



***Modelling of the Universe  
-- Simulation of the GPS (The Global  
Positioning System)***

by

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## **Abstract**

*ParAbel is a project at Agder University College aims at mathematics and physics on Internet. One of the sub projects named SimReal [2] (Simulation in 'Realfag'), which currently aims at simulating behaviours and properties of universe for both mathematics and physics studying.*

*This project focuses on simulation of the GPS which is one sub task in SimReal. In this paper, on the one hand, we will set up a GPS simulation model and use it to be a teaching material for learning mathematics and physics. On the other hand, by testing the simulation model among the college students, we will discuss about the learning effect comparing to traditional learning.*

## **Preface**

This thesis concludes the two-year Master of Science program in Information and Communication Technology (ICT) at Agder University College (AUC), Faculty of Engineering and Science in Grimstad, Norway. The workload of this thesis equals 30 ECTS and the project has been carried out from January to June 2007.

I would like to thank Professor Per Henrik Hogstad, my supervisor at Agder University College, for excellent supervision and guidance throughout the project period.

Grimstad, May 2007

Lei Liang

## Version Control

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1 **Version** indicates the version number starting at 0.1 for the first draft and 1.0 for the first review version.

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# 1 Introduction

## 1.1 Background

Modelling of the Universe is a project which currently carried on in ParAbel. It is used for helping learning college mathematics and physics on Internet [1]. It contains universe overview, planetary system, nuclear process, neutron star, black hole, HR-diagram (Hertzsprung-Russell diagram), satellite system, Gauss Law, GPS and so on.

Our project is a part of Modelling the Universe in the ParAbel/SimReal project. A simulation model will be composed for The Global Positioning System (GPS).

The first part of the project will be to compose a simulation model to simulate and hope for better understanding the behaviour of GPS.

A second part will be to incorporate the relativistic time computing in GPS to better understand a relativistic effect in our daily life.

A third part is to test this simulation program on college students and try to compare the learning effect to traditional learning.

## 1.2 Problem Description

ParAbel is a project at Agder University College aims at mathematics and physics on Internet. One of the sub projects named SimReal [2] (Simulation in 'Realfag'), which currently aims at simulating behaviours and properties of universe for both mathematics and physics studying.

Currently, progress has only partially been done in the Planetary System, in GPS model and Satellite System we only have some basic theories and research.

## 1.3 Sub Problems

### 1.3.1 GPS

As described in the beginning of this chapter, one of main research parts is the GPS model. In this simulation model of GPS, we will focus on the following problems.

First, in our GPS simulation model, we are not going to cover all the behaviours of the GPS in our real life. Our research will just concentrate on the data or information transmission and measurements. For example, how to measure the distance between a GPS satellite and a target GPS receive. Therefore, before we go further in the simulation of GPS, we have to have an overview of the theoretical study in GPS.

Second, we will investigate and discuss about GPS time [5] problem. This GPS time problem is the kernel part of a GPS system, and it is also the difficult part in this project. Comparing with other function we are going to implement, the GPS time contains advanced algorithms and techniques in physics and mathematics.

Third, the last sub problem we extracted is the GPS model simulating. We can divide this sub problem into two parts. On the one hand, we will try to compose a model with an overview of GPS. Like how the GPS satellites orbit with the earth rotating at the same time. On the other hand, we will specify a concrete animation to show the theories and methods we used to determine the target GPS receiver and calculate the data needed.

### **1.3.2 Flash programming**

In theoretical, we should just focus on the fields we work on, the programming part should not be one of the sub problems in our research. However, in this project, our results are based on a series of animations. In order to illustrate those models for a better understanding, we need to face a great many of three-dimension objects. For example, in GPS and Satellite System simulation, we will give an overview of earth rotation with the satellites moving around in their orbits.

Since we will use three-dimension as a significant part in the simulation, a following problem is that what the best programming tool for this project. As we all know, there are Java, C++, and python, and so on. Each of them has their outstanding strengths and also has weaknesses in different aspects. Which one is the best matching our models' simulation requirements? Or there are maybe other programming tools.

### **1.3.3 Take the target users into account**

In the beginning of this paper, we mentioned that all of these simulation models will be used in a practical college course. The results and effects will be evaluated by both the teachers and students. The target of this research is that, comparing with our traditional education mode, whether it is better for understanding by using these simulation models to help students in the corresponding fields.

Therefore, with considering our end users, our design will be based on the knowledge of college students, and this relevant knowledge should be covered in our models as large as possible. Nevertheless, we expect these models could also be used for advanced study and research in each domain.

## **1.4 Delimitation**

### **1.4.1 Delimitation of Project**

This project is to deal with a sub task of Modelling of the Universe. The primary target is to design a GPS simulation model. The eventual goal in our study is using the simulation model to help teaching for college student on Internet. Moreover, we add the relativistic time computing in, and this is a core part in GPS simulation model.

In the GPS model, we will design the fundamental part of the models, and an important point is that we don't need to simulate all the behaviours for each system. While, we will implement as many as we could, since these models will be tested by college students in practice. The relativistic time computing is an auxiliary behaviour part in GPS modelling. Because it refers to relativity, so it will be a main research part in our project.

#### **1.4.2 Delimitation of GPS**

In our modelling of GPS, all the efforts will focus on getting an accurate position of the target GPS receiver but not tracking the moving of it. That means, in this simulation model, what we prefer to doing is how to precisely calculate the GPS receiver's position rather than set a route for a GPS car to make the best convenient way and avoid unnecessary traffics.

The relativistic time of GPS is the kernel part in our model. However, we will take this part in to consideration when we first finish the fundamental part of the simulation model.

### **1.5 Temporary Assumptions and Hypotheses**

#### **1.5.1 The GPS model**

We assume that the simulation model of GPS can have some exterior features and properties of GPS, and can calculate the target GPS receiver's position precisely. Besides, we expect that it can examine the errors when the GPS satellites change the positions. Furthermore, we hope we can use the model to explain the relativistic time computing.

#### **1.5.2 The simulating result of the models**

In GPS model, we hope that the calculated result of the position should close to the actual value. When the positions of the GPS satellites changed, the error will not influence the calculation of the GPS receiver's position.

### **1.6 Importance of the Problem**

In our problem description, we organised the entire project as a three-step process. First is to compose simulation models for GPS. The second one is the relativistic time computing. And the last is to test our models in practical education in the college students.

Here we need to stress is the last problem. It extremely depends on the effect of solving the first two problems. The ideal result of the project is that it must be better than the traditional educating by using the animation models to help teaching in relevant fields. Consequently, whether the project is successful will much rely on the simulation models we design.

In this project, another importance of the problem is the research part. Not like a general step in a project doing the research first and then doing the implementation later. In our study, the research part will come after when we mainly fix the GPS model, and will go parallel with using



the models testing in the college students and this research will take the practical effects into account.

## **1.7 Motivation**

This project comes from Computer Science field, and my major is Information Technology Communication, In addition, this project contains knowledge in physics mathematics and Astronomy. Therefore, this research project is an integrated study, and to involving in such a challenge and exciting project will discipline my capabilities and enhance my knowledge.

Another attractive point is that the research part will be based on the practical utilization. Consequently, it will be the best way to examine my study in my Master's career, and will be an excellent start to pave my way for my future study and work.

## 2 Theory and Literature

In this chapter, we will focus on the theoretical part of the project, and cover the basic principles of GPS.

### 2.1 GPS Overview

The Global Positioning System is a constellation of 24 satellites that work in concert with a receiver on the earth to give the user their longitude, latitude, and elevation to very high accuracy [3].

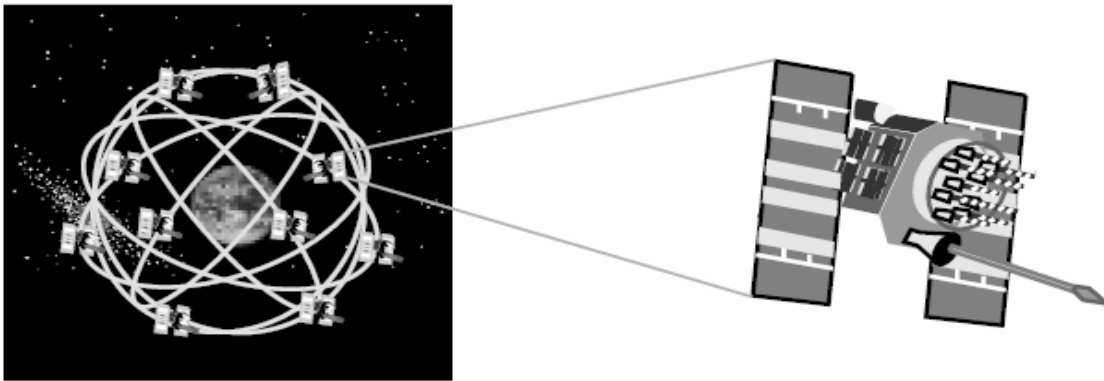


Figure 2-1

The current GPS consists of three major segments. These are the space segment (SS), a control segment (CS), and a user segment (US) [4].

#### 2.1.1 Space Segment

The Space Segment of the system consists of the orbiting GPS satellites (Space Vehicles (SV) in GPS parlance) [5]. These space vehicles send radio signals from space.

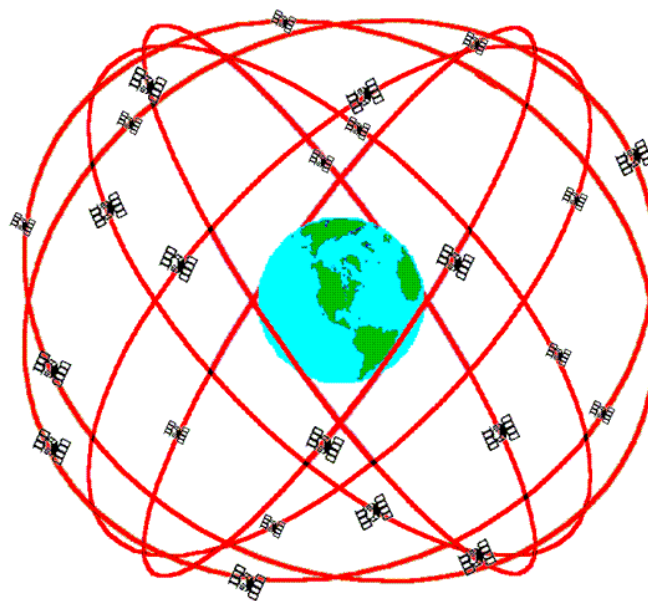
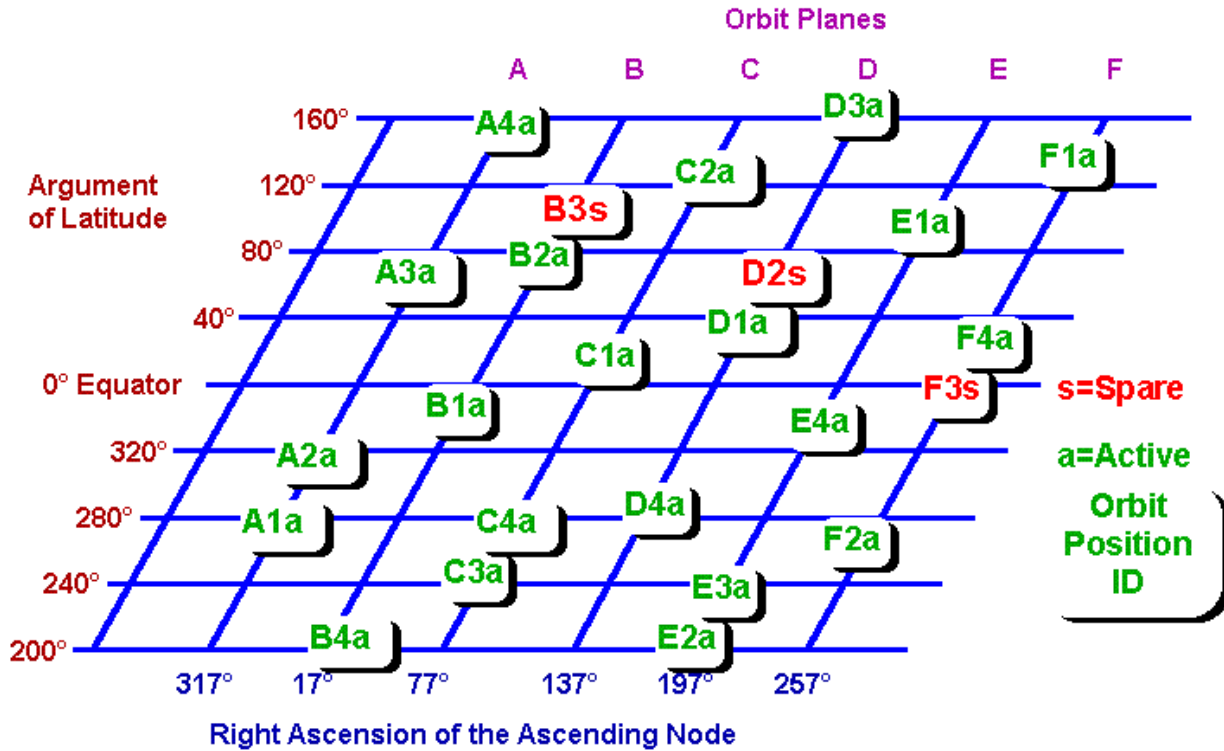


Figure 2-2

The GPS design calls for 24 SVs to be distributed equally among six circular orbital planes [6]. However, there are often more than 24 operational satellites as new ones are launched to replace older satellites [4] [5]. The orbital planes are centered on the Earth. The six planes have approximately 55° inclination (tilt relative to Earth's equator) and are separated by 60° right ascension of the ascending node.



### Simplified Representation of Nominal GPS Constellation

Figure 2-3

Orbiting at an altitude of approximately 20,200 kilometres (12,600 miles orbital radius of 26,600 km (16,500 miles)), each SV makes two complete orbits each sidereal day, so it passes over the same location on Earth once each day. The orbits are arranged so that at least six satellites are always within line of sight from almost everywhere on Earth's surface. [5]

As of April 2007, there are 30 actively broadcasting satellites in the GPS constellation. The additional satellites improve the precision of GPS receiver calculations by providing redundant measurements. With the increased number of satellites, the constellation was changed to a nonuniform arrangement. Such an arrangement was shown to improve reliability and availability of the system, relative to a uniform system, when multiple satellites fail [5].

#### 2.1.2 Control Segment

The Control Segment consists of a system of tracking stations located around the world.

The Master Control facility is located at Schriever Air Force Base (formerly Falcon AFB) in Colorado. These monitor stations measure signals from the SVs which are incorporated into orbital models for each satellite. The models compute precise orbital data (ephemeris) and SV clock corrections for each satellite. The Master Control station uploads ephemeris and clock data

to the SVs. The SVs then send subsets of the orbital ephemeris data to GPS receivers over radio signals.



**Global Positioning System (GPS) Master Control and Monitor Station Network**

*Figure 2-4*

### 2.1.3 User Segment

The GPS User Segment consists of the GPS receivers and the user community. GPS receivers convert SV signals into position, velocity, and time estimates. Four satellites are required to compute the four dimensions of X, Y, Z (position) and Time. GPS receivers are used for navigation, positioning, time dissemination, and other research.

## 2.2 Simplified method of operation of the GPS

In the previous paragraph, we have described what the GPS system is and how it designed, and the three major segments. And here we will expose some basic principles and theories.

The idea behind GPS is rather simple. If the distances from a point on the Earth (a GPS receiver) to three GPS satellites are known along with the satellite locations, then the location of the point (or receiver) can be determined by simply applying the well-known concept of resection [5]. But how can we get the distances to the satellites as well as the satellite locations?

As mentioned before, each GPS satellite continuously transmits a microwave radio signal composed of two carriers, two codes, and a navigation message. When a GPS receiver is switched on, it will pick up the GPS signal through the receiver antenna. Once the receiver acquires the GPS signal, it will process it using its built-in software. The partial outcome of the signal processing consists of the distances to the GPS satellites through the digital codes (known as the pseudoranges) and the satellite coordinates through the navigation message.

Theoretically, only three distances to three simultaneously tracked satellites are needed. In this case, the receiver would be located at the intersection of three spheres, as shown in *Figure 2-5*; each has a radius of one receiver-satellite distance and is centred on that particular satellite. From the practical point of view, however, a fourth satellite is needed to account for the receiver clock offset [7].

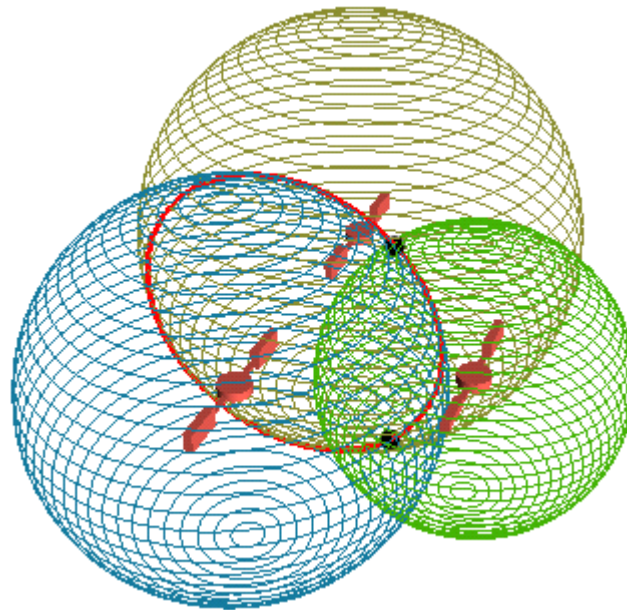


Figure 2-5

### 2.3 Time is of the essence

There are several factors will influence the precise of determination of the receiver. Like Atmospheric effects, Multi-path effects, clock errors and so on. in this project, we will only focus on the time and consider the clock errors.

