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Research Article

H_∞ Control for Networked Control Systems with Time Delays and Packet Dropouts

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This paper is concerned with the H_∞ control issue for a class of networked control systems (NCSs) with packet dropouts and time-varying delays. Firstly, the addressed NCS is modeled as a Markovian discrete-time switched system with two subsystems; by using the average dwell time method, a sufficient condition is obtained for the mean square exponential stability of the closed-loop NCS with a desired H_∞ disturbance attenuation level. Then, the desired H_∞ controller is obtained by solving a set of linear matrix inequalities (LMIs). Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

Networked control systems (NCSs) are distributed systems in which communication between sensors, actuators, and controllers is supported by a shared real-time network. Compared with conventional point-to-point system connection, this new network-based control scheme reduces system wiring and has low cost, high reliability, information sharing, and remote control [1, 2]. Nevertheless, the introduction of communication networks also brings some new problems and challenges, such as time-delay, packet dropout, quantization, and band-limited channel [3–8], which all might be potential sources of poor performance, even of instability.

Random delay and packet dropout in NCS are two major causes for the deterioration of system stability; various approaches have been developed for the NCS with random communication delays and packet dropout in [9–17]. The time delay occurs in various physical, industrial, and engineering systems and is a source of poor performance and instability of systems. In [9, 10], the uncertainties of the delays are transformed into those of the system models with uncertain parameters. The delay is limited to take finite values during a sampling period, and the NCS is ultimately modeled as a discrete-time switched system with a finite number of subsystems [11, 12]. In [13–15], the delay is assumed to be random and follows some specific distribution laws, which may not be exactly known prior in practice. And in some literature the delay is separated into a nominal part and an uncertain part; in this way, the NCS is represented as an uncertain system with norm-bounded uncertainties or polytopic uncertainties. Another important issue in NCS control problem is packet dropout; most of the NCS models are presented by using the Bernoulli random binary distributed sequence methods or the Markov chain. For NCSs, let the binary-valued function denote the data transmission status from sensor to controller and controller to actuator, respectively, where 1 means successful packet communication and 0 is the case of packet dropout [16]. Reference [17] proposes an iterative method to model NCSs with bounded packet dropout as MJLSs with partly unknown transition probabilities.

On the other hand, in view of abrupt variation in the structures, such as component failures, sudden environmental disturbance, and abrupt variations of the operating points of NCSs, it is more appropriate to model such class of systems as a special class of stochastic hybrid systems with finite operation modes. And packet dropout (time-delay) of the next sampling moment may have a close relation to the previous moment, so it is reasonable to model NCS as the Markov switched system. The mean square stabilization of a class of Markovian NCS is studied in [18], and the average dwell time (ADT) approach is applied to investigate the stability of the NCS in [19]. However, to the best of our knowledge, the problems of mean square exponential stability and control for the NCS have not been fully investigated to date. This motivates the present study.

With the motivation of the above reasons, we consider the mean square exponential H_∞ performance for NCS with random delay and packet dropout. The main contribution can be summarized as follows: (i) an NCS model with random delay and packet dropout is proposed firstly; the packet dropout process is modeled as a finite state Markov chain and the resulting closed-loop system is a Markovian switching system; (ii) the parameter-dependent Lyapunov function is applied for stability analysis and control synthesis, and sufficient conditions for the robustly mean square exponential stability of the closed-loop system are given by using the ADT method [20], where the convergence of the Markov chain is utilized; and (iii) a state feedback controller is designed by using a cone complementary linearization approach to ensure that the closed-loop system is mean square exponentially stable and achieves the disturbance attenuation level.

The paper is organized as follows. In Section 2, the NCS with packet dropouts and time-varying delays is modeled as a class of the Markovian discrete-time switched system with two subsystems. The mean square exponential stability of the closed-loop NCS with a desired H_∞ disturbance attenuation level is developed in Section 3 and the desired H_∞ controller is formulated in a set of LMIs. A numerical example is provided in Section 4. Finally, Section 5 concludes this paper.

