LCRT: A ToA Based Mobile Terminal Localization Algorithm in NLOS Environment

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Abstract—**Non line-of-sight (NLOS) propagation in range measurement is a key problem for mobile terminal localization. This paper proposes a Low Computational Residual Test (LCRT) algorithm that can identify the number of line-of-sight (LOS) transmissions and reduce the computational complexity compared with the Residual Test (RT) algorithm. LCRT is based on the assumption that when all range measurements are from LOS propagations, the normalized residual distribution follows the central Chi-Square distribution while for NLOS cases it is non-central. An optimized procedure to generate the sets of range measurements is adopted and Least Square (LS) instead of Approximate Maximum Likelihood (AML) is used during the identification of LOS propagations, resulting in reduced computation complexity. Simulation results show that the LCRT can efficiently identify the set of LOS. The correct decision rate is higher than 92% and the variances of results are approaching to the Cramer-Rao Lower Bound (CRLB) when there are more than 3 LOS propagations.**

I. Introduction

Location information of Mobile Terminal (MT) is a demand in many wireless systems. The rule of the FCC for detection of emergency calls requires all wireless providers to locate an E-911 caller with an accuracy of 100 m and 300 m for 67% and 95% of calls [1]. The location of node is also important information in ad hoc networks, Wireless Sensor Networks (WSNs) [2] and cognitive radio networks [3].

There are four widely used measurement methods for localization: Time of Arrival (ToA), Time Difference on Arrival (TDoA), Angle of Arrival (AoA) and Received Signal Strength Indicator (RSSI). In this paper, we focus on ToA. The ToA based measurement error contains measurement noise (LOS error) and NLOS error. The NLOS error, derived from the unavailability of the direct path, has been considered as a key problem in the location estimation. Therefore, it is important to mitigate or eliminate the NLOS errors in location estimation.

The elimination or mitigation of NLOS errors needs extra information. There are, on the whole, three methods to deal with the NLOS errors. The first one is to use the preknowledge of channels or NLOS conditions. If the model of channels or which path has NLOS propagation is known in advance, it is much more convenient to eliminate the NLOS errors. But the prerequisite information is always limited. The second method is to measure the propagation characteristics to get enough information on channels [4, 5] or rebuild the LOS parameters

in NLOS conditions [6] by using the measurement history based hypothesis test. However, it is not easy to acquire an accurate model or parameters for localization since wireless channels are time-varying. Repeated measurement is also energy consuming, especially in power constrained WSNs. The last method is to use larger number of Reference Nodes (RNs) in addition to the required ones. In 2-dimensional cases, 3 RNs without NLOS errors are needed to get an accurate estimation. If there are more than 3 RNs available, additional information can be used. There are two alternatives to use this method. The first alternative is to get the location with all measurements (both LOS and NLOS) and then provide weighting to minimize errors that NLOS propagation brings about [2, 7]. The second alternative attempts to identify the LOS propagations from all range measurements and then utilize them [8]. A prerequisite of these methods is that at least 3 LOS propagations are needed for an accurate localization.

In this study, the second alternative of the third method is targeted. The algorithm distinguishes the LOS propagations from all range measurements and then estimates the location with them. RT [8] is a typical method which can eliminate NLOS error efficiently. But the computational complexity of this method is quite high and it may not be suitable for computation and energy constrained circumstances, such as WSNs. Therefore, a method with energy efficiency and smaller computation complexity is needed. LCRT is proposed in this paper in order to trim the computational complexity while not to sacrifice location accuracy. LCRT uses several processes to reduce computational complexity and makes the algorithm more stable. LCRT optimizes the estimation process by selecting the range measurements set with minimum normalized residual and uses LS [9] estimation instead of the AML [10] that is used in RT in LOS propagation identification process.

The rest of paper is arranged as follows: The second section describes the new algorithm and then analyzes its characteristics. In the third section the simulation results are presented. The conclusions are given in the last section.

II. Low Computational Residual Test

In this study, we consider mobile node localizations in a 2-dimensional case. Let $\mathbf{x}=(x, y)$ be the position of a MT, (x_i, y_i) be the position of the ith RN and r_i be the distance

measurement between the MT and the ith RN. Assume that, the system (LOS) errors, are random variables following i.i.d. zero mean Gaussian distribution.