A number of time systems are used worldwide for various purposes [8]. Of these, the Coordinated Universal Time (UTC) and the GPS Time are the most important to GPS users. UTC is an atomic time scale based on the International Atomic Time (TAI). TAI is a uniform time scale, which is computed based on independent time scales generated by atomic clocks located at various timing laboratories throughout the world. In surveying and navigation, however, a time system with relation to the rotation of the Earth, not the atomic time, is desired. This is achieved by occasionally adjusting the UTC time scale by 1-second increments, known as leap seconds, to keep it within 0.9 second of another time scale called the Universal Time 1 (UT1) [8] [9], where UT1 is a universal time that gives a measure of the rotation of the Earth. Leap seconds are introduced occasionally, on either June 30 or December 31. As of July 2001, the last leap second was introduced on January 1, 1999, which made the difference between TAI and UTC time scales to be exactly 32 seconds (TAI is ahead of UTC).

GPS Time is the time scale used for referencing, or time tagging, the GPS signals. It is computed based on the time scales generated by the atomic clocks at the monitor stations and onboard GPS satellites. There are no leap seconds introduced into GPS Time, which means that GPS Time is a continuous time scale. GPS Time scale was set equal to that of the UTC on January 6, 1980. However, due to the leap seconds introduced into the UTC time scale, GPS Time moved ahead of the UTC by 13 seconds on January 1, 1999. The difference between GPS and UTC time scales is given in the GPS navigation message. Both GPS satellite and receiver clocks are offset from the GPS Time, as a result of satellite and receiver clock errors.

## 2.4 Relativistic time in GPS

According to the theory of relativity, the clocks on the satellites are affected by their speed (special relativity) as well as their gravitational potential (general relativity). For the GPS satellites, general relativity predicts that the atomic clocks at GPS orbital altitudes will tick more rapidly, because they are in a weaker gravitational field than atomic clocks on Earth's surface. The gravitational effect is greater than the effect of the speed of the satellite, so the time in satellites moves faster than time on the earth. Special relativity predicts that atomic clocks moving at GPS orbital speeds will tick more slowly than stationary ground clocks [10].

As we know GPS uses accurate, stable atomic clocks in satellites and on the ground to provide world-wide position and time determination. These clocks have gravitational and motional frequency shifts which are so large that, without carefully accounting for numerous relativistic effects, the system would not work.

In a word, time runs faster in the satellites.

## 2.5 Teaching and learning methods

### 2.5.1 Traditional teaching

Typically, teaching has focused on the passive learner where students are expected to learn and repeat back information. Authentic, practically applied learning challenges students to take a more active role in their own education, as they assimilate a strong theoretical knowledge base within the context of material that they can work with in real life. [11]

Our aim is to use the simulation models to help teaching and make the students understand better.

In some scientific subjects, it's more important to practice than just study the basic theory. Therefore, we think our simulation models will give a lot of help for the college students learning relevant knowledge. Especially, the students have the change to understand what they learning actively.

### 2.5.2 Interactive teaching

Interactive teaching is a superior method of physics instruction, and computer tools are a significant part of the interactive teacher's tool box. [12]

Interactive teaching is just giving students something to do, getting back what they have done, and then assimilating it yourself, so that you can decide what would be best to do next. [13]

For example, even the most lucid and brilliant exposition of a subject by a teacher in a lecture, may result in limited learning if the students' brains do not do the necessary work to process it. There are several possible causes why students' learning may fall short of expectations in such a situation. They may,

- not understand a crucial concept partway into the lecture and so what follows is unintelligible,

- be missing prior information or not have a good understanding of what went before, so the conceptual structures on which the lecture is based are absent,
- lack the interest, motivation, or desire to expend the mental effort to follow the presentation, understand the arguments, make sense of the positions, and validate the inferences

### **2.5.3 Computer-assisted learning**

Computer-assisted learning provides perhaps the best opportunity for student self-guided learning. It is self-paced and self-planned, with the students themselves choosing their own paths through the mass of information encompassed by the package. Successful use of such packages will not only increase students' knowledge, but will require them to develop other important skills, including self-assessment and planning of studies, information technology skills, creativity, and self-motivation. [14]

Computer-assisted learning is to convey a vast amount of information in a very short period of time. It is a powerful method of reinforcing concepts and topics first introduced to you through your textbook, and discussion in the classroom. Computer-assessed learning enables you in a powerful way to comprehend complex concepts. [15]

### 3 Research Method

This chapter focuses on how to design the simulation model of the GPS system. With the simple review and analysis the GPS system and theories in last chapter, we plan to simulate the GPS in two major parts. They are GPS overview and GPS simulation.

In GPS overview, we hope that we can make a model which contains as much features and attributes of the GPS as possible, like the orbital planes, the rotations' speed of the earth and the satellites, and so forth. On the other hand, we will consider the GPS calculation part as the kernel of this project, and it will be divided into three sub-parts. First, we will use a very simple model to illustrate the simple principles of the GPS, and how to calculate the position of the GPS receiver with just only three GPS satellites. Second, we extend the first model to an advanced situation, and in this model we will introduce one more GPS satellite to get the GPS receiver's 3 dimensions position. Furthermore, we will take the time offset of the GPS satellite into allowance. Finally, if time allowed, we will refer to the relativity in GPS system. Therefore, we hope we can associate the GPS calculation and the relativistic time together to make precise tracking.

#### 3.1 GPS overview model

In this overview part, we make an animation to illustrate the very basic information of the GPS system. The following four messages are the major elements for the overview model.

- 24 satellites in 6 Orbital Planes
- 4 Satellites in each Plane
- 20,200 km Altitudes, 55 Degree Inclination
- 11,263 km/h Velocity, 2 complete orbits each sidereal day



*Figure 3-1*

And another major factor here is to let the satellites and the earth rotate in each correct speed. Therefore, we construct our model with the parameters given above, and we plan our animation model would contain the following features

First, we fixed the objects (earth, satellites and orbits) according to the parameters and information we have already known.



Second, the rotations of the satellites and the earth will be the key part of this simulation. On the one hand, we would fix a rotation speed for the earth and give a double speed to the GPS satellites. On the other hand, in order to vivify the animation, the satellites will have correct sizes when they are rotating. When they rotate from far to close (from back to front of the earth), the sizes of the satellites will gradually enlarge.

Third, to make the animation used in a friend manner, we will add some buttons or check boxes to improve the functionalities and control the animation easily.

- 1) An INFORMATION check box to describe the messages in this overview model
- 2) There are five buttons will be used to control the rotation, there are RUN, STOP, RESET, NEXT and PREVIOUS.
- 3) Consider there are 24 satellites on 6 orbits. We will design a check box to show the orbit planes. Furthermore, it would possible to use some check boxes or buttons to hide or show the satellites in different orbit plane.
- 4) When you click the earth or satellite, you can get brief information about the current situations. For example, when you click a satellite, you may get the speed of the satellite, current position of the satellite, current orbit, and the radius of the orbit and so on.

In this GPS overview part, we only refer to the space segment, so it seems just an overview of the space segment. However, in this project, what we focus on is to simulate the basic functions of the GPS. Like how the GPS satellites distribute in different orbits, how the GPS satellites move around the earth. What we do in this project is just to simulate the exterior part of the GPS. The details inside the GPS system will just be a light part of this project. We don't care what kind of signal the satellite sends, how the GPS receivers process the signals and how the GPS control segment functions. Therefore, this overview part we designed only contains the space segment which can represent the GPS system in a very intuitive manner.

Furthermore, in last chapter, we knew that from April 2007, there are 30 actively broadcasting satellites in the GPS constellation. This will not be a simulation part in the current situation. But we will refer to it in the information to explain it. And we will consider it as a future work if this project continues.

## **3.2 GPS simulation**

### **3.2.1 Three GPS satellites determination model**

In this part, we will use three simulation models to demonstrate three different situations to determine the GPS receiver's position. These simulation models are analogous, the theories and principles we used are almost the same. The reason why we don't put them together is because we consider the different situations will lead to different results and it will make a better understanding for the users.

Before we start our models for the GPS determination, we need to clarify that we will not take any influences into account. So our simulation models of GPS system are just ideal models. However, since there are used for aiding the college student with their learning mathematics

and physics on Internet, thus, without considering the influences from non-technical area, the ideal simulation models will not do any harm for us to illustrate the methods and theories of the GPS.

As stated in the beginning in this chapter, we will divide the GPS calculation into three sub-parts. They will explain how to determine the position of GPS' receiver from a simple to advanced model. Although, the principles behind are the same. The three simulation models are:

- 1) Three stationary GPS satellites determine the position GPS receiver.
- 2) Four GPS satellites determine the position of the GPS receiver.
- 3) Deciding the position of the GPS receiver in a relativistic condition.

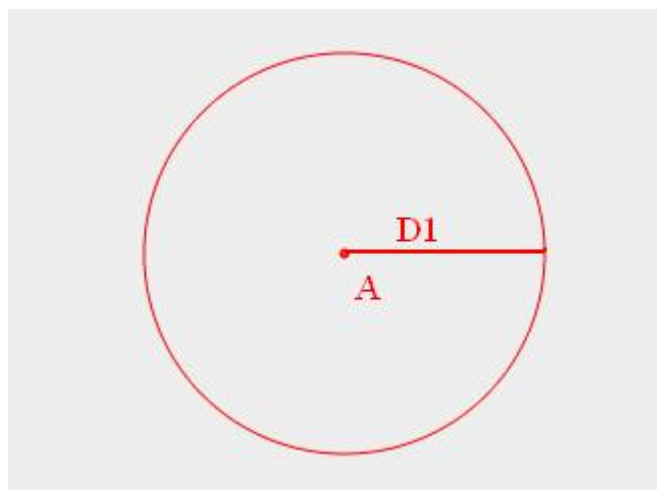
### ● Trilateration

Before we give an explicit description of GPS calculation, we will go through the simple principles how to determine the position of the GPS receiver. One of the major methods we used in this project is the Trilateration [16].

Let's first think a two dimensions condition:

- 1) Suppose that you are now in an unknown place.
- 2) You are told that your current location is a certain distance far from place  $A$ . Let's call this certain distance  $D_1$ .
- 3) Your friend in place  $B$  tells you that the distance between your current position and him is  $D_2$ .
- 4) Furthermore, the place  $C$  is  $D_3$  far from you,  $D_3$  is the distance from place  $C$  to you.

With the information above, it would be easy to get certain position where you are. Using the first two messages, we can know that you are now in a circular region centred on the place  $A$ , and the distance from you to place  $A$  is  $D_1$ . Like what we depict in the *Figure 3-2*.



*Figure 3-2*

Now we take the third condition into consideration, if we centred on place  $B$ , draw a circle with the radius  $D_2$ . Therefore, you currently have two circles and there are two intersections showing in *Figure 3-3*, we call them Point 1 and Point 2. So we can say that one of these two points is the certain position where you are. However, it is obvious that only three conditions will not enough to get the right position we need.

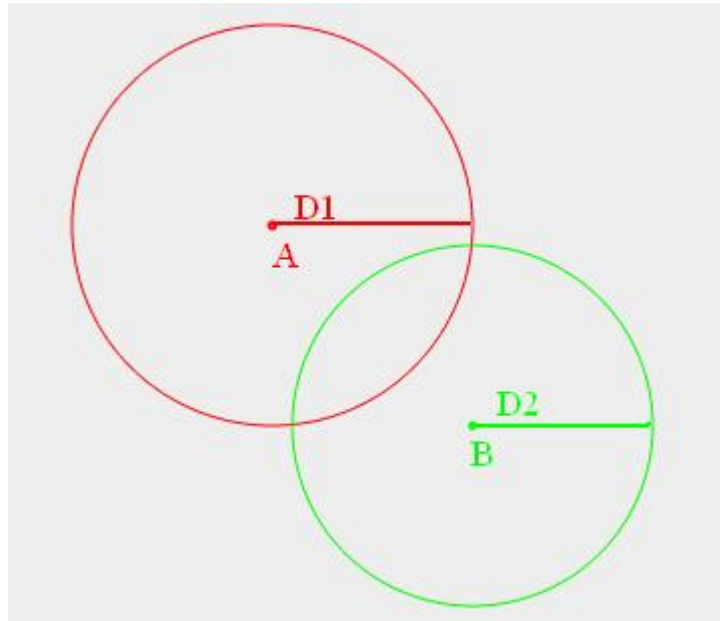


Figure 3-3

Here we add the last condition, by drawing the circle with the radius of  $D_3$  and centred on the place  $C$ , in the *Figure 3-4* we can clearly find the intersection we are looking for, and it surely that you current position is the intersection of the three circles.

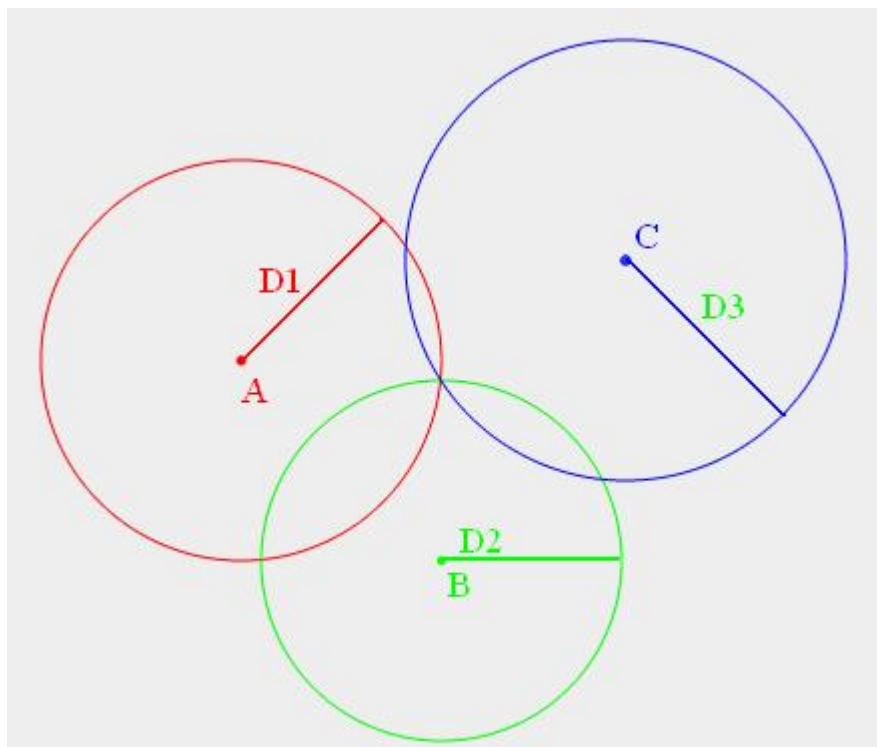


Figure 3-4

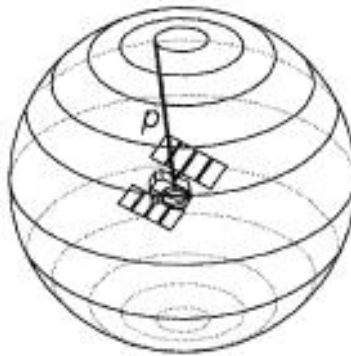
This is the simple concept of Trilateration, and our GPS determination uses the same concept, the difference is that the GPS is a three dimensions condition, and instead of circles we illustrate above, we determine the position with spheres.

● **Three GPS satellites determination model**

This model we will try to establish here is a simple one. Ground on the Trilateration concept we described before, we will introduce three GPS satellites to explain how we calculate the position of a GPS receiver.

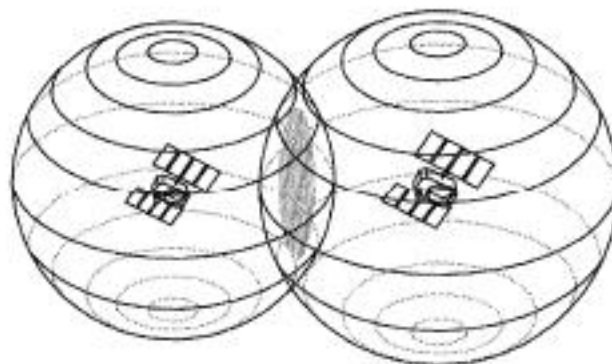
In this simple model, we assume that the clock in the receiver is synchronised with the clock in the satellite, and the ionosphere and troposphere, which slightly delay the arrival of the signal, do not exist. Furthermore, let's assume there is no measurement noise, that is, no random perturbation to the measurement, something that invariably affects all measurements to a greater or lesser degree. With a signal such measurement of the distance or range to the satellite, we can determine something about the position of the receiver: it must lie somewhere on a sphere centred on the satellite with a radius in the *Figure 3-5*. Let's call that distance  $P_1$ . Here you know that:

- 1) where satellite is in space
- 2) exactly how far we are away from it



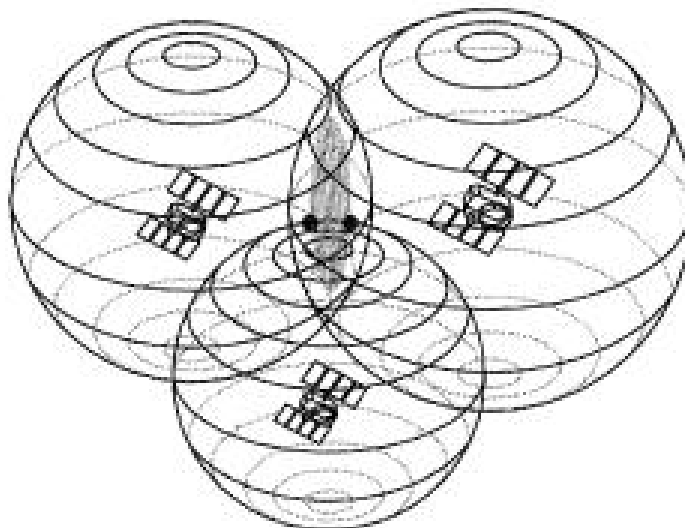
*Figure 3-5*

If we simultaneously make a range measurement to a second satellite, then our receiver must also lie on a sphere, of radius  $P_2$  centred on a sphere. The two spheres will intersect, with the location of the intersection points forming a circle, which is called a line of position. As shown in the *Figure 3-6*, we can get that only places in the universe where you can be are where the two spheres intersect (the shadow part of *Figure 3-6*).