Notation 1. Throughout the paper, the superscript "−1" and "T" stand for the inverse and transpose of a matrix, respectively; R^n denotes the n-dimensional Euclidean space and the notation $P > 0$ means that P is a real symmetric positive definite matrix. $E\{x\}$ is the expectation of the stochastic variable x. I and 0 represent identity matrix and zero matrix with appropriate dimensions in different places. In symmetric block matrices or complex matrix expressions, we use an asterisk $*$ to represent a term that is induced by symmetry and diag$\{\cdots\}$ stands for a block diagonal matrix. $\|\cdot\|$ refers to the Euclidean norm for vectors and induced 2-norm for matrix. $L_2[k_0,\infty)$ stands for the space of square integrable functions on $[k_0,\infty)$.

2. Model of Networked Control System

Consider the following system:

$$\begin{aligned} \dot{x}(t) &= A_p x(t) + B_p u(t) + f(x,t) + H_p w(t), \\ z(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$, $u(t) \in R^m$, and $z(t) \in R^P$ are the state vector, control input vector, and controlled output vector, respectively, and $w(t) \in R^d$ is the exogenous disturbance signal belonging to $L_2[0,\infty)$. A_p, B_p, H_p, and C are known real matrices with appropriate dimensions. $f:\Omega\times[t_0,\infty) \to R^n(\Omega \subset R^n)$ is the nonlinear function vector, and $f(0,t_0)=0$. f satisfies the local Lipschitz condition, that is,

$$\begin{gathered} \|f(x_1,t) - f(x_2,t)\|_2 \le \alpha\|x_1 - x_2\|_2, \\ \forall x_1, x_2 \in \Omega \subset R^n,\ \forall t \in [t_0,\infty), \end{gathered} \tag{2}$$

where $\alpha > 0$ is a known constant.

In the considered NCS, time delays exist in both channels from sensor to controller and from controller to actuator. Sensor-to-controller delay and controller-to-actuator delay are denoted by τ^{sc} and τ^{ca}, respectively. The assumptions in the above NCS are as follows:

(1) the discrete-time state-feedback controller and the actuator are event driven; the sensor is time-driven with sampling period T,

(2) the network-induced delay $\tau_k \triangleq \tau_k^{sc} + \tau_k^{ca}$ satisfies $0 < \tau_{\min} \le \tau_k \le \tau_{\max} < T$,

(3) the zero-order hold device does not update the output value until the new value arrives.

The output value of the discrete-time state-feedback controller corresponding to $x(k)$ is denoted by

$$u(k) = Kx(k). \tag{3}$$

Consider the plant input:

$$u(k) = \begin{cases} \hat{u}(k) & \text{if } \hat{u}(k) \text{ and } x(k) \text{ is successfully transmitted,} \\ u(k-1) & \text{if } \hat{u}(k) \text{ or } x(k) \text{ is lost during transmission.} \end{cases} \tag{4}$$

Discretizing system (1) in one period, we can obtain the discrete state equation of the NCS:

$$\begin{aligned} x(k+1) = {} & Ax(k) + B_0(\tau_k)u(k) \\ & + B_1(\tau_k)u(k-1) + \tilde{f}(x,k) + Hw(k), \end{aligned} \tag{5}$$

where $A = e^{A_pT}$, $B_0(\tau_k) = \int_0^{T-\tau_k} e^{A_ps}ds B_p$, $B_1(\tau_k) = \int_{T-\tau_k}^{T} e^{A_ps}ds B_p$, $H = \int_0^T e^{A_ps}ds H_p$, and $\tilde{f}(x,k) = \int_0^T e^{A_ps}ds f(x,k)$.

By using the Jordan form of the matrix A_p, $B_0(\tau_k)$ is rewritten as [21]

$$B_0(\tau_k) = F_0 + \sum_{i=1}^{\nu} \eta_i(\tau_k) F_i \tag{6}$$

with $\nu \le n$, where F_0 and F_i are constant matrices, $\eta_i(\tau_k) = e^{a(T-\tau_k)}\cos(b(T-\tau_k))$ and the eigenvalue of A is $\lambda = a + ib$.