A. Traditional RT Algorithm

The RT algorithm [8] has 2 steps. The first step is iteratively verifying the set of range measurements if it is without NLOS errors. In this step, it verifies the range measurements sets according to the size of the sets from large to small one by one until a qualified set is found. In each verification round, the global computation results of the range measurements set, which uses all the possible subsets, are used and AML is adopted for each subset. The second step is to estimate the location of MT with the qualified set by AML.

The computation of RT can be reduced. In the first step, verifying every range measurements set is time consuming and we can find an intermediate set that has the higher probability without NLOS errors and then verify it if it is qualified. In the verification procedure, the global computation of the set is not necessary because if it has a NLOS error, the set and a portion of its subsets also have some biases compared with the situation that there is no NLOS error. And AML used in iteration is an accurate but complex method. We adopt a simpler method, LS estimation, to substitute AML in the verifying procedure. In the following paragraphs, LS and LCRT which are to reduce the computational complexity are presented.

B. LS Estimation and Residual

The LS [9] estimator is a basic method used to determine the location of a MT. That is

$$\theta = \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = arg \ \ min_{\mathbf{x}} \sum_{i \in S} (r_i - \sqrt{(x - x_i)^2 + (y - y_i)^2})^2 \quad (1)$$

where θ is the estimation that minimizes the sum of the residual squares over the range measurements set S. If defining $Res(\mathbf{x}, S)$ as the sum of the residual squares of $\mathbf{x}$ over the range measurements set S [7],

$$Res(\mathbf{x}, S) = \sum_{i \in S} (r_i - \sqrt{(x - x_i)^2 + (y - y_i)^2})^2. \quad (2)$$

θ could be written as :

$$\theta = arg \ \ min_{\mathbf{x}} \ Res(\mathbf{x}, S). \quad (3)$$

The normalized Res, $\overline{Res}(\theta, S)$, is obtained by

$$\overline{Res}(\theta, S) = Res(\theta, S) / size \ of \ S. \quad (4)$$

When there is a NLOS error in a range measurements set, the normalized Res of the estimation is likely to be larger than the normalized Res when there is not a NLOS range measurement [7]. An accurate estimation tends to have a smaller $\overline{Res}(\theta, S)$.

When all range measurements are LOS, the LS estimations of θ should be a Gaussian distribution with the mean of true positions and variance which is a little larger than Cramer-Rao Lower Bound (CRLB). We use a simulation to illustrate it. The topology is as follows. The RNs are located at $(6, 0)$, $(3, -6)$, $(-3, -5)$, $(-6, -1)$, $(-4, 6)$, $(0, 5)$, $(4, 6)$ and the target node

is at $(-2, 1.6)$. The simulation results of the LS estimator is from 100000 independent trials for noise $\sigma = 0.01$. The pdf of $\hat{x}$ in θ is given in Fig. 1. From the simulation results we can see that the mean of simulated results is (-2.0000, 1.60000) which fits the true value well and the simulated variance is (3.7965e-005, 3.2341e-005) while the CRLB is (3.3263e-005, 2.5948e-005).

C. LCRT Description

In LCRT, there are 3 steps on the whole: the first step is to find out the intermediate set of range measurements with minimum $\overline{Res}(\theta, S)$ and the second step is to verify the set found in the first step if it is formed by LOS measurements. The above 2 steps are used iteratively to find a range measurements set without NLOS errors. The last step is to use the set verified to estimate the location of the MT by AML.

Let N be the number of range measurements from RNs. Assume that the number of range measurements is equal to the number of the RNs, which means that the distances between MT and RNs are measured only once for each RN. The same as reference [8], let there be all together N=7 RNs, and a portion or all measurements are LOS range measurements. Let D be the number of range measurements that have LOS propagations in all N range measurements. Obviously, $D \leq N$.

The flowchart for the LCRT algorithm is shown in Fig. 2, and it works as follows.

1) Find Intermediate Set with Minimum $\overline{Res}(\theta, S)$: In the first step, because the set of range measurements with smaller $\overline{Res}(\theta, S)$ has smaller chance contaminated by NLOS errors, the intermediate set of range measurements with minimum $\overline{Res}(\theta, S)$ is required for the next verification procedure. Note that the "intermediate" here means that the set with minimum $\overline{Res}(\theta, S)$ is not from a global searching. It is a set with minimum $\overline{Res}(\theta, S)$ among the range measurements sets which contain the same number of range measurements. For example, when it is 7 out of 7 range measurements, there is only one set and it is also the one with minimum $\overline{Res}(\theta, S)$. When it is 5 out of 7 range measurements, there are C_7^5 sets and the set with minimum $\overline{Res}(\theta, S)$ should be found out.