*Figure 3-6*

Like what we do in the two dimensions, here we also add a third simultaneous range measurement,  $P_3$ , gives us a third sphere that intersects the other two at two points only, illustrated in the *Figure 3-7*. One of these points can be dismissed immediately as being the location of our receiver because it will lie far out in space. So, the simultaneous measurement of the ranges to three satellites provides sufficient information to determine a position fix in three dimensions.



*Figure 3-7*

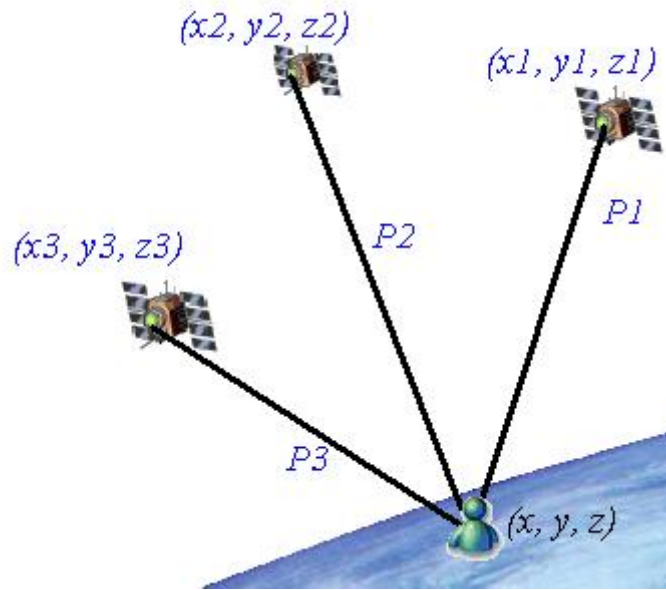
With the short description of how to determine the GPS receiver's position with three simultaneous satellites, we can get three parameters, they are  $P_1$ ,  $P_2$  and  $P_3$ , each of them represents the distances from each satellite to the GPS receiver. But only these pieces of information are not enough to determine the receiver's position. We need more conditions to make the determination.

### ● Mathematics in GPS

As we mentioned in previous chapter, GPS receivers receive signals from GPS satellite. We put what kinds of signals aside. These signals contain the data we need. One of the most significant data is the position of the satellites which can locate the three-dimensional coordinates. Another important data is the time which can help us to calculation the distances between the receiver and the satellite. Now we have:

- 1) The current positions of three satellites. Let's use  $L_1(x_1, y_1, z_1)$ ,  $L_2(x_2, y_2, z_2)$  and  $L_3(x_3, y_3, z_3)$  represent the locations of satellites.
- 2) From the signal we also get the time data. We can capture two useful values from these time data. Let's call them t-send and t-recv, then we subtract them and get a new time value which means the total time consumed from signal sending to signal reaching the receiver. We name the three total times as  $t_1$ ,  $t_2$  and  $t_3$ .
- 3) The speed of the light is a constant value which always be notated by  $c$ . and now we multiply the speed of the light with the total time, we will get the distance from receiver to each satellite. The three distance values are  $P_1 = c * t_1$ ,  $P_2 = c * t_2$  and  $P_3 = c * t_3$

Now using the messages we get from the signals, we could determine the position of the receiver, as shown in *Figure 3-8*.



*Figure 3-8*

Assume this unknown position is  $R$ , and it is represented with a three-dimensional coordinates  $(x, y, z)$ . So we can get an equation below:

$$P_1 = \sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} \quad (3-01)$$

And it will easy to get the other two equations. Thus combine the three together we could the following set of equations:

$$P_1 = \sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2}$$

$$P_2 = \sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2} \quad (3-02)$$

$$P_3 = \sqrt{(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2} \quad (3-03)$$

In these three equations, there are three unknown parameters which are  $x$ ,  $y$  and  $z$ . It is no problem that to work them out, and by resolve this set of equations we will get the position of the GPS receiver.

After we get the three-dimensional coordinates  $(x, y, z)$  we have one more thing to deal with. That means we have to convert this  $(x, y, z)$  to latitude and longitude using trigonometry and to elevation using the distance formula between the centre of the earth and the point  $(x, y, z)$  and subtracting the radius of the earth.

Now, we first start with the latitude. *Figure 3-9* illustrate how we process it.

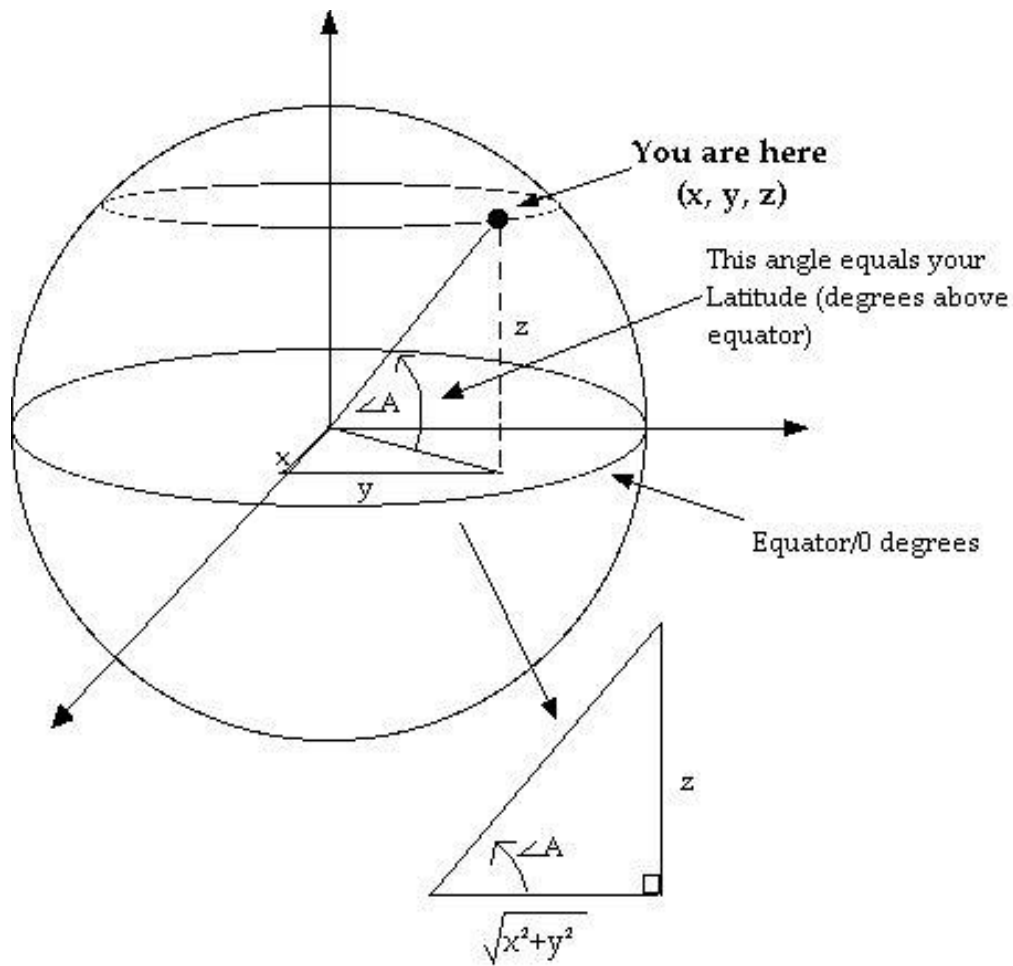


Figure 3-9

From the figure, we can easy to get that:

$$\tan A = \frac{z}{\sqrt{x^2 + y^2}} \quad (3-04)$$

And resolve angle  $A$  from equation 3-04, we could get:

$$A = \arctan \frac{z}{\sqrt{x^2 + y^2}} \quad (3-05)$$

The angle  $A$  is the latitude we want.

Second, we get the result of longitude.

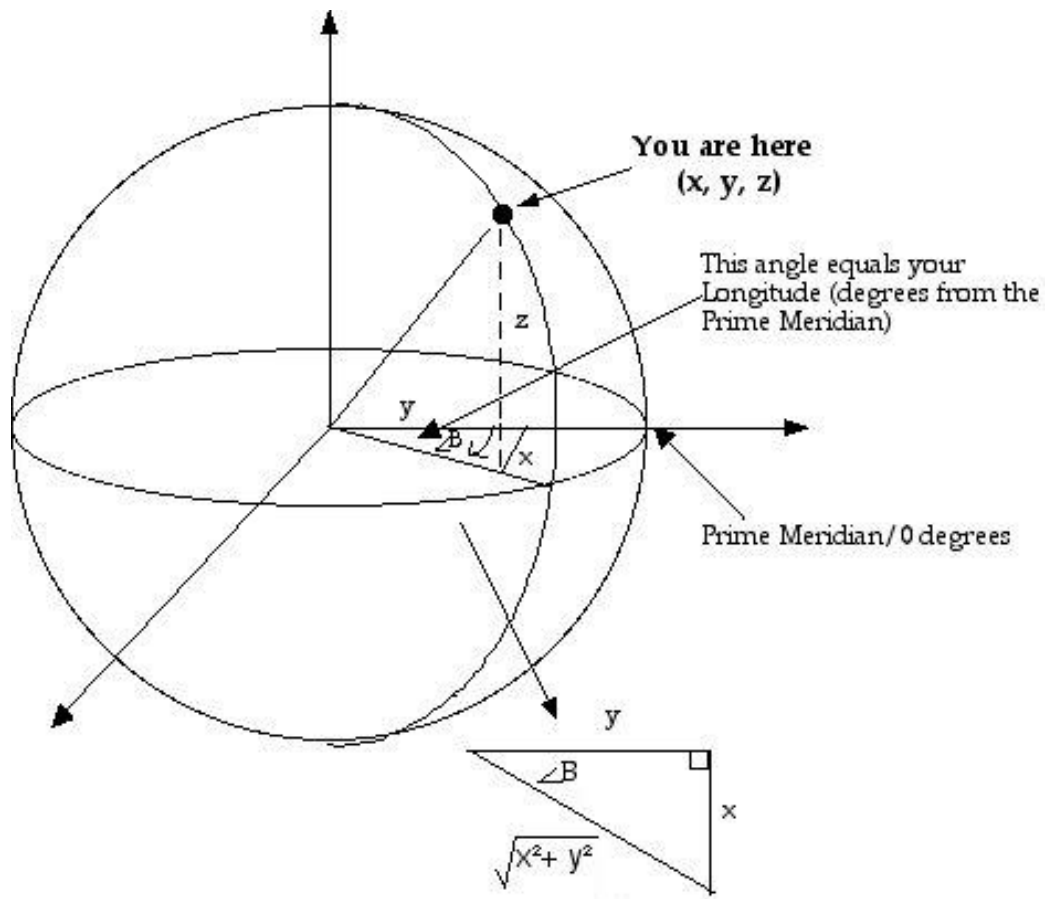


Figure 3-10

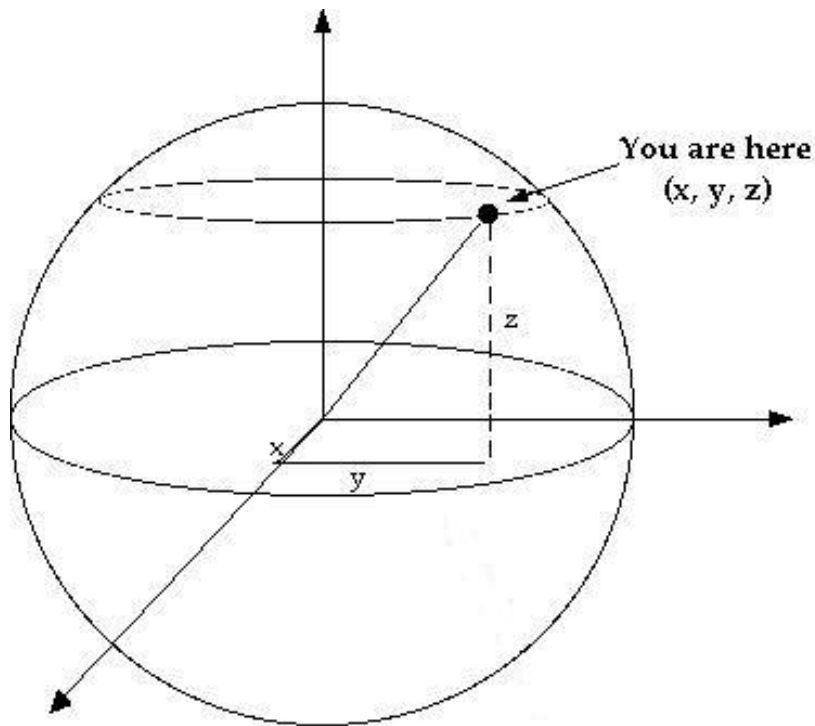


Figure 3-11



As described in *Figure 3-10*, the angle  $B$  is the longitude, so from the figure we can get the longitude by get the value of  $B$ :

$$\tan B = \frac{x}{y} \quad (3-06)$$

Resolve  $B$  from equation (3-06), then it will get:

$$B = \arctan \frac{x}{y} \quad (3-07)$$

Finally, we can get the elevation of the receiver, as shown in the *Figure 3-11*. The elevation is the distance from the receiver to the centre of the earth. Let's call the radius of the earth is  $R_E$ . Thus, the elevation  $E$  is:

$$E = R - R_E \quad (3-08)$$

$$E = \sqrt{x^2 + y^2 + z^2} - R_E \quad (3-09)$$

#### ● The simulation model

The discussion, how to get the GPS receiver by using three-dimensional Trilateration concept, is just the principle we solve the calculation part. In order to make an understanding and explicit explanation for how the GPS system works, we must combine the calculation and the basic functions together. So based on this opinion, we think this simulation model will be constructed in four steps.

Firstly, as the aim of this simulation model is to give a simple understanding of how the GPS system works, we design this model to be a stationary one with only three GPS satellites to determine the location of the GPS receiver. Thus, in this model, we use three satellites, receiver and earth to consist the frame. Here we will stress that the receiver is an unknown factor, so in our simulation model, it will be possible that we can control the position of the receiver.

Secondly, the aim of this model is to determine the correct position of the receiver, so once the receiver appears somewhere on earth, the satellites would simultaneously send signals to the receiver. At the same time when the signals were sent, there are three timers to record the start times for those three satellites, and when the signals reach the receiver, the arrival times will also be recorded. So by multiple the speed of the light and the total time of sending signal (arrival time subtract start time), we can get the distance of each satellite.

Thirdly, since we know the positions of the receivers, we could use the equations 3-01, 3-02, 3-03 to get the position  $(x, y, z)$  of the receiver, and it could convert to latitude, longitude and elevation.

Finally, there will be a controller plays the role of control segment. After every 24 hours, there will be signals send to satellites to correct the time because the time on satellite is quick than it on earth.

With the analysis above, we hope our simulation model would have the following features and attributes:

- 1) To control the appearance of the receiver, we will design a button to control it. By clicking the button, the receiver could be seen in different place on earth.
- 2) Another button to control the signals sending from satellites. When the receiver is on a certain place, once you click this button, signals will send to receiver. These signals' speed is the speed of light. We will slow them down in our design. On the one hand, it makes clear that those signals are sent from the satellites to the receiver, not from the receiver to the satellites. On the other hand, the user can see that the signals reach the receiver in different time because of the distance between the receiver and each satellite.
- 3) After every 24 hours, the controller sends signal to satellites to check the time. In order to make a better understanding, we also slow down the speed of the signal which is the speed of the light.
- 4) When sending the signal, there will be text field shows the time when the signal sends and the time when the signal reaches the receiver. Therefore, we can see the signals are sent at the same time, but received within different times. Besides, once get the received time, the distance from each satellite to the receiver will be showed in each text field. Furthermore, the receiver's position will be calculated and showed in latitude, longitude and elevation as well as in three-dimensional coordinate.
- 5) A check box will be used to illustrate how we apply the three-dimensional Trilateration to determine the location of the receiver.
- 6) To make a better understanding of the simulation model, we will have another two check boxes. One of them is an explanation about how this model functions and how to control it. Another one is some information about the theories of how the GPS system works.