Then, $\{B_0(\tau_k) \mid k \in N\}$ is a subset of co(F) with

$$\begin{aligned} F &= \left\{F_0 + \sum_{i=1}^{\nu} \tilde{\eta}_i F_i \mid \tilde{\eta}_i = \overline{\eta}_i,\ \underline{\eta}_i,\ i = 1,2,\ldots,\nu\right\} \\ &= \left\{\tilde{F}_i \mid i = 1,2,\ldots,2^{\nu}\right\}, \end{aligned} \tag{7}$$

where $\overline{\eta}_i = \max \eta_i(\tau_k)$, $\underline{\eta}_i = \min \eta_i(\tau_k)$, F is the set of vertices, and $\text{co}(\cdot)$ denotes the convex hull. Thus we obtain

$$B_0(\tau_k) = \sum_{i=1}^{2^v} \xi_i(k) \widetilde{F}_i, \tag{8}$$

with $\sum_{i=1}^{2^v} \xi_i(k) = 1$, $\xi_i(k) \in [0, 1]$.

Defining an augmented vector $\overline{x}(k) = [x^T(k)\ u^T(k-1)]^T$, during each sampling period, two cases may arise, which can be listed as follows.

S_1: no packet dropout happens; (5) can be written as

$$\begin{aligned} \overline{x}(k+1) &= \overline{A}_1(k)\overline{x}(k) + \overline{f}(x,k) + \overline{H}w(k), \\ z(k) &= \overline{C}\overline{x}(k), \end{aligned} \tag{9}$$

where

$$\begin{aligned} \overline{A}_1(k) &= \begin{bmatrix} A + B_0(\tau_k)K & B_1(\tau_k) \\ K & 0 \end{bmatrix} \\ &= \begin{bmatrix} A & B - B_0(\tau_k) \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} B_0(\tau_k) \\ I \end{bmatrix} [K\ \ 0], \\ B &= \int_0^T e^{A_p s} ds B_p, \qquad B_1(\tau_k) = B - B_0(\tau_k), \\ \overline{f}(x,k) &= \begin{bmatrix} \widetilde{f}(x,k) \\ 0 \end{bmatrix}, \qquad \overline{H} = \begin{bmatrix} H \\ 0 \end{bmatrix}, \qquad \overline{C} = [C\ \ 0]. \end{aligned} \tag{10}$$

Substituting (8) into (9) gives rise to

$$\overline{A}_1(k) = \sum_{i=1}^{2^v} \xi_i(k) A_i, \tag{11}$$

where

$$\begin{aligned} A_i &= \widetilde{A}_i + \widetilde{B}_i [K\ \ 0] = \widetilde{A}_i + \widetilde{B}_i \overline{K}, \\ \widetilde{A}_i &= \begin{bmatrix} A & B - \widetilde{F}_i \\ 0 & 0 \end{bmatrix}, \qquad \widetilde{B}_i = \begin{bmatrix} \widetilde{F}_i \\ I \end{bmatrix}, \qquad \overline{K} = [K\ \ 0]. \end{aligned} \tag{12}$$

S_2: packet dropout happens; (5) can be written as

$$\begin{aligned} \overline{x}(k+1) &= \overline{A}_2(k)\overline{x}(k) + \overline{f}(x,k) + \overline{H}w(k), \\ z(k) &= \overline{C}\overline{x}(k), \end{aligned} \tag{13}$$

where $\overline{A}_2 = \left[\begin{smallmatrix} A & B \\ 0 & I \end{smallmatrix}\right]$.

From (2)–(9), the nonlinear uncertainty $\overline{f}(x,k)$ satisfies

$$\overline{f}^T(x,k)\overline{f}(x,k) = \widetilde{f}^T(x,k)\widetilde{f}(x,k) \le \overline{x}^T(k) U^T U \overline{x}(k), \tag{14}$$

where U is a known constant positive-definite matrix.

By the above analysis and assumptions, we can see that networked control system can be described by the following switched system with two modes:

$$\begin{aligned} \overline{x}(k+1) &= \overline{A}_{\sigma(k)}(k)\overline{x}(k) + \overline{f}(x,k) + \overline{H}w(k), \\ z(k) &= \overline{C}\overline{x}(k), \end{aligned} \tag{15}$$

where $\sigma(k)$ is called a switching signal. $\sigma(k) = 1$ represents no packet dropout, while $\sigma(k) = 2$ implies packet dropout. The switching characteristics between the two modes are often assumed as the Markov chain, and π_{rl} is transition probability from mode r to l, $r, l = 1, 2$; therefore, $\sigma(k)$ of the Markov chain has ergodicity and satisfied the following condition:

$$\lim_{n\to\infty} \pi_{rl}^{(n)} = \pi_l, \quad r, l = 1, 2, \tag{16}$$

where π_l is the limitation of state l. So $\{\pi_1, \pi_2\}$ is the stationary distribution of the Markov chain.