2) Verification of Set without NLOS Error: In the second step, it verifies if the set with minimum $\overline{Res}(\theta, S)$ is formed by LOS measurements. The process of generating subsets of the set for verifying if it is the one formed by LOS measurements is the same as the method used in LCC-Rwgh [2]. The process is as follows:
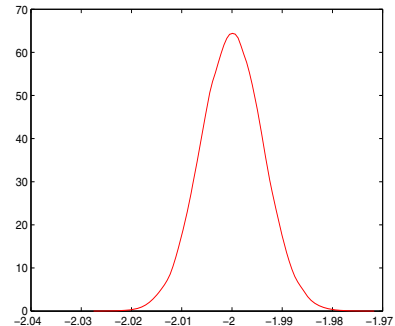

Fig. 1.   Pdf of the LS estimation.

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a) Suppose there are all together U range measurements in the range measurements set that needs verification. Get all the subsets which contain U-1 range measurements from all these U range measurements. Let the number of the subsets be K, obviously, $K = C_U^{U-1} = U$. Each subset is represented by RN index set $\{S_i, i = 1, 2, \ldots K\}$. For each subset, computes the LS estimation θ and the corresponding $\overline{Res}(\theta, S)$. From the K subsets, find the one that has the minimum $\overline{Res}(\theta, S)$ and let this subset S be S_{min}.

b) Let the number of elements in S_{min} be P. Pick out every element in P respectively to form P new range measurements subsets, and the new index set is $\{S_i, i = 1, 2, \ldots P\}$. For the P new subsets, compute the location estimations by LS estimation and the corresponding $\overline{Res}(\theta, S)$, and then find the one that has the minimum $\overline{Res}(\theta, S)$ and let the S be S_{min}.

c) If $P > 3$, go to step b); if $P=3$, go to step d). Note that there are not enough range measurements in 2-Dimensional cases for location estimation if $P < 3$.

d) Compute the location estimation via LS estimation and its $\overline{Res}(\theta, S)$ with all the U range measurements, i.e., the C_U^Uth estimation.

In the process discussed above, it computes, with the LS estimation, a total number of

$$M = (U^2 + U)/2 - 5 \quad U \geq 3 \quad (5)$$

estimations of θ. For example, if $U=7$, there are all together 23 different estimations of θ. Let these estimations be $\theta(k)$, $k = 1 \ldots 23$, with $\theta(23)$ the C_7^7th estimation.

Then, the LCRT computes the square of the normalized residuals of $\theta(M)$ denoted in [8]. Note that it is different from

$\overline{Res}(\theta, S)$ discussed in Section II. B.

$$\chi_x^2(k) = [\hat{x}(k) - \hat{x}(M)]^2 / B_x(k) \quad (6)$$
$$\chi_y^2(k) = [\hat{y}(k) - \hat{y}(M)]^2 / B_y(k), k = 1, 2 \ldots M - 1, \quad (7)$$

where $B_x(k)$ and $B_y(k)$ are the approximation of the CRLB of θ from the kth set which produces $\theta(k)$. $\hat{x}(M)$ and $\hat{y}(M)$ are the result of C_U^Uth estimation. The CRLB is the theoretical value which requires the true θ, which is impossible to get in practice. So the $\theta(k)$ is used as a substitute to produce $B_x(k)$ and $B_y(k)$ [8].

Since the result of LS estimation is a Gaussian distribution, the random variables in (6) and (7) are an approximate central Chi-Square distribution of one degree of freedom in LOS cases. If in NLOS cases, the $\theta(23)$ and some other $\theta(k)$ will contain biases and the pdf will be non-central. Let $\chi_{x,y}^2 = \chi_x^2 + \chi_y^2$. Fig. 3 plots the pdf of $\chi_{x,y}^2$ when $N=7$. The results are from 1000 independent trials for system noise $\sigma=10$ m and NLOS error 500 m. The figure shows that when $D=7$, the pdf is a typical central Chi-Square distribution while $D=6$ has some biases. When $D=5$ and less, the distributions are similar to that in $D=6$.

