# The Influence of Spatial Correlation and Severity of Fading on the Statistical Properties of the Capacity of OSTBC Nakagami-$m$ MIMO channels


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***Abstract*—This paper deals with the analysis of statistical properties of the capacity of spatially uncorrelated orthogonal space-time block coded (OSTBC) Nakagami-$m$ multiple-input multiple-output (MIMO) channels. We have derived exact closed-form expressions for the probability density function (PDF), cumulative distribution function (CDF), level-crossing rate (LCR), and average duration of fades (ADF) of the channel capacity. We have also investigated the statistical properties of the approximated capacity of spatially correlated OSTBC Nakagami-$m$ MIMO channels. The results are studied for different values of the fading parameter $m$, corresponding to different fading conditions. It is observed that an increase in the MIMO dimension[1] or a decrease in the severity of fading increases the mean channel capacity. While, a significant decrease in the mean channel capacity is observed with an increase in the spatial correlation. The correctness of theoretical results is confirmed by simulations.**


## I. Introduction

Provision of multiple antennas at the receiver and transmitter allows the design of multiple-input multiple-output (MIMO) systems to exploit spatial diversity in order to increase the spectral efficiency and to acquire a diversity gain [1]. One promising method to achieve the desired capacity is to use space-time coding techniques, such as space-time trellis codes (STTC) [2] or space-time block codes (STBC) [3], [4]. One of the advantages of using OSTBC is that it transforms MIMO fading channels into equivalent single-input single-output (SISO) channels [5]. Moreover, being orthogonal in structure, maximum likelihood decoding can be applied at the receiver that results in a significant decrease in the complexity of the receiver structure, compared to the prevailing coding techniques (e.g., STTC) [4]. Studies pertaining to the analysis of the capacity of OSTBC MIMO channels can be found in [6], [7]. The outage performance and the error probability analysis of OSTBC MIMO systems have been studied in [8]–[10].

In this paper, we have extended the analysis of the statistical properties of the capacity of uncorrelated OSTBC Rayleigh MIMO channels presented in [11] to uncorrelated OSTBC Nakagami-$m$ MIMO channels. The Nakagami-$m$ distribution can be considered as a more general channel model compared to a Rayleigh channel as it incorporates scenarios where the fading can be more (or less) severe than Rayleigh fading. Moreover, the one-sided Gaussian and the Rayleigh distribution are inherently included in the Nakagami-$m$ distribution as special cases, i.e., for $m = 0.5$ and $m = 1$, respectively.

We have derived analytical expressions for the PDF, CDF, LCR, and ADF of the channel capacity of uncorrelated OSTBC Nakagami-$m$ MIMO channels. The mean value and spread of the channel capacity has been analyzed with the help of the PDF of the channel capacity. On the other hand, the analysis of the LCR and ADF of the channel capacity is very helpful to study the temporal behavior of the channel capacity. The LCR of the channel capacity provides the information regarding the expected number of up-crossings (or down-crossings) of the channel capacity through a certain threshold level in a time interval of one second. While, the ADF of the channel capacity describes the average duration of the time intervals over which the channel capacity is below a given level [12], [13]. We have studied the above mentioned statistical quantities for different values of the fading parameter $m$ and for different MIMO dimensions. It is observed that an increase in the MIMO dimension or a decrease in the severity of fading results in an increase in the mean channel capacity. The results for the PDF, CDF, LCR, and ADF of the capacity of Rayleigh channels can be readily obtained as a special case from the findings in [11] by setting $m = 1$. We have also investigated the capacity of spatially correlated OSTBC Nakagami-$m$ MIMO channels. For such channels, we have derived an approximate expression for the channel capacity. Thereafter, the expressions for the PDF, CDF, LCR, and ADF of the channel capacity are found. It is observed that the spatial correlation significantly reduces the mean channel capacity. We have verified the theoretical results by simulations, whereby a very good fitting is observed.

The rest of the paper is organized as follows. In Section II,

[1]Throughout this paper, we will refer to the MIMO dimension as $N_R \times N_T$, where $N_R$ is the number of receive antennas and $N_T$ denotes the number of transmit antennas.