For an arbitrary switching sequence $\sigma(k)$ and any given integer $k > 0$, let k_0 imply the initial time, and $k_0 < k_1 < k_2 < \cdots k_q < \cdots < k$, $q \ge 1$ represent the switching instants. Denote $T^1[k_0, k)$ as the all sequence of the time period in which subsystem 1 is active during the time interval $[k_0, k)$. Similarly, $T^2[k_0, k)$ represents the all period sequence that subsystem 2 is active during the time interval $[k_0, k)$.

Lemma 1 (Schur complement [22]). *For a given matrix* $S = \left[\begin{smallmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{smallmatrix}\right]$, *where* S_{11}, S_{22} *are square matrices, the following conditions are equivalent:*

(1) $S < 0$;

(2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2 (see [23]). *The stochastic stability in discrete time implies the stochastic stability in continuous time.*

Definition 3 (see [24]). The closed-loop system (15) is mean square exponentially stable with $w(k) = 0$, if there exists $\delta > 0$, $0 < \beta < 1$, such that

$$E\left\{\|\overline{x}(k)\|^2\right\} < \delta\beta^{k-k_0} E\left\{\|\overline{x}(k_0)\|^2\right\} \tag{17}$$

for all initial condition $(\overline{x}(k_0), \sigma(k_0))$.

Definition 4 (see [25]). For any $k > k_0 \ge 0$, let $N_\sigma[k_0, k)$ denote the total number of the switching of $\sigma(k)$ during the interval $[k_0, k)$. If

$$N_\sigma[k_0, k) \le N_0 + \frac{k - k_0}{T_a} \tag{18}$$

holds for a given $N_0 \ge 0$, $T_a > 0$, then the constant T_a is called the average dwell time and N_0 is the chatter bound. For simplicity, we choose $N_0 = 0$ without loss of generality.

Definition 5 (see [20]). Given scalars $\gamma > 0$ and $0 < \lambda < 1$, the closed-loop system (15) is robustly exponentially stable with an exponential H_∞ performance γ if the following conditions are satisfied:

(a) the closed-loop system (15) with $\omega(k) \equiv 0$ is exponentially stable;

(b) under the zero-initial condition, it holds that

$$\sum_{k=k_0}^{\infty} E\left\{\lambda^k z^T(k) z(k)\right\} < \gamma^2 \sum_{k=k_0}^{\infty} E\left\{w^T(k) w(k)\right\}, \quad \forall w(k) \in L_2[k_0, \infty). \tag{19}$$

3. Main Results

The following theorems present a sufficient condition for the mean square stability of the considered system and the H_∞ controller design method.

3.1. Stability Analysis. In this subsection, sufficient conditions for the existence of mean square exponential stability of system (15) with $\omega(k) \equiv 0$ are given in the following theorem.

Theorem 6. *System* (15) *is mean square exponentially stable with a decay rate* λ^ρ, *if there exist positive definite matrices* P_i, Q, *scalars* $\mu \ge 1$, λ_1, *and* λ_2, *such that*

$$\begin{bmatrix} A_i^T\left(\pi_{11}P_j+\pi_{12}Q\right)A_i - \lambda_1 P_i + U^T U & A_i^T\left(\pi_{11}P_j+\pi_{12}Q\right) \\ * & \left(\pi_{11}P_j+\pi_{12}Q\right) - I \end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} \overline{A_2}^T\left(\pi_{21}P_j+\pi_{22}Q\right)\overline{A_2} - \lambda_2 Q + U^T U & \overline{A_2}^T\left(\pi_{21}P_j+\pi_{22}Q\right) \\ * & \left(\pi_{21}P_j+\pi_{22}Q\right) - I \end{bmatrix} < 0 \quad i, j = 1, 2, 3, \ldots, 2^v, \tag{21}$$

$$\frac{1}{\mu} Q \le P_i \le \mu Q, \tag{22}$$

$$0 < \lambda < 1, \tag{23}$$

$$\max\left\{\pi_{12}, \pi_{21}\right\} < -\frac{\ln \lambda}{\ln \mu}, \tag{24}$$

where $\lambda = \lambda_1^{\pi_{21}/(\pi_{12}+\pi_{21})} \lambda_2^{\pi_{12}/(\pi_{12}+\pi_{21})}$, $\rho = 1 + \max\{\pi_{12}, \pi_{21}\} \cdot (\ln \mu / \ln \lambda)$.