### 3.2.2 Four GPS satellites determination model

The first model is an ideal model without any disturbance, we assumed that the clock in the GPS receiver was synchronised with the clocks in the satellites. However, this assumption is not correct.

In this model, we take the time into account. Time plays a very important role in positioning with GPS. The GPS signal is controlled by accurate timing devices, the atomic satellite clocks. In addition, measuring the ranges (distances) from the receiver to the satellites is based on both the receiver and the satellite clocks. GPS is also a timing system, that is, it can be used for time synchronization.

When a GPS receiver is switched on, its clock will, in general, be mis-synchronised by an unknown amount with respect to the satellites clocks. Furthermore, the atomic clocks in the satellites are synchronised with each other and to a master time scale – called GPS time – only to within about a millisecond. The range measurements the receiver makes are biased by the receiver and satellite clock errors and therefore are referred to as pseudorange.

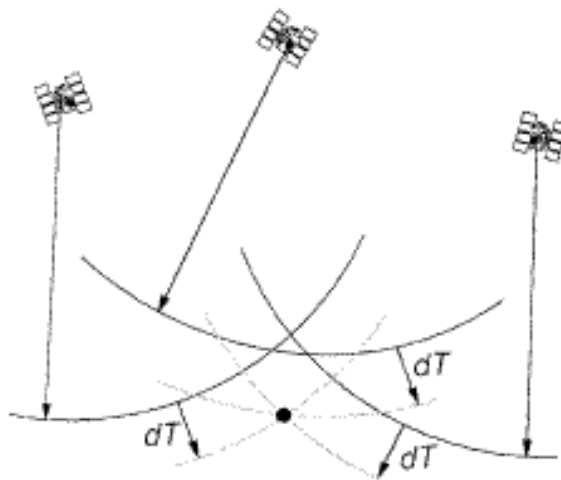
A timing error of a millisecond would result in an error in position of about 300 kilometres, clearly an intolerable amount. System operators conceivably could better synchronise the

satellite clocks by frequently sending them adjustment commands from the ground, but atomic clocks have been found to actually keep better time if they are left alone and their readings are corrected. The United States Naval Observatory monitors the GPS satellite clocks and determines the offsets and drifts with respect to GPS time. These parameters subsequently are uploaded to the satellites and transmitted as part of the navigation message broadcast by satellites. A GPS receiver uses these satellite clock offset values to correct the measured pseudoranges.

Nonetheless, we still have the receiver clock error to deal with. Because of this error, the three spheres with the radii equal to the measured pseudoranges corrected by the satellite clock offsets will not intersect at a common point. However, if the receiver clock error,  $dT$ , can be determined, then the pseudoranges can be corrected and the position of the receiver determined. The situation we compressed into two dimensions is illustrated in *Figure 3-12*.

Comparing with three unknown parameters in our previous model, we now have one more which is the receiver clock error,  $dT$ . So, actually we have four unknown parameters that we must determine:

- 1) Latitude
- 2) Longitude
- 3) Elevation
- 4) Receiver clock offset



*Determination of receiver clock offset ( $dT$ ) and true user position (intersection of shaded lines) from the intersection of spheres centred the satellites; pseudoranges (distances) are shown by arcs of solid lines*  
**Figure 3-12**

Now, it is impossible to precisely determine the values of four parameters by only using three measurements. The way to solve this problem is that we simultaneously add one more satellite to measure.

Therefore, we have the following set of equations:

$$P_1 = \sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} - c \times dT \quad (3-10)$$

$$P_2 = \sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2} - c \times dT \quad (3-11)$$

$$P_3 = \sqrt{(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2} - c \times dT \quad (3-12)$$

$$P_4 = \sqrt{(x_4 - x)^2 + (y_4 - y)^2 + (z_4 - z)^2} - c \times dT \quad (3-13)$$

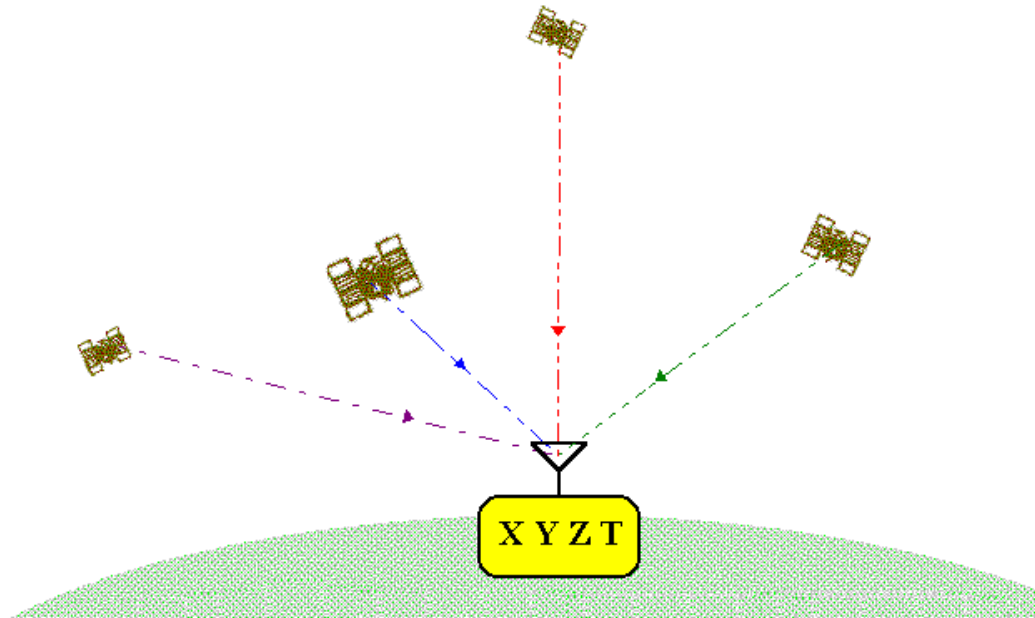


Figure 3-13

In the set of equations above, we have to note that:

- 1)  $P_1, P_2, P_3$  and  $P_4$  represent the distances from the satellites to receiver.
- 2)  $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$  and  $(x_4, y_4, z_4)$  represent the positions of the satellites.
- 3) The  $c$  represents the velocity of the light.
- 4) The parameter,  $dT$ , is the receiver clock error.
- 5)  $(x, y, z)$  represents the unknown position of the receiver.

Then we can get a more precise position  $(x, y, z)$ . And convert these  $x, y$  and  $z$  to latitude, longitude and elevation.

#### ● The simulation model with four satellites

So, this simulation will be based on the previous model, we could take this model as an advanced one. The principles and concepts are almost the same with our first model, and the only difference is the time offset. But the determination of the measurements is more precise, the functions are more powerful. Therefore, we will construct this simulation model in the same way we did in the first model.

First, compare with the first model, we would have four GPS satellites to be used to tracking and calculate the position of the receiver.

Second, we will use the rotation function in this model. So we can control and enable the earth and the satellites rotate.

Third, to determine the position of the receiver, the satellites send signals to the receiver with the time information and their current positions. Then the receiver can get the right position by dealing with the information and get the three-dimensional position and the time offset.

Finally, there will be a controller plays the role of control segment. After every 24 hours, there will be signals send to satellites to correct the time because the time on satellite is quick than it on earth.

To design this model, we will almost inherit the features and attributes in our first GPS simulation model and the overview model. Thus, comparing with the GPS overview model, this model will have the following new features:

- 1) We can enable and control the rotations of the earth and the satellites.
- 2) It's able to calculate the time error of the receiver.

### 3.2.3 Relativity in GPS model

#### ● Special Relativity and General Relativity

One of the effects of Einstein's Special Relativity theory is time dilation. Clocks moving with high velocity run slower than clocks with the smaller relative velocity. Therefore, clocks in the GPS satellites will run slower compare to the clocks on the Earth because GPS satellites have a large velocity. In more details the rates that clocks tick compare to the static clock are given by the formulas:

$$T_E^2 = \left(1 - \frac{V_E^2}{c^2}\right) \times T_S^2 \quad (3-14)$$

And

$$T_G^2 = \left(1 - \frac{V_G^2}{c^2}\right) \times T_S^2 \quad (3-15)$$

Where  $T_E$  is proper time of the clock on the surface of the Earth,  $V_E$  is the clock's linear velocity due to the rotation of Earth,  $T_S$  is proper time of static clock,  $T_G$  is proper time of the clock in the GPS satellite, and  $V_G$  is linear velocity of the GPS satellite in the orbit. Therefore the ratio at which clock will tick could be derived from the formulas above:

$$\frac{T_E^2}{T_G^2} = \frac{\left(1 - \frac{V_E^2}{c^2}\right) \times T_S^2}{\left(1 - \frac{V_G^2}{c^2}\right) \times T_S^2} \quad (3-16)$$

This effect has a fixed value. Clocks in the GPS satellites run slow by 6 millionth of a second per day.

The effect of General Relativity is that clocks at the higher altitude above the Earth run faster than the clocks on the surface of the Earth. From the Schwarzschild metric we could calculate the rates at which clocks of particular interest tick compare to the clocks at infinite distance from

source of gravitational field. Let's compare the rates of two static clocks: one on the surface of the Earth and other in the GPS satellite. For a static clock:

$$dT^2 = -dS^2 = \frac{\left(1 - \frac{R}{r}\right)}{dT_i^2} \quad (3-17)$$

Where  $T$  is proper time of the clock we interested in,  $T_i$  is proper time of the clock infinite distance away, and

$$R = \frac{2MG}{c^2} \quad (3-18)$$

Where  $M$  is mass of the Earth,  $G$  is Newtonian gravitation constant and  $c$  is speed of light. Therefore the ratio of rates at which clock on the surface of Earth and in GPS satellite will tick is given by formula:

$$\frac{dT_E^2}{dT_G^2} = \frac{1 - \frac{R}{rE}}{1 - \frac{R}{rG}} \quad (3-19)$$

Where  $rE$  is radius of Earth and  $rG$  is distance from the centre of the Earth to GPS satellite. This effect also has a fixed value. The clocks in the GPS satellites run fast by 45 millionths of second per day. Therefore combining Special and General Relativity effects we conclude that clock in GPS satellites run fast by 39 millionth of a second per day which if not taken into account could produce position error of 12 kilometres.

- **The simulation model with relativity**

As we described before, the three GPS simulation models have the same structure. So we can design the model in the previous two models. The new feature in our models is when the satellites rotate around the earth, the clocks on the satellites run fast than the clock on the receiver on the earth.

## 4 Implement

In the first chapter, we said that our project is a sub task in SimReal of ParAbel group. Currently, most tasks are related to mathematics and physics, and a great amount of them is to simulate the phenomena in physics and mathematics' formulas. So, there is a lot of works to do with the programming. In SimReal, most of the programs are based on actionscript 2.0 [4-1] in Flash 8 Professional [4-2].

In chapter 3, we discuss that we will design four models to illustrate a simple GPS system and how it determines the receiver's position. However, in this chapter we will talk the significant parts of our models instead describing how we design the models from one to another.

### 4.1 The Implement of the GPS overview model

In this model, as we stress in chapter 3, the chief task is to simulate rotations of earth and satellites.

First, in order to make a rotation earth, we have two ways to implement the rotation of the earth. One of these methods is to take a three-dimension object to represent earth rotation. The other one is to use a set of serial pictures to instead real rotation.

In our design, we will apply the second method. The reason is that:

- 1) To program a three-dimension earth is not a simple task because it contains different information in different angle.
- 2) In Flash, you can add a set of serial two-dimensional pictures to corresponding frames to consist an animation, shown in *Figure 4-1*. Once you convert them to a symbol, it then can be regarded as a rotate object.

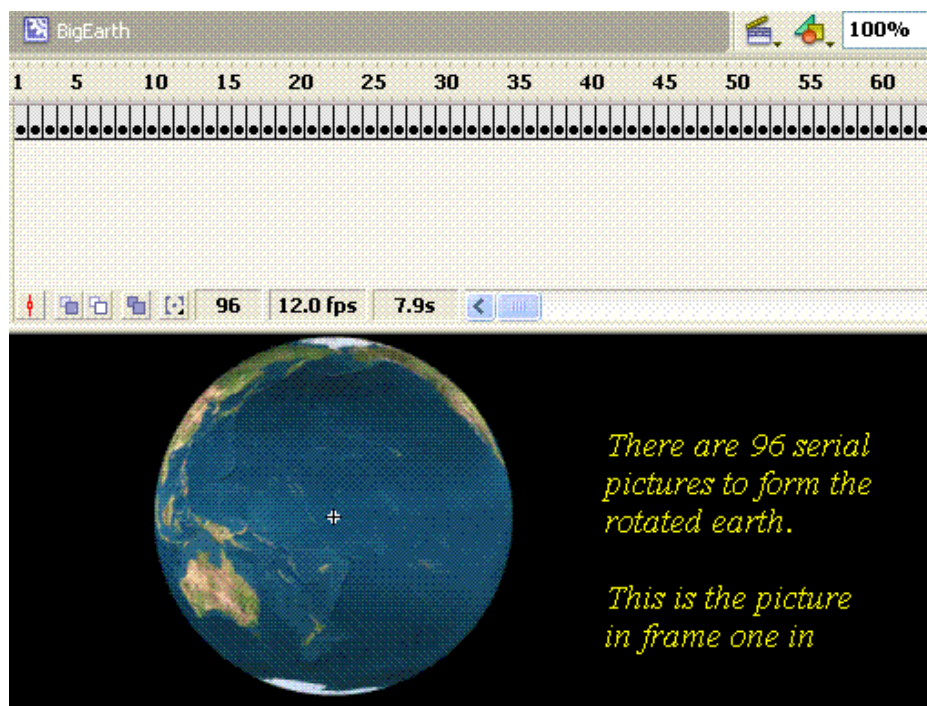
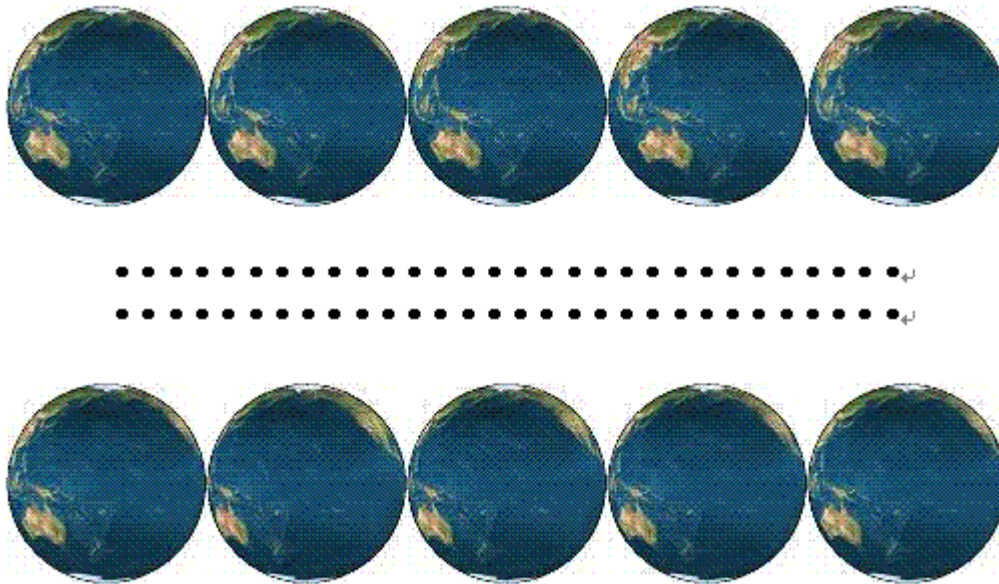


Figure 4-1

Therefore, we can use many serial pictures to consist a simple animation to simulate the rotation of the earth, although it's a fake three-dimensional earth. In *Figure 4-2*, we enumerate the first and the last five pictures in our total 96 serial pictures.



*Figure 4-2*

As we finish the rotation of earth, we will think the rotation of the satellites. Because the satellites are rotating around the earth, so it will give the users an effect that the satellites turn to large gradually from the back to front of the earth, and when they move to the back of the earth, they cannot be shown on screen.

In our simulation, we use the cosine and sine functions to change the positions of the satellites in  $x$  (from left to right) and  $y$  (from bottom to top) coordinates regularly. Both of them are used to initiate the positions of the satellites and to control the orbit planes. However, it is not enough because either can we hide the satellites when they move to the back of the earth or scale them. Therefore, we have to add a new parameter to control the rotation and simulation the three-dimensional effect. In our model we add a  $z$  coordinate to control the position from back to front. On the one hand, this  $z$  coordinate is used to control the size of satellites, and on the other hand, it also help to hide the satellites when it rotate to the back of the earth. *Figure 4-3* shows what kind of effect we expect. In this figure, all the satellites are on one orbit plane, we can clearly find that the different sizes of different satellites and one of them is rotating to the back of the earth.

When we simulate the rotations of the earth and the satellites, we add a time parameter to each of them. So, with the time going, we can see the satellites in different places, and the earth rotates. And this time parameters will be used to control the moving of those objects.

When we fix this problem of rotations, our models will benefit from it. In the GPS overview model, we illustrate the rotations by using 24 GPS satellites moving around a self-rotate earth.



Moreover, in the models of how the GPS system works, we will illustrate how we measure the pseudoranges.