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we define briefly the capacity of OSTBC Nakagami-$m$ MIMO channels. Section III deals with the derivation of the statistical properties of the capacity of uncorrelated OSTBC Nakagami-$m$ MIMO channels. In Section IV, the statistical properties of the approximate capacity of spatially correlated OSTBC Nakagami-$m$ MIMO channels are investigated. Section V aims at the validation and analysis of the obtained results with the help of simulations. Finally, the conclusions are given in Section VI.

## II. The Capacity of Spatially Uncorrelated OSTBC Nakagami-$m$ MIMO Channels

In this article, we have considered a MIMO system with $N_T$ transmit and $N_R$ receive antennas. The complex random channel gains are represented by $h_i(t)$ $(i = 1, 2, ..., N_R N_T)$. Moreover, we have assumed that the stochastic processes $h_i(t)$ are mutually uncorrelated and the envelope $|h_i(t)|$ follows a Nakagami-$m$ distribution

$$p_{|h_i|}(z) = \frac{2m^m z^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{mz^2}{\Omega}}, \quad z \geq 0 \tag{1}$$

for $i = 1, 2, ..., N_R N_T$, where $\Omega = E\left\{|h_i(t)|^2\right\}$, $m = \Omega^2 / \text{Var}\left\{|h_i(t)|^2\right\}$, and $\Gamma(\cdot)$ represents the gamma function [14]. The capacity of OSTBC MIMO systems can be expressed as [15]

$$C(t) = \log_2 \left(1 + \frac{\gamma_s}{N_T} \mathbf{h}^H(t)\mathbf{h}(t)\right) \quad \text{(bits/sec/Hz)} \tag{2}$$

where $\mathbf{h}(t)$ represents the $N_R N_T \times 1$ complex channel gain vector with entries $h_i(t)$ $(i = 1, 2, ..., N_R N_T)$, $(\cdot)^H$ denotes the Hermitian operator, and $\gamma_s$ is the signal-to-noise ratio (SNR). The channel capacity $C(t)$ given by (2) can be written as

$$C(t) = \log_2 \left(1 + \gamma_s' \sum_{i=1}^{N_R N_T} \chi_i^2(t)\right) \quad \text{(bits/sec/Hz)} \tag{3}$$

where $\gamma_s' = \gamma_s / N_T$ and $\chi_i^2(t) = |h_i(t)|^2$ $(i = 1, 2, ..., N_R N_T)$. Due to the assumption that the envelope $|h_i(t)|$ is Nakagami-$m$ distributed, the squared envelope $\chi_i^2(t)$ follows the gamma distribution. Let $\Xi(t) = \sum_{i=1}^{N_R N_T} \chi_i^2(t)$, then the PDF $p_\Xi(z)$ of $\Xi(t)$ can be expressed as [16]

$$p_\Xi(z) = \frac{z^{N_R N_T m-1} e^{-\frac{z}{\beta}}}{\beta^{N_R N_T m}\Gamma(N_R N_T m)}, \quad z \geq 0 \tag{4}$$

where $\beta = \Omega / m$ is a parameter of the Nakagami-$m$ distribution. In order to derive the expressions for the LCR of the OSTBC Nakagami-$m$ MIMO channel capacity, we require the joint PDF $p_{\Xi\dot{\Xi}}(z, \dot{z})$ of $\Xi(t)$ and $\dot{\Xi}(t)$ at the same time $t$. In this article, the time derivative of a process is denoted by a raised dot. The joint PDF $p_{\Xi\dot{\Xi}}(z, \dot{z})$ can be expressed as [17]

$$p_{\Xi\dot{\Xi}}(z, \dot{z}) = \frac{z^{N_R N_T m-3/2} e^{-\frac{z}{\beta}} e^{-\frac{\dot{z}^2}{8\beta_N z}}}{2\beta^{N_R N_T m}\Gamma(N_R N_T m)\sqrt{2\beta_N \pi}} \tag{5}$$

for $z \geq 0$ and $|\dot{z}| < \infty$, where under isotropic scattering conditions $\beta_N$ is given by [18]

$$\beta_N = 2(\pi f_{\max})^2. \tag{6}$$

In the next section, we will derive the expressions for the PDF, CDF, LCR, and ADF of the OSTBC Nakagami-$m$ MIMO channel capacity.