Proof. For the system (15), define the following Lyapunov function:

$$V_{\sigma(k)}(\overline{x}(k), \xi(k)) = \overline{x}^T(k) \widetilde{P}_{\sigma(k)} \overline{x}(k), \tag{25}$$

where

$$\widetilde{P}_{\sigma(k)} = \sum_{i=1}^{2^v} \xi_i(k) P_i, \quad \text{for } \sigma(k) = 1, \qquad \widetilde{P}_{\sigma(k)} = Q, \quad \text{for } \sigma(k) = 2. \tag{26}$$

For subsystem 1, it follows from (15) that

$$\begin{aligned}
&\Delta V_1[\overline{x}(k+1)] \\
&= E\left[V_1(\overline{x}(k+1), \xi(k+1))\right] - \lambda_1 V_1(\overline{x}(k), \xi(k)) \\
&= \overline{x}^T(k+1)\left(\pi_{11}\sum_{j=1}^{2^v}\xi_j(k+1)P_j + \pi_{12}Q\right)\overline{x}(k+1) \\
&\quad - \lambda_1 \overline{x}^T(k)\left(\sum_{i=1}^{2^v}\xi_i(k)P_i\right)\overline{x}(k) \\
&\le \left(\sum_{i=1}^{2^v}\xi_i(k)A_i\overline{x}(k) + \overline{f}(x,k)\right)^T \\
&\quad \times \left(\pi_{11}\sum_{j=1}^{2^v}\xi_j(k+1)P_j + \pi_{12}Q\right) \\
&\quad \cdot \left(\sum_{i=1}^{2^v}\xi_i(k)A_i\overline{x}(k) + \overline{f}(x,k)\right) \\
&\quad - \lambda_1 \overline{x}^T(k)\left(\sum_{i=1}^{2^v}\xi_i(k)P_i\right)\overline{x}(k) \\
&\quad + \overline{x}^T(k)U^T U\overline{x}(k) - \overline{f}^T(x,k)\overline{f}(x,k) \\
&= \sum_{i=1}^{2^v}\xi_i(k)\sum_{j=1}^{2^v}\xi_j(k+1)\begin{bmatrix}\overline{x}(k) \\ \overline{f}(x,k)\end{bmatrix}^T \Theta \begin{bmatrix}\overline{x}(k) \\ \overline{f}(x,k)\end{bmatrix},
\end{aligned} \tag{27}$$

where

$$\Theta = \begin{bmatrix} A_i^T\left(\pi_{11}P_j + \pi_{12}Q\right)A_i - \lambda_1 P_i + U^T U & A_i^T\left(\pi_{11}P_j + \pi_{12}Q\right) \\ * & \left(\pi_{11}P_j + \pi_{12}Q\right) - I \end{bmatrix}. \tag{28}$$

From inequality (20), one obtains

$$E\left[V_1(\overline{x}(k+1), \xi(k+1))\right] < \lambda_1 V_1(\overline{x}(k), \xi(k)). \tag{29}$$

In the same way, for subsystem 2, we obtain

$$\begin{aligned}
&\Delta V_2[\overline{x}(k+1)] \\
&= E\left[V_2(\overline{x}(k+1), \xi(k+1))\right] - \lambda_2 V_2(\overline{x}(k), \xi(k)) \\
&= \overline{x}^T(k+1)\left(\pi_{21}\sum_{j=1}^{2^v}\xi_j(k+1)P_j + \pi_{22}Q\right)\overline{x}(k+1) \\
&\quad - \lambda_2 \overline{x}^T(k) Q \overline{x}(k)
\end{aligned}$$

$$\leq \left(\overline{A_2}\overline{x}(k) + \overline{f}(x,k)\right)^T \times \left(\pi_{21}\sum_{j=1}^{2^v}\xi_j(k+1)P_j + \pi_{22}Q\right) \cdot \left(\overline{A_2}\overline{x}(k) + \overline{f}(x,k)\right) - \lambda_2\overline{x}^T(k)Q\overline{x}(k) + \overline{x}^T(k)U^TU\overline{x}(k) - \overline{f}^T(x,k)\overline{f}(x,k); \quad (30)$$

then

$$E\left[V_2(\overline{x}(k+1),\xi(k+1))\right] < \lambda_2 V_2(\overline{x}(k),\xi(k)). \quad (31)$$