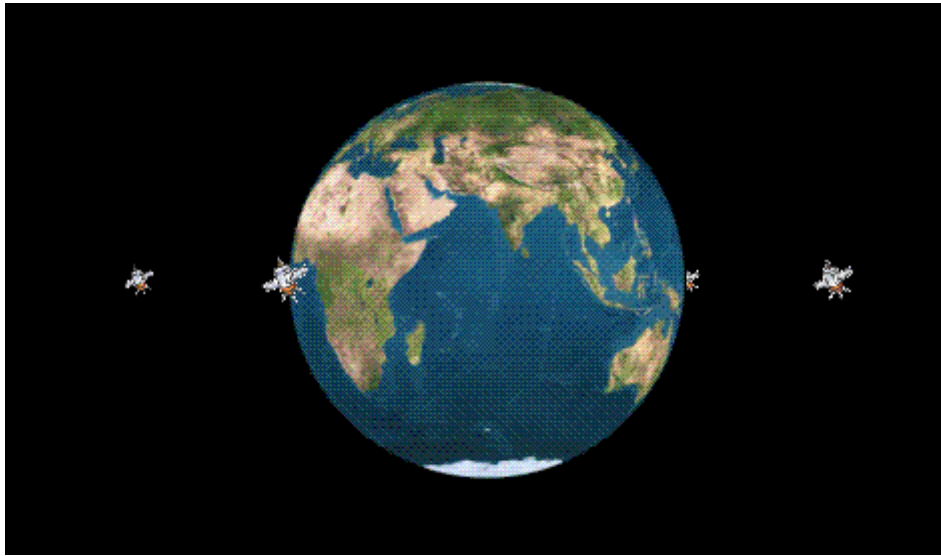


Figure 4-3

As we said, we will use the time parameters to control the rotations of the earth and the satellites. In the GPS overview model, we design five buttons to control the movement of the objects, they are:

- 1) A "RUN" button to start the rotating of the earth and the satellites.
- 2) A "STOP" button to cease the animation.
- 3) A "NEXT" button and a "PREVIOUS" button to show a forward step or a backward step.
- 4) A "RESET" button to stop the animation and make all the objects to their initial positions.

Furthermore, we will have a simple "information" text field to show what the model wants to demonstrate. And what we do here is to put a check box, and once you tick it, you will get some information about the simulation model, like what shows in *Figure 4-4*.

Overview of the GPS

- 24 satellites in 6 Orbital Planes
- 4 Satellites in each Plane
- 20,200 km Altitudes, 55 Degree Inclination
- 11,263 km/h Velocity, 2 complete orbits each sidereal day

As of April 2007, there are 30 actively broadcasting satellites in the GPS constellation

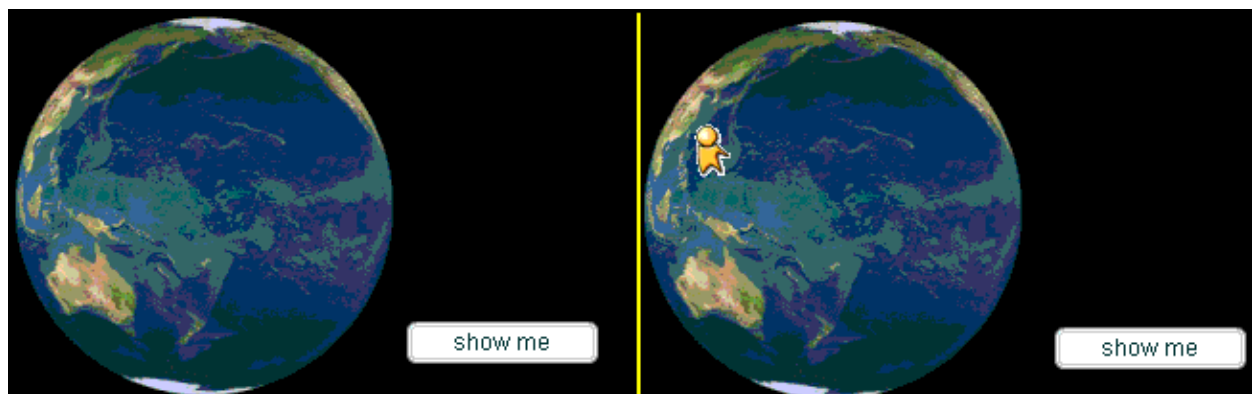
Info

Figure 4-4

## 4.2 The Implements of GPS simulation models

Now, we will explain how we deal with the GPS simulation models. First, let's see how to get the receiver shown.

In the beginning, we only show the earth and the GPS satellites on the animation, and then we click the button "Show me", the receiver will automatically shown on the animation in an unknown position. For this unknown position, it can appear in any position. So consider this situation, we use a random function to achieve it. *Figure 4-5* shows when we click the button, the receiver shows in the animation.



*Figure 4-5*

As the receiver shown on the screen, the satellites send signals to it. In our simulation model, we use another button, "where I am", to control the sending of the signals.

### Sending the signal

In the GPS simulation models, we will the process of sending signals to illustrate how the GPS system works. There are two kinds of signals.

- 1) Once the receiver wants to know its position, the GPS satellites send the signals to it.
- 2) Every 24 hours, the control segment will send signals back to GPS satellites to check and correct the GPS time.

In the second condition, the satellites' position can be detected when the receiver the signals from satellites, and the position of control segment is also in the certain location. Therefore, when we simulate the process when the signals sent from control segment to satellites, we can easily to draw lines from control segment to the satellites, as shown in *Figure 4-6*.

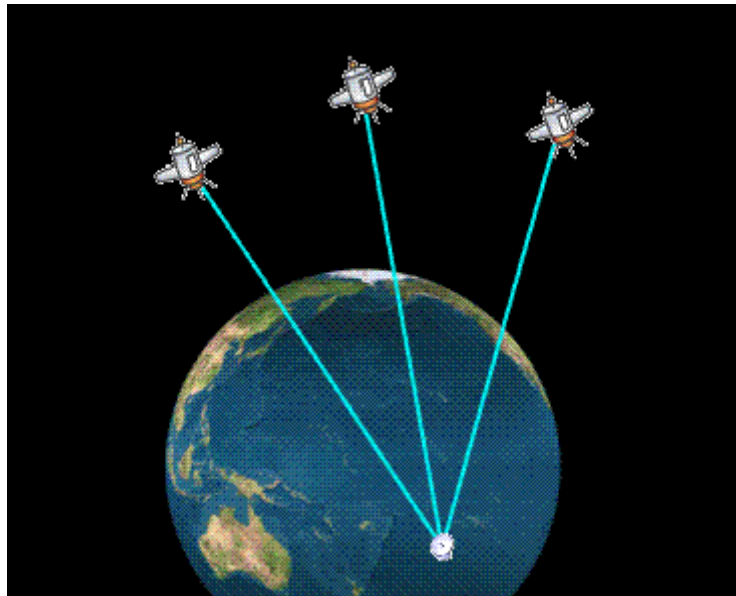


Figure 4-6

However, the first condition is more complex than the second condition. Not like the positions of the satellites and control segment, the receiver's position is unknown; we have to determine where it is. So the problem comes. In Flash actionscript, if you want draw a line, you must have both the point where the line starts and the point where it ends. But when sending signal from satellites to receiver, we only have the position of the satellites where the signals send, we can not decide where the signals reach.

When we discuss how the receiver shows on earth, we use a random function in Flash actionscript, even it has a different position in three-dimensional coordinate every time when we refresh the models, it do has a position which contains the x, y and z values in its coordinate. So, every time when the GPS receiver's position is initiated, we use a constant variable to hold the value of receiver's position. Thus, in one process of determining the position of the receiver, we will able to send signals from the satellites to the receiver, as shown in *Figure 4-7*.

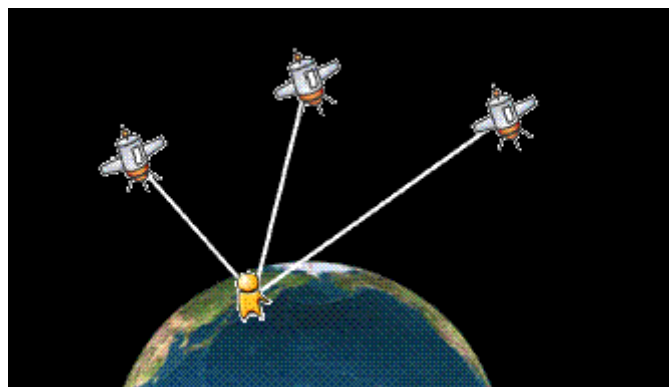


Figure 4-7

Now we will talk how we do the calculation. When the signals reach the receiver, our model will calculate the position if the receiver. This calculation is based on two sets of equations (3-01) to (3-03) and (3-10) to (3-13).

We convert (3-01), (3-02) and (3-03) to following equations:

$$P_1^2 = (x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 \quad (4-01)$$

$$P_2^2 = (x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2 \quad (4-02)$$

$$P_3^2 = (x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2 \quad (4-03)$$

Then it is easy to get:

$$P_1^2 = x_1^2 + y_1^2 + z_1^2 + x^2 + y^2 + z^2 - 2x_1x - 2y_1y - 2z_1z \quad (4-04)$$

$$P_2^2 = x_2^2 + y_2^2 + z_2^2 + x^2 + y^2 + z^2 - 2x_2x - 2y_2y - 2z_2z \quad (4-05)$$

$$P_3^2 = x_3^2 + y_3^2 + z_3^2 + x^2 + y^2 + z^2 - 2x_3x - 2y_3y - 2z_3z \quad (4-06)$$

Now we transpose the equations above:

$$2x_1x + 2y_1y + 2z_1z = x_1^2 + y_1^2 + z_1^2 + x^2 + y^2 + z^2 - P_1^2 \quad (4-07)$$

$$2x_2x + 2y_2y + 2z_2z = x_2^2 + y_2^2 + z_2^2 + x^2 + y^2 + z^2 - P_2^2 \quad (4-08)$$

$$2x_3x + 2y_3y + 2z_3z = x_3^2 + y_3^2 + z_3^2 + x^2 + y^2 + z^2 - P_3^2 \quad (4-09)$$

We know here the  $(x_1, y_1, z_1)$ ,  $(x_2, y_2, z_2)$  and  $(x_3, y_3, z_3)$  are the positions of the satellites, and  $p_1$ ,  $p_2$  and  $p_3$  are the distances from satellites to the receiver.  $(x, y, z)$  is the receiver's unknown position we want. But it seems difficult to get the answers from equations (4-07), (4-08) and (4-09), as the reason that the power of the equations is two, and we cannot convert them to linear equations to get the values of the unknown parameters.

If we will initiate the receiver on the surface of the earth, and the unknown value  $(x, y, z)$  is a certain position on the earth. So, in the right part of the equations,  $x^2 + y^2 + z^2$  is the square of the radius of the earth. Moreover,  $x_1^2 + y_1^2 + z_1^2$ ,  $x_2^2 + y_2^2 + z_2^2$  and  $x_3^2 + y_3^2 + z_3^2$  are invariable values.  $P_1^2$ ,  $P_2^2$  and  $P_3^2$  are the squares of the distances from satellites to receiver. Therefore, the right parts of the equations are certain values, the set of equations can be solved easier.

We convert the equations to the following:

$$2x_1x + 2y_1y + 2z_1z = C_1 \quad (4-10)$$

$$2x_2x + 2y_2y + 2z_2z = C_2 \quad (4-11)$$

$$2x_3x + 2y_3y + 2z_3z = C_3 \quad (4-12)$$

And

$$C_1 = x_1^2 + y_1^2 + z_1^2 + x^2 + y^2 + z^2 - P_1^2 \quad (4-13)$$

$$C_2 = x_2^2 + y_2^2 + z_2^2 + x^2 + y^2 + z^2 - P_2^2 \quad (4-14)$$

$$C_3 = x_3^2 + y_3^2 + z_3^2 + x^2 + y^2 + z^2 - P_3^2 \quad (4-15)$$

Therefore, we can get the following equation:

$$\begin{bmatrix} 2x_1 & 2y_1 & 2z_1 \\ 2x_2 & 2y_2 & 2z_2 \\ 2x_3 & 2y_3 & 2z_3 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} \quad (4-16)$$

So we can get the value  $(x, y, z)$ . By using the equations (3-05), (3-07) and (3-09), we can get the latitude, longitude and elevation. But we initiate the receiver on the surface of the earth. The result of the elevation should be zero.

However, it does not work if we want to solve the equations (3-10), (3-11), (3-12) and (3-13). Because if we convert the equations, we can get:

$$P_1^2 = \left[ \sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} - c \times dT \right]^2 \quad (4-17)$$

$$P_2^2 = \left[ \sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2} - c \times dT \right]^2 \quad (4-18)$$

$$P_3^2 = \left[ \sqrt{(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2} - c \times dT \right]^2 \quad (4-19)$$

$$P_4^2 = \left[ \sqrt{(x_4 - x)^2 + (y_4 - y)^2 + (z_4 - z)^2} - c \times dT \right]^2 \quad (4-20)$$

If we get rid of the square brackets, we will get:

$$P_1^2 = (x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 + (c \times dT)^2 - 2\sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} \times c \times dT \quad (4-21)$$

$$P_2^2 = (x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2 + (c \times dT)^2 - 2\sqrt{(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2} \times c \times dT \quad (4-22)$$

$$P_3^2 = (x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2 + (c \times dT)^2 - 2\sqrt{(x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2} \times c \times dT \quad (4-23)$$

$$P_4^2 = (x_4 - x)^2 + (y_4 - y)^2 + (z_4 - z)^2 + (c \times dT)^2 - 2\sqrt{(x_4 - x)^2 + (y_4 - y)^2 + (z_4 - z)^2} \times c \times dT \quad (4-24)$$

It is obviously that we have square of  $dT$  and as well  $dT$  in the equations. If we try to use the method we deal with the simulation model without time offset, it would be very difficult.

In chapter 3, we discussed a lot about the Trilateration to determine the receiver's position. In our models, when the receiver's position has been confirmed, we can show how the Trilateration works in our determining the position of the receiver.

First, when we try to determine the position of the receiver, we send signals from satellites. Then we can get the positions of the satellites. Moreover, the information contains time data, so we are able to calculate the distances from the satellites to the receiver. These distances are the radii of the spheres centred on the satellites. And two spheres can determine a certain range of the position of the receiver, and one more can select two points, one of them is the position we want. Adding the sphere of the earth, we can get the receiver's position, as shown in *Figure 4-8*.