## III. Statistical Properties of the Capacity of Spatially Uncorrelated OSTBC Nakagami-$m$ MIMO Channels

The channel capacity $C(t)$ presented in (2) can be considered as a mapping of a random vector process $\mathbf{h}(t)$ to another random process, namely $C(t)$. Hence, the PDF $p_C(z)$ of the channel capacity $C(t)$ can be found using the PDF $p_\Xi(z)$ in (4) and by applying the concept of transformation of random variables [19] as follows

$$\begin{aligned} p_C(r) &= \frac{2^r \ln(2)}{\gamma_s'} p_\Xi\left(\frac{2^r - 1}{\gamma_s'}\right) \\ &= \frac{2^r (2^r - 1)^{N_R N_T m-1} \ln(2) e^{-\frac{2^r - 1}{\beta\gamma_s'}}}{(\beta\gamma_s')^{N_R N_T m}\Gamma(N_R N_T m)}, \quad r \geq 0. \end{aligned} \tag{7}$$

The CDF $F_C(r)$ of the channel capacity can be found using

$$F_C(r) = \int_0^r p_C(x) dx. \tag{8}$$

By substituting (7) in (8) and doing some algebraic manipulations, the CDF of the channel capacity can be expressed as

$$F_C(r) = 1 - \frac{\Gamma(N_R N_T m, \frac{1}{\beta\gamma_s'})}{\Gamma(N_R N_T m)}, \quad r \geq 0 \tag{9}$$

where $\Gamma(\cdot, \cdot)$ denotes the generalized gamma function [14]. The LCR $N_C(r)$ of the channel capacity can be obtained by solving the following integral [13]

$$N_C(r) = \int_0^\infty \dot{z} p_{C\dot{C}}(r, \dot{z}) d\dot{z}, \quad r \geq 0 \tag{10}$$

where $p_{C\dot{C}}(z, \dot{z})$ is the joint PDF of $C(t)$ and $\dot{C}(t)$. The joint PDF $p_{C\dot{C}}(z, \dot{z})$ can be obtained using (5) and by applying the concept of transformation of random variables [19] as follows

$$\begin{aligned} p_{C\dot{C}}(z, \dot{z}) &= \left(\frac{2^z \ln(2)}{\gamma_s'}\right)^2 p_{\Xi\dot{\Xi}}\left(\frac{2^z - 1}{\gamma_s'}, \frac{2^z \dot{z} \ln(2)}{\gamma_s'}\right) \\ &= \left(\frac{2^z \ln(2)}{\gamma_s'}\right)^2 \frac{(2^z - 1/\gamma_s')^{N_R N_T m-3/2} e^{-\frac{(\dot{z}2^z \ln(2))^2}{8\beta_N \gamma_s'(2^z - 1)}}}{2\beta^{N_R N_T m}\Gamma(N_R N_T m)\sqrt{2\beta_N \pi}} \\ &\quad \times e^{-\frac{2^z - 1}{\beta\gamma_s'}}, \quad z \geq 0, |\dot{z}| < \infty. \end{aligned} \tag{11}$$

By inserting (11) in (10), the final expression for the LCR of the channel capacity can be written as

$$N_C(r) = \sqrt{\frac{2\beta_N}{\pi}} \frac{(2^r - 1/\gamma_s')^{N_R N_T m-1/2} e^{-\frac{2^r - 1}{\beta\gamma_s'}}}{\beta^{N_R N_T m}\Gamma(N_R N_T m)}, \quad r \geq 0. \tag{12}$$

The ADF $T_C(r)$ of the channel capacity can now be obtained using [13]

$$T_C(r) = \frac{F_C(r)}{N_C(r)} \tag{13}$$

where $F_C(r)$ and $N_C(r)$ are given by (9) and (12), respectively.