In the simulated pdf of χ_x^2 and χ_y^2, only 1% or less $\chi_{x,y}^2$ random variable (r.v.) are larger than 4.93 in LOS condition. Here we define $TH=4.93$ as the threshold and the LOS measurements can be determined by it. For example, if 1% or less of the $\chi_{x,y}^2$ r.v. are larger than 4.93, $D=7$; otherwise, verify if $D < 7$. Let p_{od} be the probability of over-determination; i.e., when $D < 7$ but the LCRT decides that $D=7$. Similarly, p_{ud} is the probability of under-determination; i.e. when $D=7$ but the decision is $D < 7$. Under-determination is preferred because it results in a smaller number of LOS RNs for localization while over-determination contains NLOS RNs which result large errors. The reason for choosing 4.93 (1%) as the threshold is as follows: if the value of threshold is smaller, the percent of $\chi_{x,y}^2$ larger than TH becomes larger. Because the variance of θ is larger than CRLB, which will introduce a larger fluctuation in $\chi_{x,y}^2$ value, the percentage vibrates in different trails, which will cause a higher rate of under-determination. When the threshold is small enough ($TH < 0.001$), the percent of LOS and NLOS has the same trends, which introduces high rate of over-determination. If the threshold is larger, the percentage is smaller. Because the number of sets that can be used is limited (especially when N is small), it is not easy to distinguish the
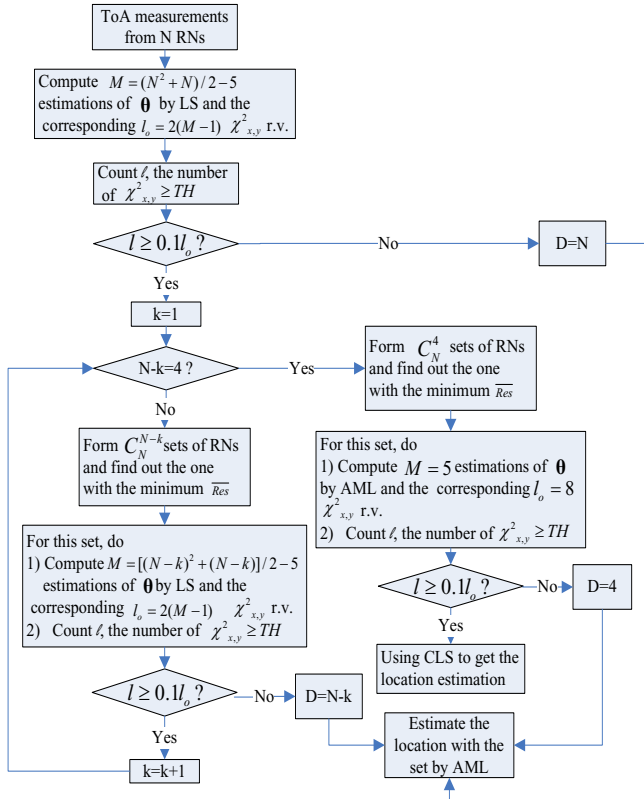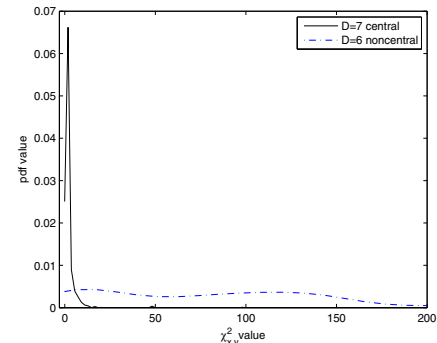

Fig. 2. Flowchart of LCRT.

Fig. 3. The pdf of $D=7$ and $D=6$.

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sets in LOS or in NLOS with smaller percentage, especially when the value of NLOS error is also small, which will cause higher rate of over-determination.

For ease of computation, in practice, 10% is chosen in LCRT. That is, for N=7, if the number of $\chi^2_{x,y}$ larger than 4.93 is less than $0.1 \times 2 \times 22 \approx 5$, the LCRT decides that D=7. Otherwise, it verifies if D=6.

To summarize, the steps for verifying if D=7 are as follows:

a) Do $M = (U^2 + U)/2 - 5$ LS estimations of θ and compute the corresponding $\chi^2_{x,y}$.

b) Count the number of $\chi^2_{x,y}$, named l, that are larger than TH=4.93.

c) If $l < 5$, D=7 and then using AML for computing the final answer with all these range measurements; Otherwise, verify if D=6.

To verify if D=6, form C_7^6 sets of range measurements, six per set. For each set, computes its θ and the corresponding $\overline{Res}$. Find the set with minimum $\overline{Res}$ from these 7 sets and verify if this set is the one without NLOS via the method discussed above. If $l < 0.1 \times 2 \times 15 = 3$, then D=6; otherwise verify if D=5. The steps for verifying D=5 are similar to the D=6 case.