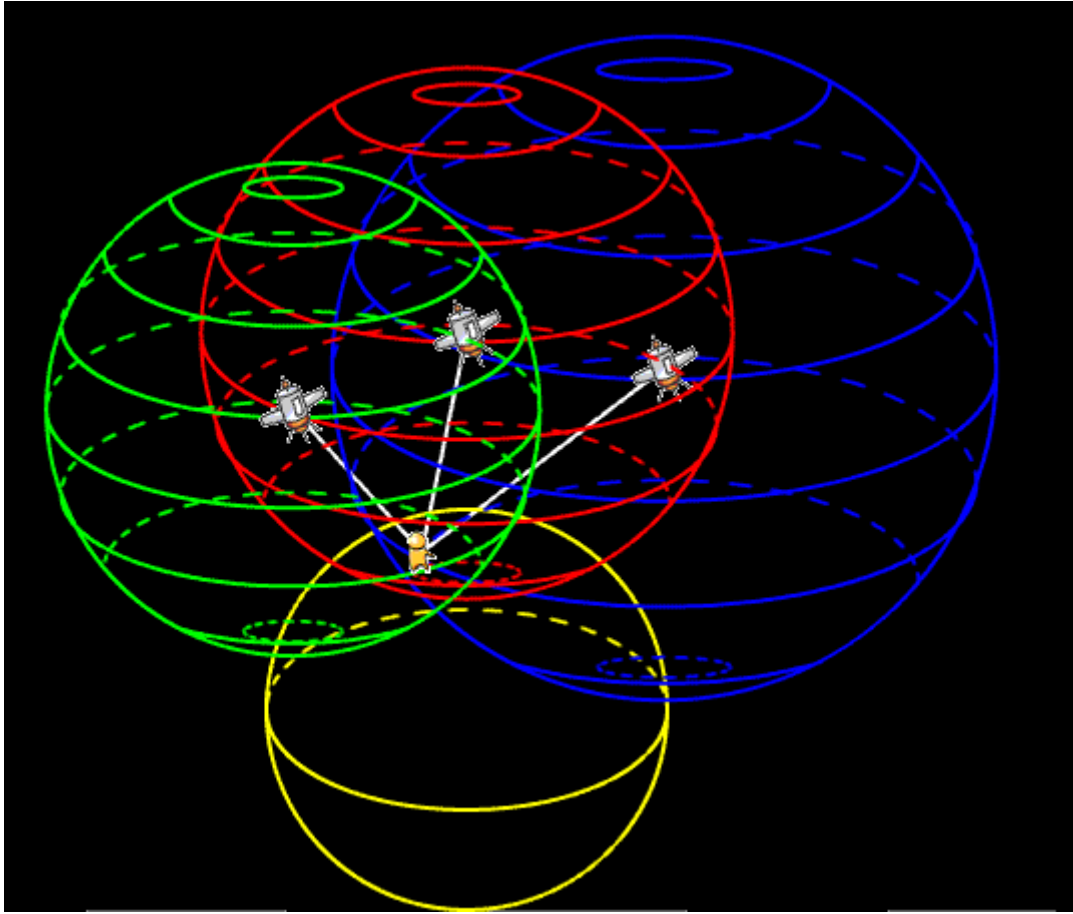


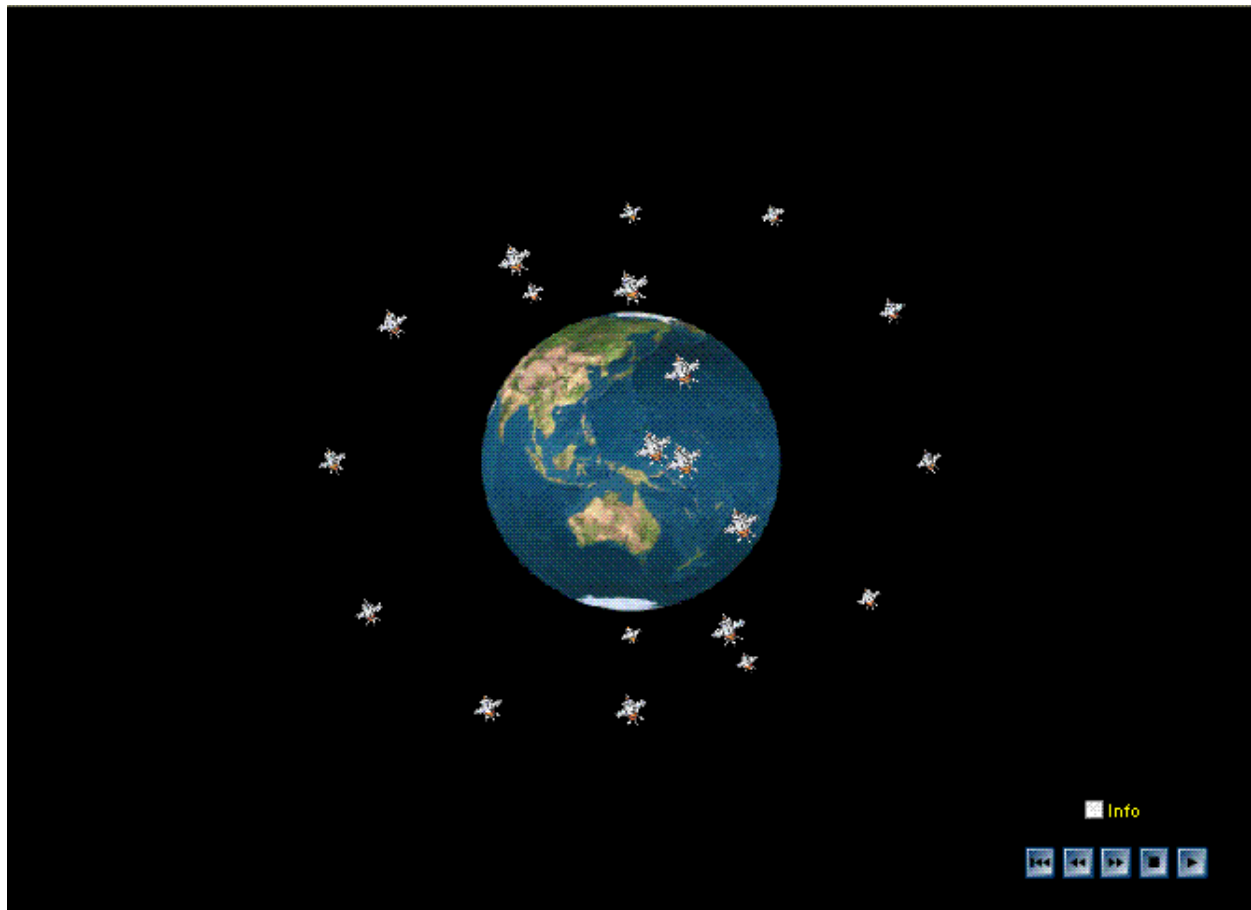
Figure 4-8

## 5 Result

In this chapter, we will test our simulation models we designed.

### 5.1 The Result of the GPS overview model

The GPS overview model contains 31 objects, 6 controlling objects and 25 movable objects, as shown in *Figure 5-1*. The 25 movable objects are stationary when the model is initiated. When we click the “RUN” button in the right and bottom corner, the earth and the 24 GPS satellites begin their own rotations. Every 4 satellites in one orbit plane, and there are total 6 orbits. The other 4 buttons are used to control the rotation of the earth and the 24 GPS satellites. Any time you click the “RESET” button, the earth and the satellites will return to the original places. Other three buttons “STOP”, “NEXT” and “PREVIOUS” are used to stop the animation to check the detail about where the satellites are, and what the next or previous positions they are.



*Figure 5-1*

We have the check box “info” to control the model to show the information in this GPS overview model. *Figure 5-2* shows the effect when we tick the check box.

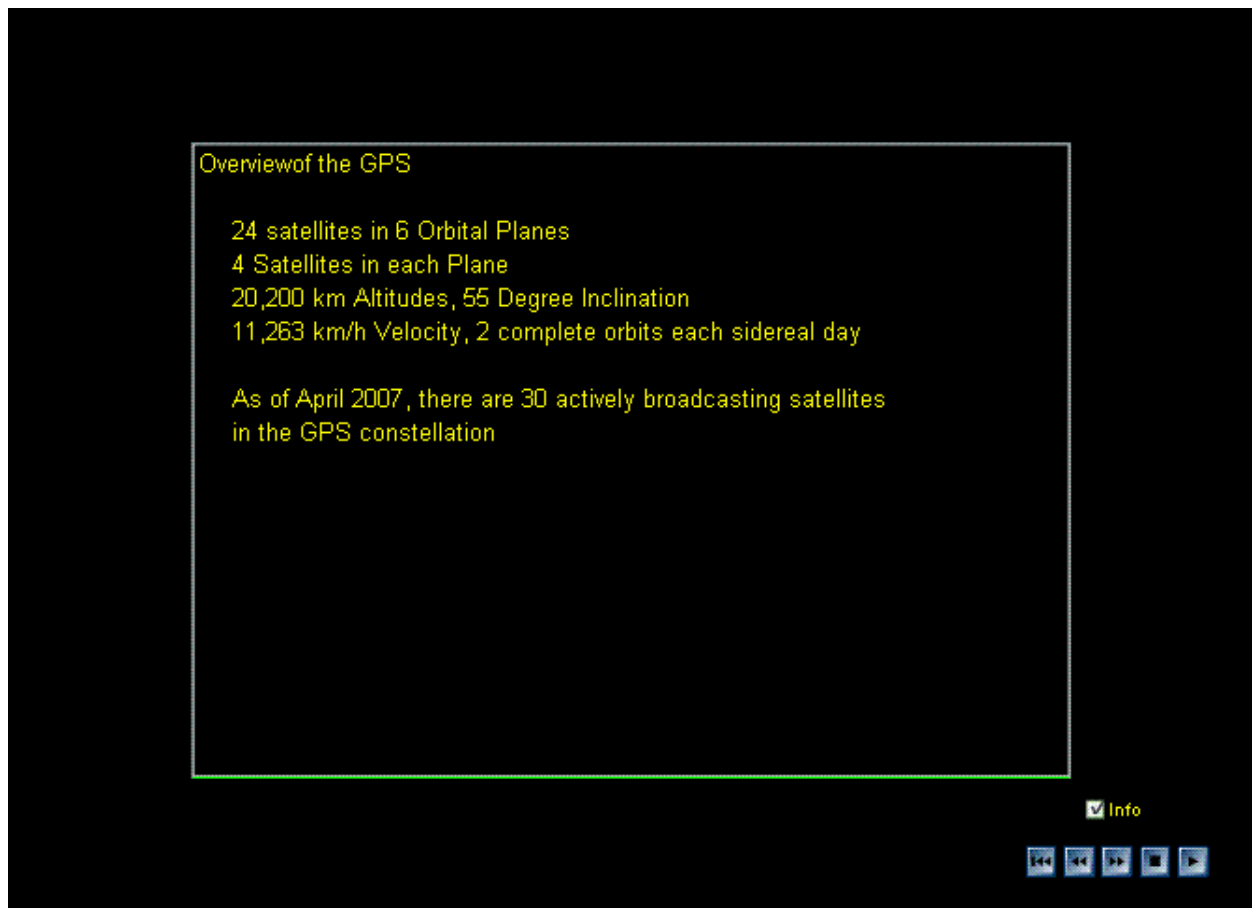


Figure 5-2

## 5.2 The Results of the GPS simulation models

In the GPS simulation model with three satellites, we have the following objects:

- 1) Three GPS satellites, the earth, the GPS receiver and the control segment.
- 2) Five buttons and three check boxes to control the model.
- 3) 14 pairs of text fields and labels to record the values or information

When we start this model, the “help” check box can be ticked to see how to use the model to determine the receiver’s position, as shown in *Figure 5-3*. And the “info” check box also provides some information about how to determine the receiver’s position, as shown in *Figure 5-4*.



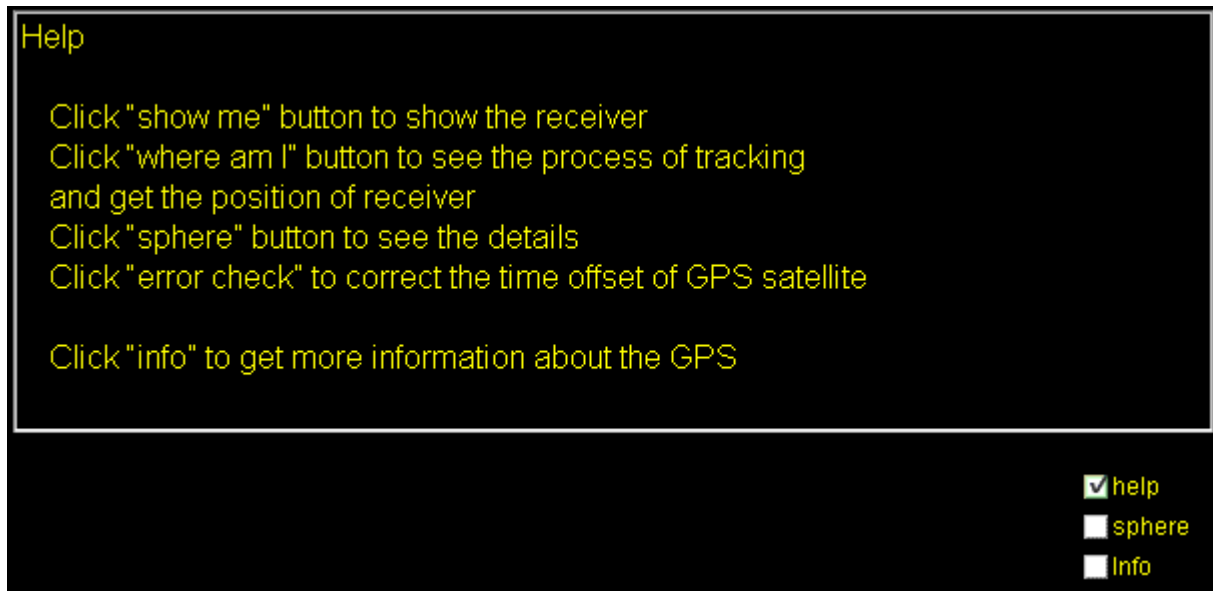


Figure 5-3

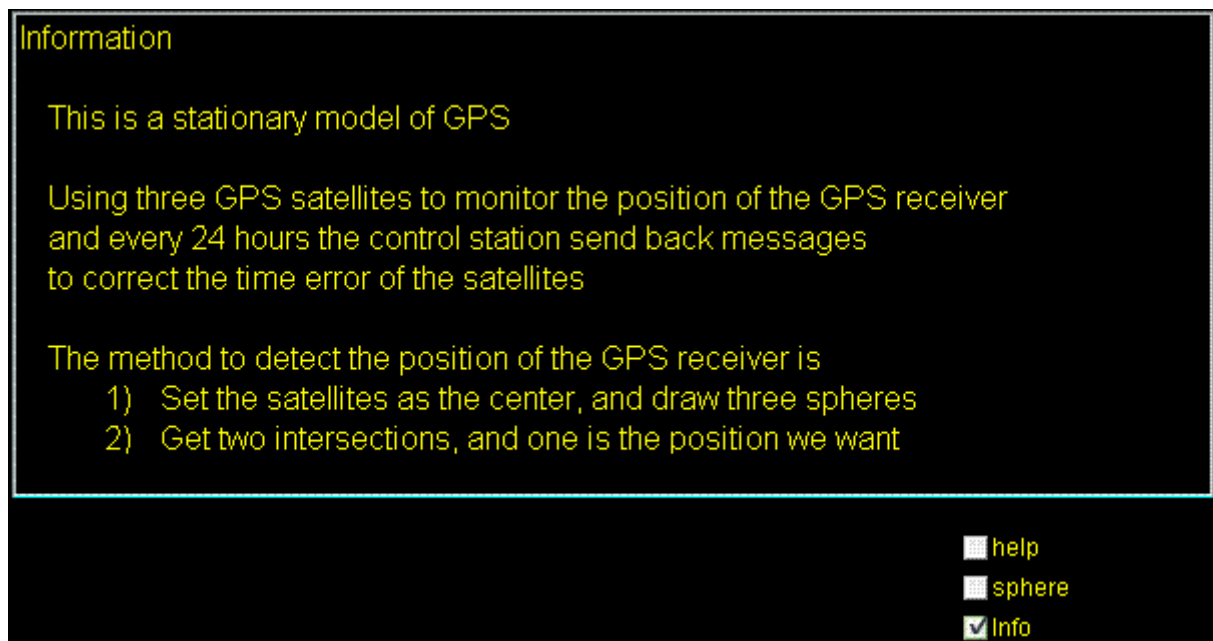


Figure 5-4

- 1) We click the button “show me”, the receiver would be initiated in some place on the surface of the earth. In our model, we only allow the receiver appears in the northern part of the earth because the satellites are near to the northern.
- 2) When we click the button “where I am”, the signals send from three satellites at the same time with the same speed. Because the distances from the satellites to the receiver are seldom the same, so every signal reaches the receiver in different time. Therefore, we get the time information depends on when the signals reach the receiver. Like Figure 5-5 depicts, all the signals send from the satellites at the same time, we just think all of them start at zero, when one of them reaches the receiver; the corresponding data are obtained in the text fields.

- 3) When the signals reach the receiver, and all the data for determining the receiver's position are collected, we can finally get the position of the receiver, as shown in Figure 5-6.
- 4) In our model, instead sending signals from control segment to satellites every 24 hours, we use a button to control this function. Once we click the "error check" button, signals will fend from the control segment to the satellites, as shown in *Figure 5-7*.