## IV. High SNR approximation of the Channel Capacity of Spatially Correlated OSTBC Nakagami-$m$ MIMO Channels

It is widely reported in the literature that a spatial correlation between the sub-channels of a MIMO channel has a significant influence on the channel capacity. At high SNR, the channel capacity $C_{app}(t)$ of spatially correlated OSTBC MIMO channels can be approximated as [15]

$$C_{app}(t) \approx \log_2 \det\left(\gamma_s' \mathbf{h}^H(t)\mathbf{h}(t)\right) + \alpha_R \tag{14}$$

where

$$\begin{aligned} \alpha_R &= \log_2 \det(R_{Rx}) + \log_2 \det(R_{Tx}) \\ &= \log_2\left(\prod_{i=1}^{N_R} \lambda_{R_{Rx}}^{(i)}\right) + \log_2\left(\prod_{i=1}^{N_R} \lambda_{R_{Tx}}^{(i)}\right). \end{aligned} \tag{15}$$

Here $R_{Rx}$ $(R_{Tx})$ is the full rank $N_R \times N_R$ $(N_T \times N_T)$ receiver (transmitter) correlation matrix, $\det(\cdot)$ denotes the matrix determinant, and $\lambda_{R_{Rx}}^{(i)}$ $\left(\lambda_{R_{Tx}}^{(i)}\right)$ represent the eigenvalues of the receiver (transmitter) correlation matrix. In (14), $\log_2 \det\left(\gamma_s' \mathbf{h}^H(t)\mathbf{h}(t)\right)$ denotes the high SNR approximation of the channel capacity $C(t)$ of OSTBC MIMO channels and $\alpha_R$ can be considered as a correction term added to the high SNR approximation due to the spatial correlation. The receive and transmit antenna correlations under isotropic scattering conditions can be expressed in closed form as [20]

$$\rho_{p,q}^T(\delta_{pq}) = J_0(2\pi\delta_{pq}/\lambda_s) \tag{16a}$$

$$\rho_{m,n}^R(d_{mn}) = J_0(2\pi d_{mn}/\lambda_s) \tag{16b}$$

where $\rho_{p,q}^T(\delta_{pq})$ $\left(\rho_{m,n}^R(d_{mn})\right)$ represents the transmit (receive) antenna correlation and $J_0(\cdot)$ is the Bessel function of the first kind of order zero. In (16a) and (16b), $\delta_{pq}$ $(d_{mn})$ represents the spacing between the receive (transmit) antenna elements and $\lambda_s$ is the wavelength of the transmitted signal. The statistical properties of the approximated channel capacity $C_{app}(t)$ of spatially correlated OSTBC Nakagami-$m$ MIMO channels presented in (14) can be found by following a similar procedure as developed in the previous section for the channel capacity $C(t)$ of spatially uncorrelated OSTBC Nakagami-$m$ MIMO channels. The PDF, CDF, and LCR of the channel capacity $C_{app}(t)$ can be approximated as

$$p_{C_{app}}(r) \approx \frac{2^{r-\alpha_R}\left(2^{r-\alpha_R}\right)^{N_R N_T m-1} \ln(2) e^{-\frac{2^{r-\alpha_R}}{\beta\gamma_s'}}}{(\beta\gamma_s')^{N_R N_T m}\,\Gamma(N_R N_T m)}, \quad r \geq 0 \tag{17}$$

$$\begin{aligned} F_{C_{app}}(r) \approx\ & \frac{\ln(2)}{(\beta\gamma_s')^{N_R N_T m}\,\Gamma(N_R N_T m)} \int_0^r 2^{x-\alpha_R} e^{-\frac{2^{x-\alpha_R}}{\beta\gamma_s'}} \\ & \times \left(2^{x-\alpha_R}\right)^{N_R N_T m-1} dx, \quad r \geq 0 \end{aligned} \tag{18}$$

$$N_{C_{app}}(r) \approx \frac{\left(2^{r-\alpha_R}/\gamma_s'\right)^{N_R N_T m-1/2} e^{-\frac{2^{r-\alpha_R}}{\beta\gamma_s'}}}{\sqrt{\pi/2\beta_N}\,\beta^{N_R N_T m}\Gamma(N_R N_T m)}, \quad r \geq 0, \tag{19}$$

respectively. The ADF of the channel capacity $C_{app}(t)$ can be found by substituting (18) and (19) in (13).

