce the complexity of the MS, multiuser transmission (MUT) techniques can be invoked at the base station (BS) [1]–[5]. Widely used linear preprocessing techniques, such as the minimum mean square error (MMSE) and the zero-forcing (ZF) MUT arrangements, were

Manuscript received October 11, 2007; revised March 23, 2008, May 30, 2008, and June 10, 2008. First published June 24, 2008; current version published February 17, 2009. The review of this paper was coordinated by Prof. N. Arumugam.

The authors are with the School of Electronics and Computer Science, University of Southampton, Southampton SO17 1BJ, U.K. (e-mail: wl03r@ecs.soton.ac.uk; lly@ecs.soton.ac.uk; lh@ecs.soton.ac.uk).

Digital Object Identifier 10.1109/TVT.2008.927728