In case of verifying if D=4, because the percent vibrates a lot with the different locations of RNs, AML is adopted in the verifying process instead of LS estimation and the threshold is smaller accordingly, that is 2.46.

3) Localization by AML: The above 2 steps can introduce a set of range measurements without NLOS errors if it exists. In the third step, the AML is used for the final location estimation by this set since its performance is closer to CRLB compared with other algorithms when there is no NLOS error.

Because in 2-dimensional cases, the minimum number of range measurements needed for localization is 3, there is no subset for verification if D=3. Several methods are test in case of D=3, such as finding the set with minimum $\overline{Res}$ from all C_7^3 the sets and then estimating the location with AML. Finally, Constrained Least Squares (CLS) [11] is adopted to get the location estimation if $D < 4$. The performance of CLS is much stable in case of D=3 while other methods tested including the one used in RT are not very stable.

D. The Complexity Analysis and Characteristics of LCRT

1) The LCRT uses the set of range measurements which has the smallest $\overline{Res}$ in the group instead of all these possible sets. This will reduce the number of sets that needs verification. Although finding set with minimum $\overline{Res}$ involve extra computation compared with RT, considering the number of sets that need verification, it is beneficial.

2) In the verification procedure, LCRT uses the optimized steps to reduce the number of range measurements subsets while in RT it uses all the possible subsets. This will reduce the number of estimations, especially when the probability of LOS range measurements is large. For example, when verifying if D=7 and 6, in RT it needs 99 AML estimations while in LCRT it only needs 23 LS estimations.

3) In the verification procedure, LS is adopted instead of AML in RT because LS is much simpler and also efficient which can reduce the computation. AML is an iterative method which is much more complicated than LS. For example, for 10000 tests of these 2 methods via Intel Core 2 Duo CPU T8300 with 2G RAM, LS uses 1.3281 second while AML consumes 34.3594 seconds.

4) When the set of LOS range measurements is decided, the AML is used to get the final estimation because the precision of AML is higher.

5) When $D < 4$, the CLS is adopted to get a stable result.

Because the threshold value is from the situation when the RNs are distributed sparsely around the edges of the simulation area, in practices, the RNs are also needed to be deployed as mentioned above. Other distribution of RNs, such as disposed in a line along a side of potential MTs, may bring some changes of threshold if it is still using 1%.

III. SIMULATIONS AND NUMERICAL RESULTS

The performance of LCRT is evaluated through simulations using Matlab 7. There are seven RNs located in a 12×10 area and their locations are placed at $(-6, 5)$, $(-4, -3)$, $(-5, 0)$, $(1, -5)$, $(5, 4)$, $(0, 6)$, $(6, -4)$. The target node is located at $(0, 2)$. All units are in kilometers. For a given trial, the distance measurements for LOS [8] are $r_i = r_i^0 + \epsilon_i$, where r_i^0 is the true distance and measurement errors, ϵ_i, are i.i.d., zero mean Gaussian r.v. with variance σ^2. σ varies from 4 m to 18 m. For NLOS [8], $r_i = r_i^0 + \epsilon_i + \alpha_i$, where α_i is an additional error, which follows uniform distribution between 0.2-1.6 km. The range measurements contaminated by NLOS errors are chosen from all the range measurements randomly.

A. Percentage of Dimension Decided by LCRT

In the above presented simulation environment, the percent of the dimension decided by LCRT is given in Table 1. It is from 400 independent trials with $\sigma = 10$ m. From the table we can observe that the percent of under-determination is less than 8% and over-determination does not happen in the simulated scenarios.

TABLE I
PERCENT OF DIMENSION DECIDED BY LCRT.

Dimension of LOS	D=7	D=6	D=5	D=4
Under-determination	4%	4.25%	7%	7.25%
Accuracy	96%	95.75%	93%	92.75%
Over-determination	0%	0%	0%	0%

B. Performance Comparison of LCRT with Other Algorithms

The performance of the algorithms, including AML, LS, CLS, RT, LCRT are measured by the Mean Square Error (MSE) [8] from 200 independent trials in different dimension of LOS.

$$MSE = \sum_{i=1}^{200} \left\| \hat{X}(i) - X \right\|^2 / 200, \qquad (8)$$

where $\hat{X}(i)$ is the estimated location and X is true location. Simulation results are shown in Fig. 4 and Fig.5.