## V. Numerical Results

In this section, we will discuss the analytical results found in the previous section and their validity will be tested by simulations. For the natural values of $2 \times m$, the Nakagami-$m$ distributed waveforms are generated by employing the following model [21]

$$|h_i(t)| = \sqrt{\sum_{k=1}^{2\times m} r_{i,k}^2(t)} \tag{20}$$

where $r_{i,k}(t)$ $(i = 1, 2, ..., N_R N_T$ and $k = 1, 2, ..., 2m)$ are zero-mean real-valued uncorrelated Gaussian distributed random processes with variance $\sigma_0^2$, and $m$ is the parameter of the Nakagami-$m$ distribution. In order to generate these Gaussian distributed waveforms $r_k(t)$, we have applied the sum-of-sinusoids model [22]. The model parameters are calculated from the generalized method of exact Doppler spread ($\text{GMEDS}_1$) [23]. The number of sinusoids used for the generation of the Gaussian distributed waveforms $r_k(t)$ was selected to be $N_k = 29 + k$. The maximum Doppler frequency $f_{\max}$ was 91 Hz, the SNR $\gamma_s$ was chosen to be 15 dB, $\sigma_0^2 = 1$, and the parameter $\Omega$ for the Nakagami-$m$ distribution was set to be $2 \times m$. The transmit and the receive antenna spacings are taken to be $0.4\lambda_s$. Finally, using (3) and (14), the simulation results for the statistical properties of the channel capacity $C(t)$ and $C_{app}(t)$ of OSTBC Nakagami-$m$ MIMO channels are found.

The PDF and CDF of the channel capacity $C(t)$ for $2 \times 2$, $4 \times 4$, and $6 \times 6$ MIMO channels are shown in Figs. 1 and 2, respectively for different values of the fading parameter $m$. It is observed that as the severity of fading decreases (i.e., increasing the value of $m$), the mean channel capacity increases for all MIMO dimensions. However, the spread of the PDF decreases. Moreover, it can also clearly be seen that increasing the MIMO dimension results in a prominent increase in the channel capacity. The LCR and ADF of the channel capacity $C(t)$ for $2 \times 2$, $4 \times 4$, and $6 \times 6$ MIMO channels are shown in Figs. 3 and 4, respectively for different values of the fading parameter $m$. Figure 3 shows that as the MIMO dimension or the value of the fading parameter $m$ increases, the spread of the LCR curve gets narrower. Moreover, for low MIMO dimensions (e.g., $2 \times 2$) with small values of $m$, high LCR is observed at lower signal levels. However, for large MIMO dimensions (e.g., $6 \times 6$) with large values of $m$, a high LCR is observed at high signal levels. The ADF of the channel capacity, on the other hand, decreases with an increase in the MIMO dimension or the fading parameter $m$ at low and medium signal levels.

Figures 5–8 aim at the comparative analysis of the statistical properties of the channel capacity of the uncorrelated OSTBC Nakagami-$m$ MIMO channels and the approximated channel

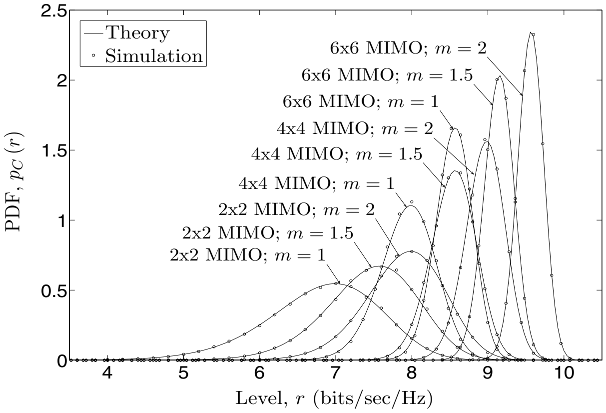


Fig. 1. The PDF $p_C(r)$ of the capacity of OSTBC Nakagami-$m$ MIMO channels.