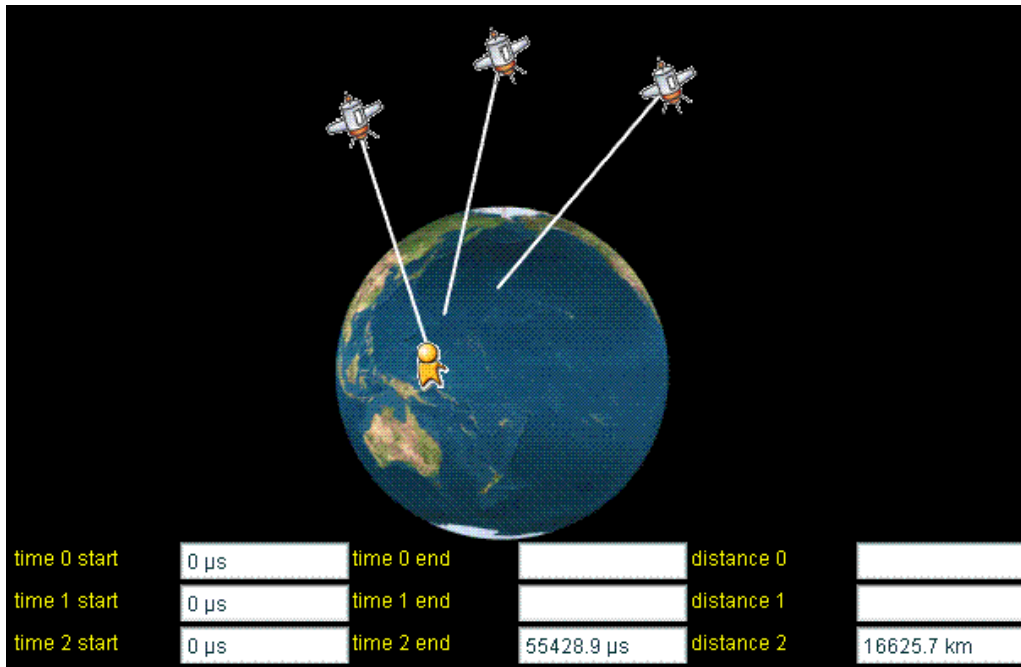


Figure 5-5

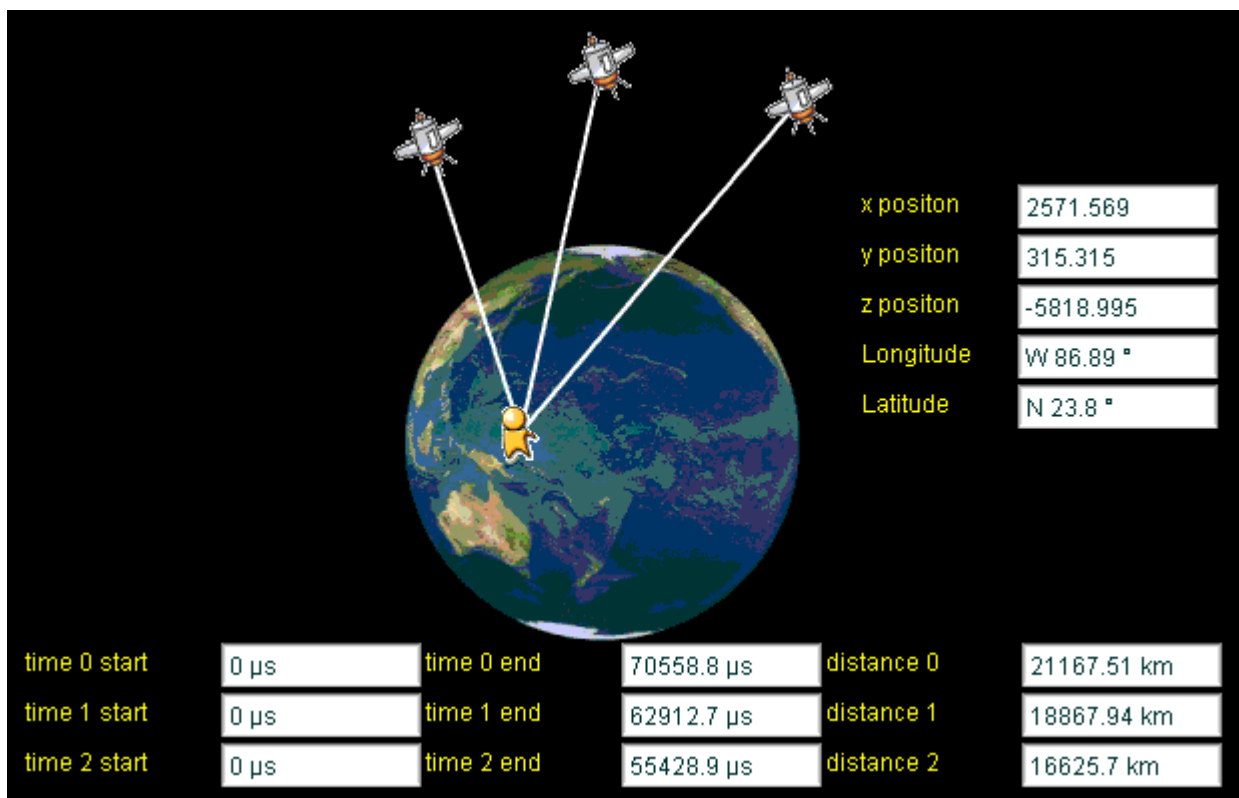


Figure 5-6

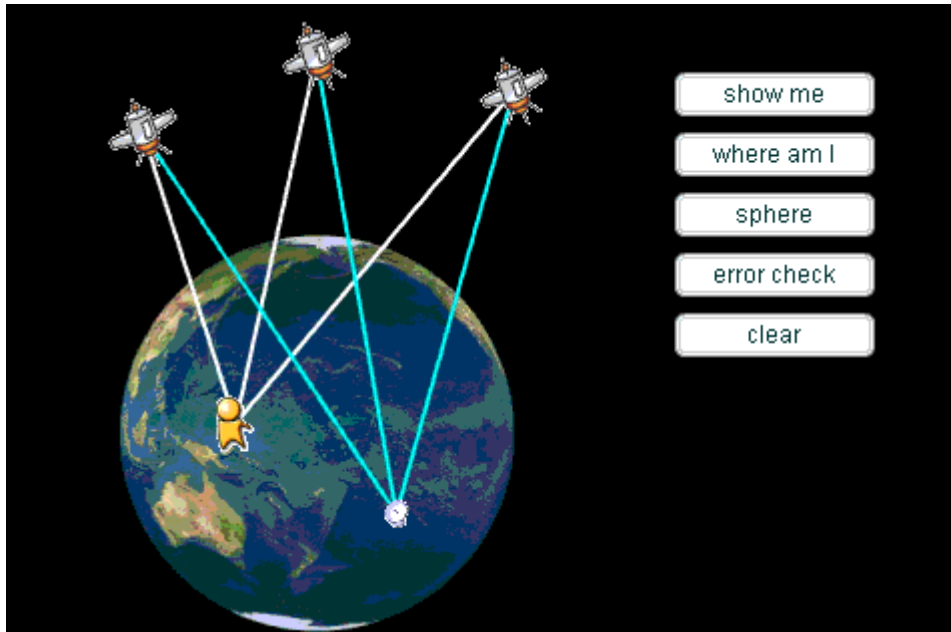


Figure 5-7

When the position of the receiver has been find, we can click the “sphere” button or tick the “sphere” check box to reveal the detail that how we use the Trilateration to get the receiver’s position, as shown in Figure 5-8.

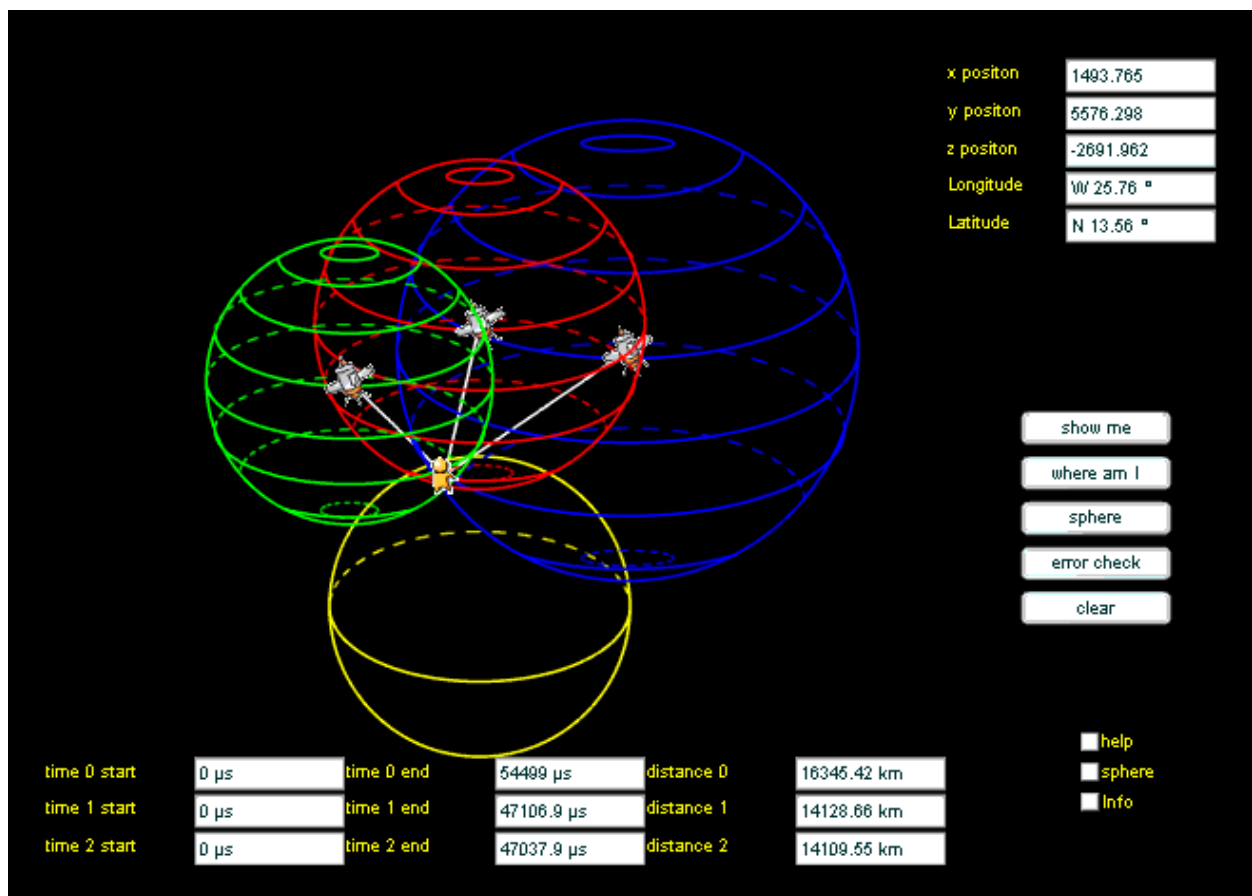
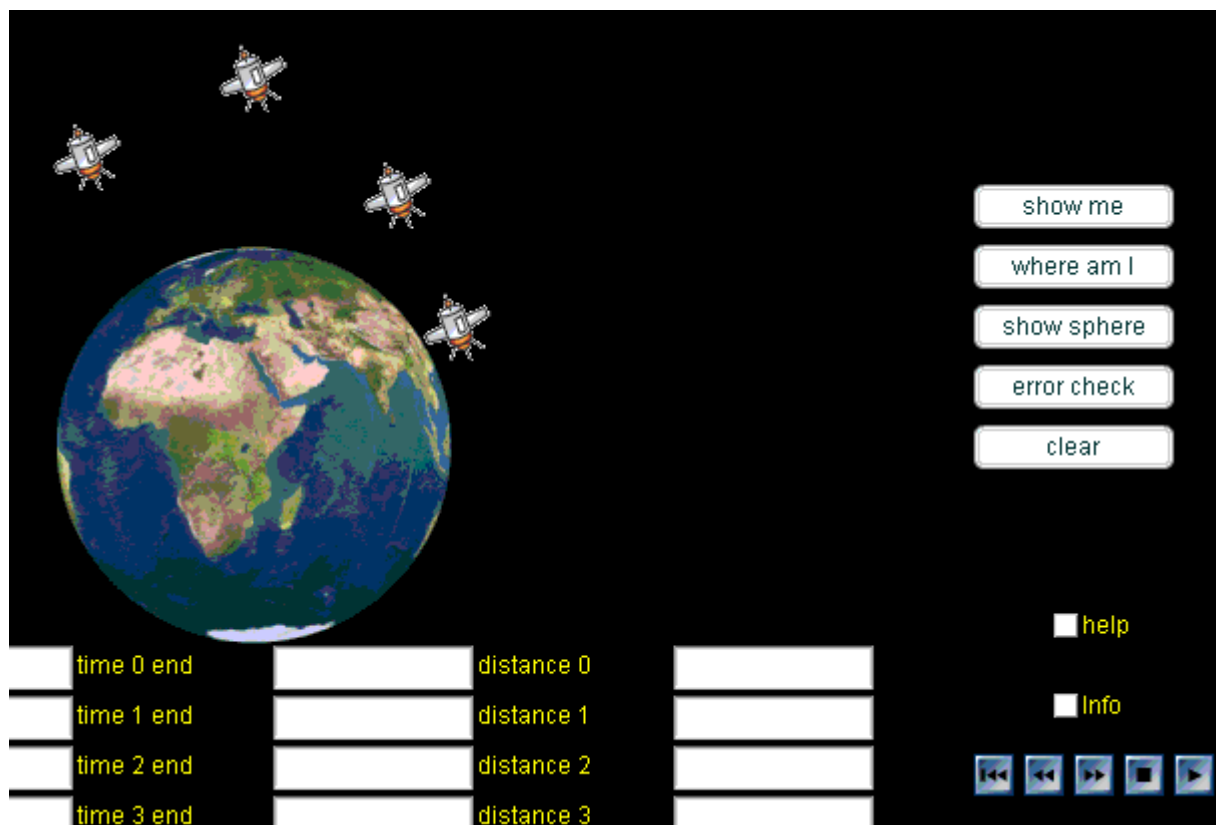


Figure 5-8

Now, we will see what the GPS simulation model with time offset functions. In this model, we have 4 satellites to determine the position of the receiver.

Most objects and features are the same with the model with three satellites, so we will not mention the same parts. Like how to initiate the receiver's position, how to send signals from the satellites to the receiver and so on.

In this model, we add the function from the GPS overview model. The four GPS satellites together with the earth can be controlled. When we click the "RUN" button, the satellites and the earth rotate, as shown in *Figure 5-9*. In this situation, we can not determination about the receiver. Only that you stop the rotations and click the "RESET" button to set the position to the original state.



*Figure 5-9*

Because the principles and concepts are almost the same, so this model is more or less similar with the model with three satellites.

- 1) Click "show me" button to initiate the receiver.
- 2) Click "where am I" button to see the process of tracking and determine the position of receiver.
- 3) Click "sphere" button to see the details about how Trilateration works in this model, as shown in *Figure 5-10*.
- 4) Click "error check" to correct the time offset of GPS satellite.

In chapter 4, we failed to find a solution for the set of equations (4-17), (4-18), (4-19) and (4-20). Therefore, we are not able to get the time error of the receiver. So, it impossible to compare the result we get by three satellites with the result we get in this model.

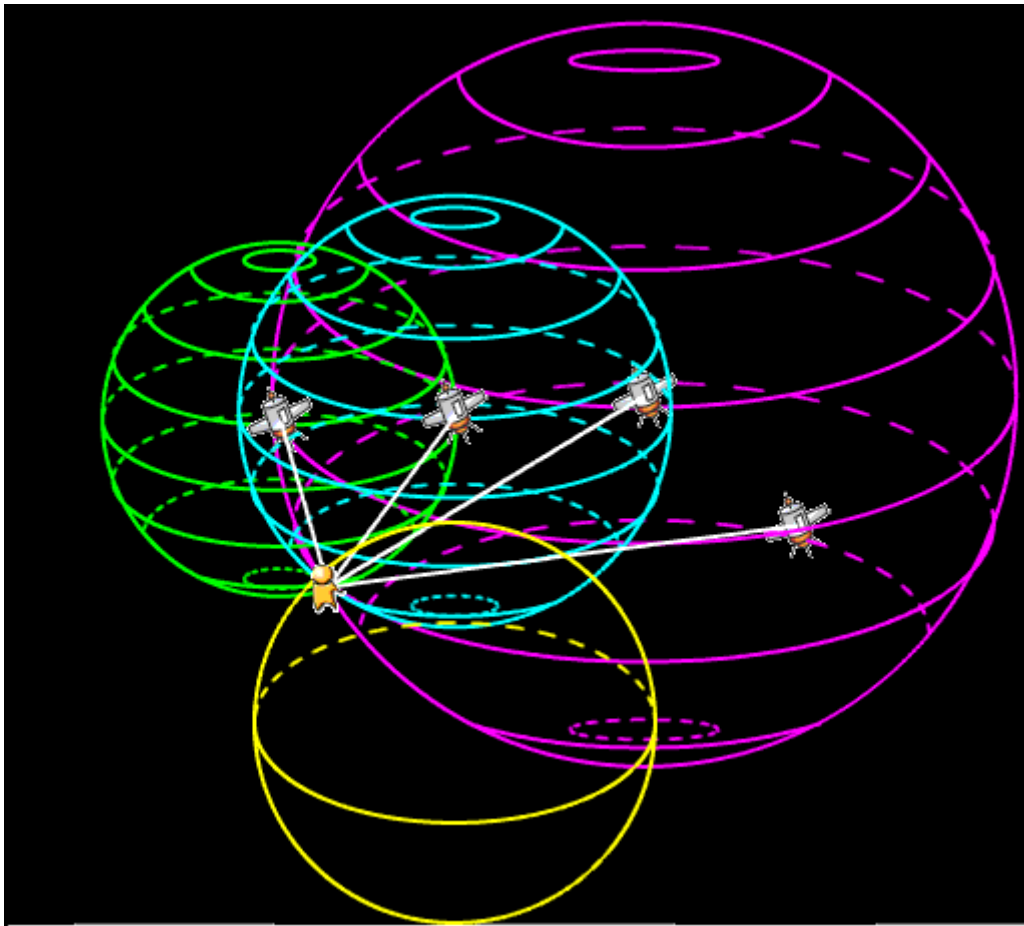


Figure 5-10

## 6 Discussion

This chapter we are going to discuss about the simulation models we designed, the problem we meet, and the experience in this project.

### 6.1 What we have done

#### 6.1.1 GPS overview model

In the GPS overview model, the most significant part is to simulate the rotations of the earth and the satellites. In general, we think the model we designed has these important features we expect.

- 1) The earth and the satellites rotate properly. When the satellites rotate to the back of the earth, they are hidden by the earth, when they rotate from far to close, the sizes increase smoothly.
- 2) There are 6 orbits, and each of them contains 4 GPS satellites.
- 3) It's possible to use buttons to control the rotations of the earth and the satellites.
- 4) A short explanation about the overview of the GPS can help the animation to be more understandable.

However, there are some drawbacks.

- 1) Although the GPS satellites rotate around the earth, we only can control them to move or to stop. It would be fancy that the velocities of the rotations can be controlled. The rotation speeds of the earth and the satellites can change in a certain range.
- 2) Because the rotation of the earth is simulated by consist a set of serial pictures together, we can not to make the rotation to slow. If the rotation of the earth is too slow, then the rotation will seem not continuous.
- 3) The tracks of the satellites on a certain orbit in the animation are in a line, so if we want to show the orbits of the satellites, we will only have the lines not ovals which can represent the orbit planes better.

In *Figure 6-1*, satellites are initiated in their own orbits. Some of them are behind the earth, so they are not visible. The sizes are different because the depths ( $z$  coordinate) are different. The five buttons only use to control the movements of the satellites and the earth.

In the GPS simulation models, originally, we planned to design three simple GPS models to illustrate the simple idea that how does the GPS system works. These three simple models are: three GPS satellites determination model, four GPS satellites determination model and relativity in GPS model. But we only designed two models without the relativity in GPS model.