Fig. 4 gives the performance of algorithms without NLOS error in all the 7 range measurements (D=7). To better compare
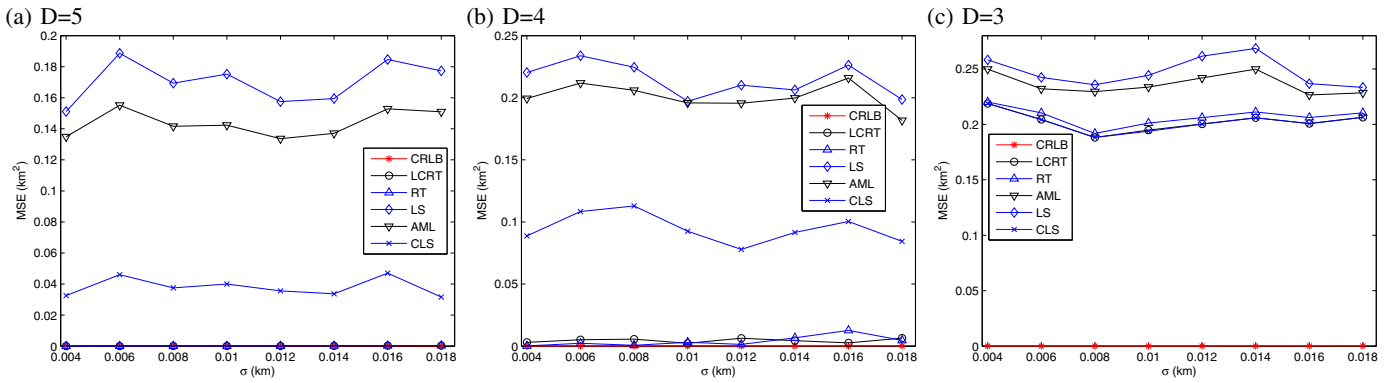
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Fig. 5. Simulation results of localization algorithms with NLOS errors.

the performance of the investigated algorithms, the CRLB is also plotted in the figure as a reference. As shown in Fig. 4, the CLS performs worst while other algorithms have close performance. Because there are some under-determinations, the performance of LCRT and RT is slightly worse than AML. Among these algorithms, AML performs best, especially when σ is large. That is the reason that AML is adopted to get the location estimation when the set of range measurements with LOS is decided in LCRT.

Fig. 5 (a) (b) (c) gives the performance of the investigated algorithms with NLOS errors when D equals to 5, 4 and 3 respectively. In Fig. 5 (a), both RT and LCRT have similar performance which is in accordance with CRLB because both of them can find the set without NLOS errors efficiently and then use AML which is close to CRLB when there are no NLOS errors to get the final estimation. Although LCRT has low computational complexity in this circumstance, it achieves close performance to RT. AML and LS use the set with all the possible range measurements which contains NLOS errors thus the performance of these algorithms is degraded. CLS is better than AML and LS since it is robust to NLOS errors. In Fig. 5 (b), because CLS is used when there is under-determination, the performance of LCRT is not as good as it is in case of D=5. Nevertheless, it is stable compared with other methods and still close to CRLB. The performance of the algorithms when D=3 is shown in Fig. 5 (c). The performance of the algorithms, which is larger than CRLB, is similar. The performance of RT is slightly worse than that of CLS. As CLS is used when $D < 4$ in LCRT, the performance of CLS and

LCRT is the same if there is no over-determination.

IV. CONCLUSIONS

This paper presents a localization algorithm LCRT which is robust to NLOS errors with lower computational complexity compared with RT. LCRT first finds the set of range measurements with minimum $\overline{Res}$, then verifies the set if it is all LOS propagation and finally estimates the location with the set verified. Optimized steps and LS instead of AML are used to reduce the computational complexity. Simulation results provide reasonable evidence on the NLOS errors mitigation effect of LCRT. The result is close to the CRLB when $D > 3$.

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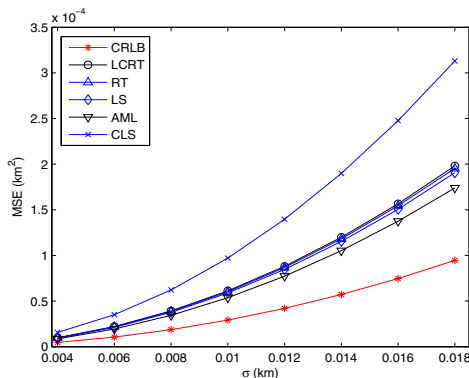
Fig. 4. Simulation results of localization algorithms without NLOS errors.

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