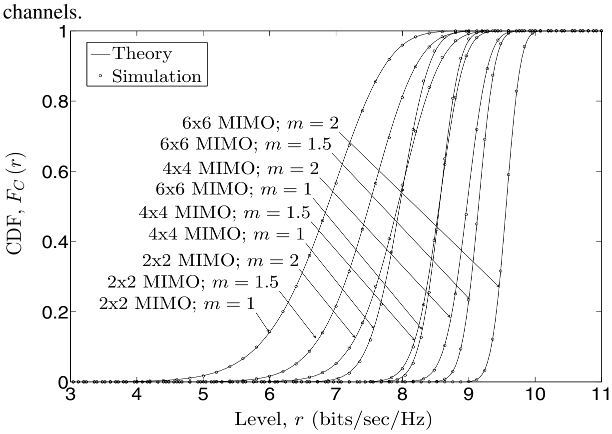


Fig. 2. The CDF $F_C(r)$ of the capacity of OSTBC Nakagami-$m$ MIMO channels.

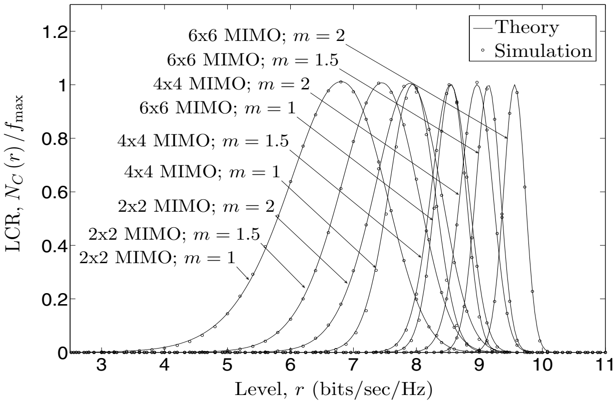


Fig. 3. The normalized LCR $N_C(r)/f_{\max}$ of the capacity of OSTBC Nakagami-$m$ MIMO channels.

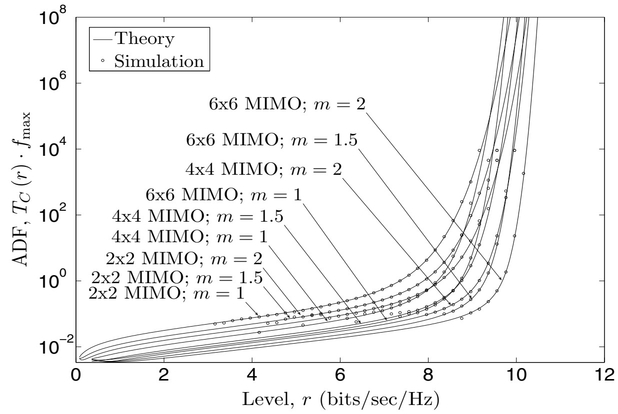


Fig. 4. The normalized ADF $T_C(r) \cdot f_{\max}$ of the capacity of OSTBC Nakagami-$m$ MIMO channels.

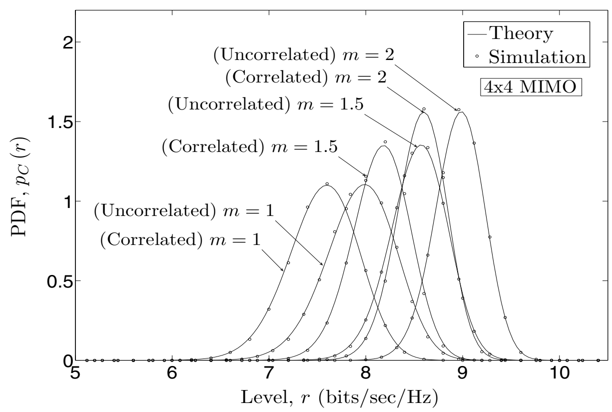


Fig. 5. The PDF $p_C(r)$ of the capacity of spatially correlated OSTBC Nakagami-$m$ MIMO channels.