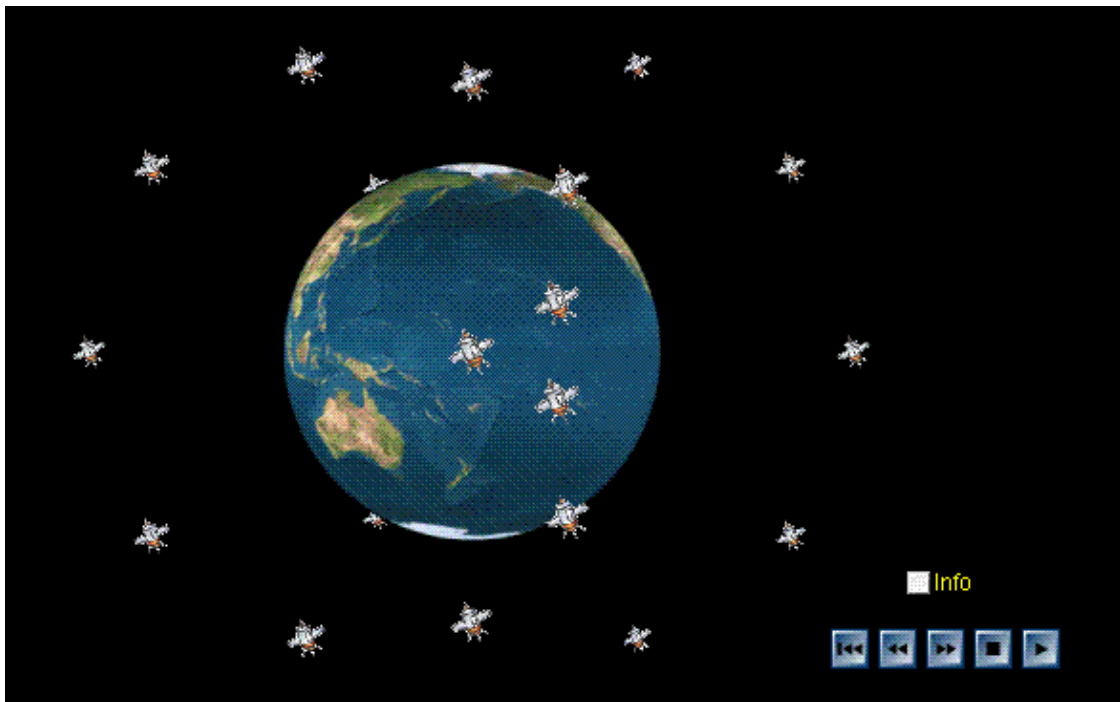


Figure 6-1

### 6.1.2 Three GPS satellites determination model

This model contains the principles and concepts of how the GPS system works. The aim that we use three satellites to determine the position of the receiver is going to demonstrate the fundamental theories of how to find the receiver in an efficient way. Like described in Figure 6-2:

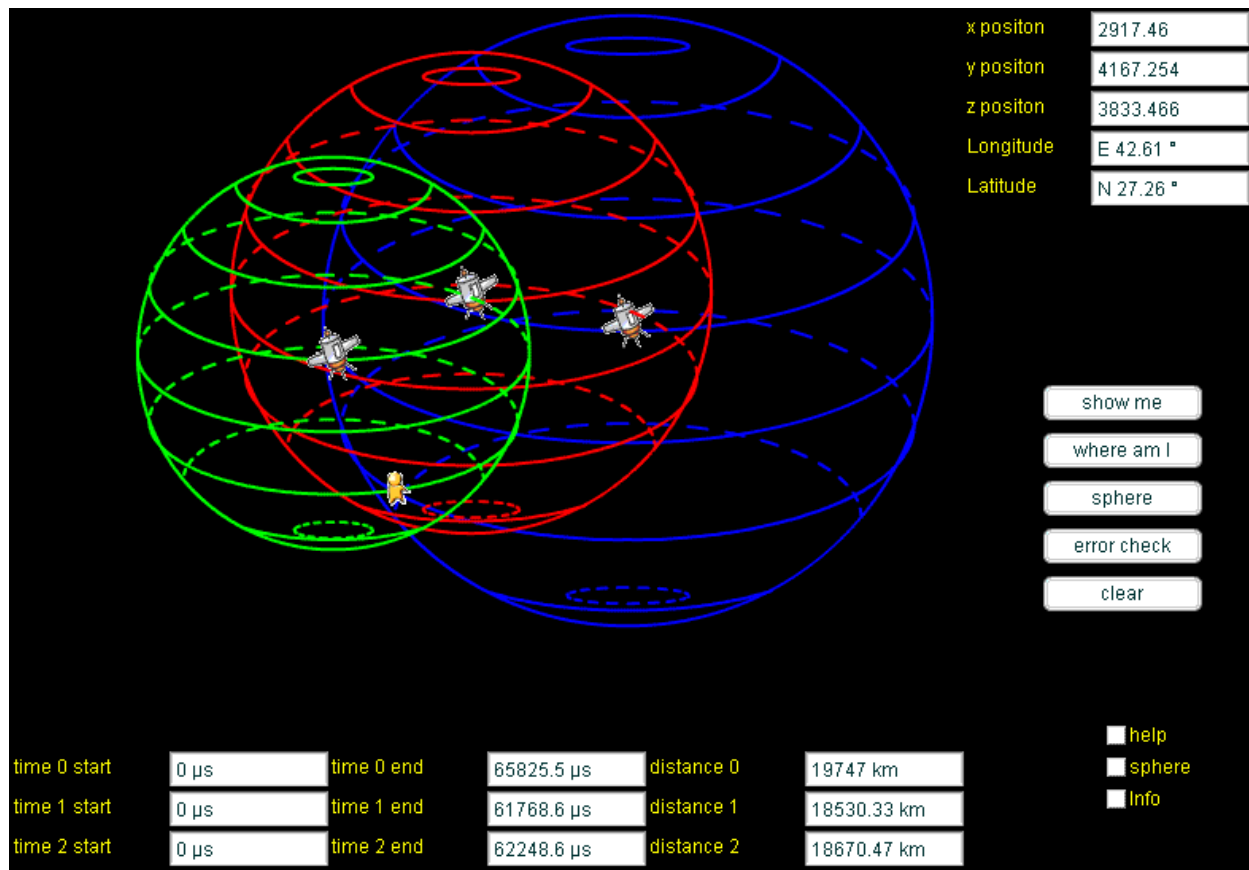


Figure 6-2

- 1) We can determine the position of the receiver when the signals reach the receiver. By calculating the information obtained in the text fields and the position of the satellites, we can get the position both in three-dimensional coordinate and the latitude and longitude. Here we do not mention the elevation. The reason is we initiate the receiver on the surface of the earth. Therefore, every time we determine the position of the receiver, the elevation of the receiver is zero.
- 2) When we want to reveal how we determine the receiver's position, we use three dimensions Trilateration to find the receiver. In *Figure 6-2*, we can see the three spheres have some overlapping, and we can find the receiver is in that overlapping area.

However, there are weaknesses in this model.

First, every time that we start to send signals from the satellites to the receiver, the start time of each GPS satellite is zero.

Second, when the signals reach the receiver, the corresponding data are shown in the text fields. It's truly the calculation of the receiver's position will not be influenced, but it's a bit difficult to distinguish from each satellite. *Figure 6-3* will easy to understand if we put corresponding data to each satellite.

Third, we initiate the receiver in the northern or the earth, the reason is we can easy to get the integrate spheres. Therefore, the results we determined are always in the northern semi-sphere. Moreover, in the animation we simulate the sending signals from satellites to receiver. We actually get the three-dimensional position of the receiver. and the calculation of the receiver's position is just like to check the determination result and the three-dimensional position we have already got are the same.



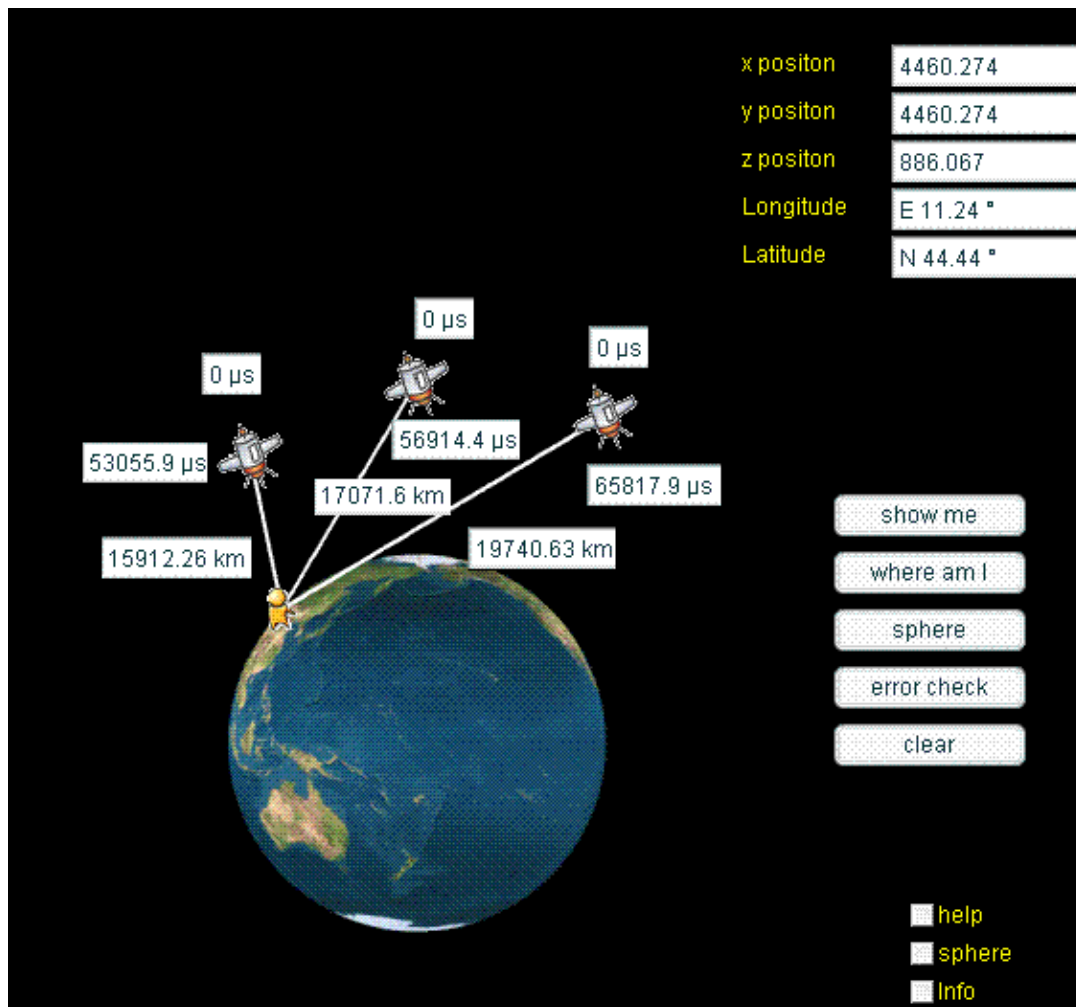


Figure 6-3

### 6.1.3 Four GPS satellites determination model

We introduce four GPS satellites to determine the position of the receiver in this model. The reason is because we want to have a more precise result than using three satellites. As we discussed in chapter 3, the time on the earth is slower than the time on the GPS satellite. If we want to get the receiver's position accurate, we need to consider the time error on the receiver (time offset). The principles are what we described in chapter 3. We can use the set of equations (from (3-10) to (3-13)) to get the time offset together with the position of the receiver.

However, when we implement the model we designed, we met the problem to solve a complex set of equations. And the new features in the model are barely that we add one more GPS satellite to help with getting the time offset and we can control the movements of the earth and the satellite.

So in the *Figure 6-4*, the calculation result does not consider the time offset in the receiver. the time offset is not calculated in this model.

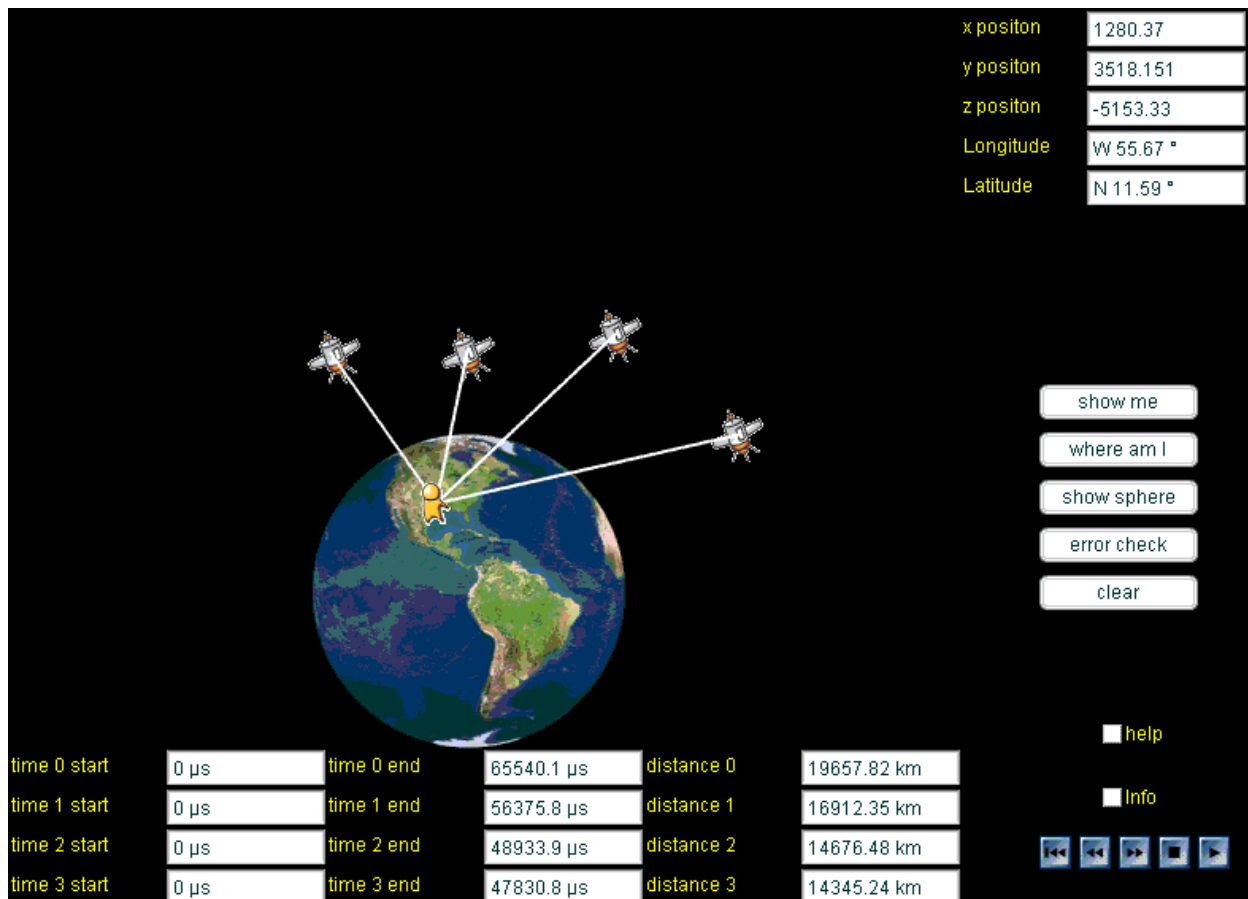


Figure 6-4

In simulating the determination of the receiver, we successfully get part of the results by using three-dimensional Trilateration. Nevertheless, we failed to get the result by using four satellites to determine the position of the receiver and also we have not implemented the relativistic time in our model. The reason is because we experienced a long time to make a vivid rotated earth and also to find a solution about the linear algebra.

## 6.2 Future work

Regard to the future work, the idea is that: On the one hand, we should improve the functionality and the integrality. On the other hand, it will be great to add some decorations to make the models more reality.

But first, in current, we only finished the GPS overview model, the three satellites determination model and part of the four satellites determination model. The relativity of GPS model we don't have time to do it. Thus, it should be urgent to finish these two tasks.

Second, in order to improve the functionality and the integrality, we would focus on the following viewpoints:

- 1) Combine the three simulation models to be one single model, and this single model should contain all the functions in each original model.
- 2) To make the models more friendly. In our designed model, we give help information to users to understand to simulation. And we hope that we could improve the model with more friendly functions.

- 3) If the GPS overview model and the simulation model can be merge to one big functional model, then it would be more fancy.

Finally, we hope that we could so some decoration to our simulation models. This will make the animation more charming and attractive.

## 7 Conclusion

Regard to this project, we simulate the construction of the GPS, and how the GPS satellites determine the receiver. The model simply implements the functions of GPS, but it does not cover the entire requirement we described. And as the reason that we failed in finishing the simulation model, we also meet the problem to test the model among the college student. However, regard to the description, we basically achieve the requirement.

## Appendices

### Glossary & Abbreviations

<b>CS</b>	<b>control segment</b>
<b>HR-diagram</b>	<b>Hertzprung-Russell diagram</b>
<b>GPS</b>	<b>The Global Positioning System</b>
<b>SS</b>	<b>space segment</b>
<b>SV</b>	<b>Space Vehicles</b>
<b>TAI</b>	<b>International Atomic Time</b>
<b>US</b>	<b>user segment</b>
<b>UT1</b>	<b>Universal Time 1</b>
<b>UTC</b>	<b>Coordinated Universal Time</b>

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