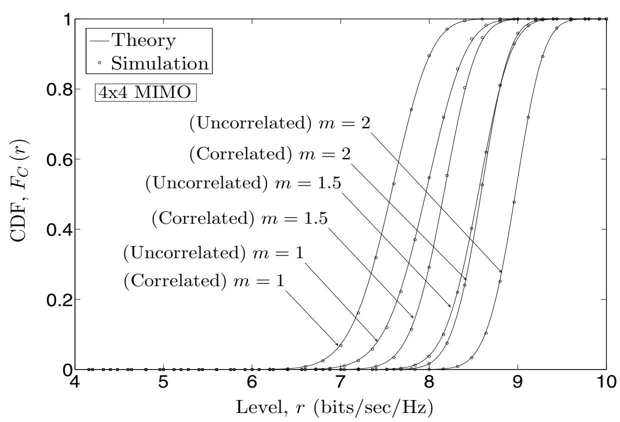


Fig. 6. The CDF $F_C(r)$ of the capacity of spatially correlated OSTBC Nakagami-$m$ MIMO channels.

capacity of the correlated OSTBC Nakagami-$m$ MIMO channels. The PDF and CDF of the channel capacity of $4 \times 4$ MIMO channels are shown in Figs. 5 and 6, respectively, for different values of the fading parameter $m$. It can clearly be observed that the spatial correlation has a noticeable influence on the mean channel capacity. The LCR and ADF of the channel capacity are presented in Figs. 7 and 8, respectively. It is apparent that the spatial correlation shifts the maximum value of the LCR to lower signal levels. Hence, for correlated systems the LCR of the capacity is higher for low signal levels compared to uncorrelated systems. On the other hand, the ADF of the channel capacity of correlated systems is higher for all the signal levels compared to uncorrelated systems. For all the cases studied in this paper, the analytical results are found to be in very good correspondence with the simulation results.

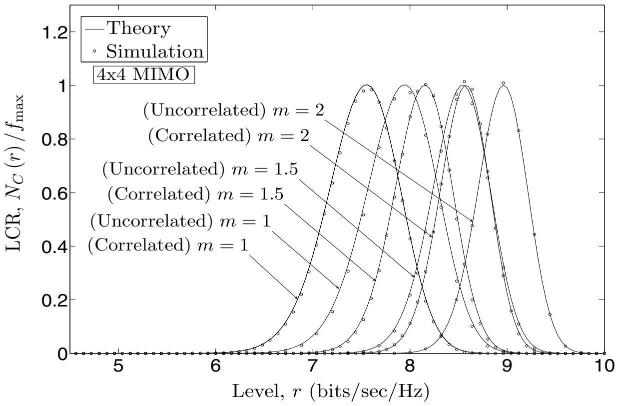


Fig. 7. The normalized LCR $N_C(r)/f_{\max}$ of the capacity of spatially correlated OSTBC Nakagami-$m$ MIMO channels.

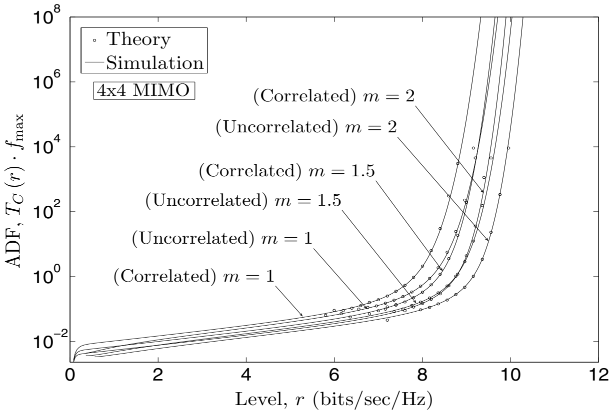


Fig. 8. The normalized ADF $T_C(r) \cdot f_{\max}$ of the capacity of spatially correlated OSTBC Nakagami-$m$ MIMO channels.

## VI. Conclusions

In this article, we have studied the statistical properties of the capacity of uncorrelated OSTBC Nakagami-$m$ MIMO channels. It is observed that the serverity of fading has a significant influence on the capacity of OSTBC systems. Specifically, an increase in the MIMO dimension or a decrease in the severity of fading results in an increase in the mean channel capacity. However, it results in a decrease in the ADF of the channel capacity at low and medium signal levels. We have also investigated the statistical properties of the approximated channel capacity of spatially correlated OSTBC Nakagami-$m$ MIMO channels. It is observed that the spatial correlation reduces the mean channel capacity of OSTBC Nakagami-$m$ MIMO channels. The validity of all analytical results has been verified by simulations.

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