Considering the condition (22), we get that

$$\begin{aligned} V_1(\overline{x}(k),\xi(k)) &\leq \mu V_2(\overline{x}(k^-),\xi(k^-)), \\ V_2(\overline{x}(k),\xi(k)) &\leq \mu V_1(\overline{x}(k^-),\xi(k^-)). \end{aligned} \quad (32)$$

Then for $k_q < k < k_{q+1}$, we get

$$\begin{aligned} &E\left[V_{\sigma(k)}(\overline{x}(k),\xi(k))\right] \\ &< E\left[\mu\lambda_{\sigma(k_q)}^{(k-k_q)}V_{\sigma(k_{q-1})}\left(\overline{x}(k_q),\xi(k_q)\right)\right] \\ &< E\left[\mu\lambda_{\sigma(k_q)}^{(k-k_q)}\lambda_{\sigma(k_{q-1})}^{(k_q-k_{q-1})}V_{\sigma(k_{q-1})}\left(\overline{x}(k_{q-1}),\xi(k_{q-1})\right)\right] \\ &\vdots \\ &< E\left[\mu^{N_\sigma[k_0,k)}\lambda_1^{T^1[k_0,k)}\lambda_2^{T^2[k_0,k)}V_{\sigma(k_0)}(\overline{x}(k_0),\xi(k_0))\right]. \end{aligned} \quad (33)$$

Note that the Markov chain is stationary (16); then

$$\begin{aligned} E\left[T^1[k_0,k)\right] &= \pi_1(k-k_0), \\ E\left[T^2[k_0,k)\right] &= \pi_2(k-k_0). \end{aligned} \quad (34)$$

Therefore, we can obtain that

$$\begin{aligned} &E\left[V_{\sigma(k)}(\overline{x}(k),\xi(k))\right] \\ &< E\left[\mu^{N_\sigma[k_0,k)}\lambda_1^{\pi_1(k-k_0)}\lambda_2^{\pi_2(k-k_0)}V_{\sigma(k_0)}(\overline{x}(k_0),\xi(k_0))\right] \\ &= E\left[\mu^{N_\sigma[k_0,k)}\lambda_1^{(\pi_{21}/(\pi_{12}+\pi_{21}))(k-k_0)} \times\lambda_2^{(\pi_{12}/(\pi_{12}+\pi_{21}))(k-k_0)}V_{\sigma(k_0)}(\overline{x}(k_0),\xi(k_0))\right] \\ &= E\left[\mu^{N_\sigma[k_0,k)}\lambda^{(k-k_0)}V_{\sigma(k_0)}(\overline{x}(k_0),\xi(k_0))\right] \\ &= E\left[\lambda^{N_\sigma[k_0,k)(\ln\mu/\ln\lambda)}\lambda^{(k-k_0)}V_{\sigma(k_0)}(\overline{x}(k_0),\xi(k_0))\right] \\ &= E\left[\left(\lambda^{1+(N_\sigma[k_0,k)/(k-k_0))\cdot(\ln\mu/\ln\lambda)}\right)^{(k-k_0)}\right] \\ &\quad\times E\left[V_{\sigma(k_0)}(\overline{x}(k_0),\xi(k_0))\right]. \end{aligned} \quad (35)$$

From Definition 4 we have that

$$\frac{k-k_0}{N_\sigma[k_0,k)} \geq T_a. \quad (36)$$

And from [24], we can get $1/E(T_a) \leq \max\{\pi_{12},\pi_{21}\}$; then combining (23) and (24), we can know that

$$0 < \lambda^{1+(N_\sigma[k_0,k)/(k-k_0))\cdot(\ln\mu/\ln\lambda)} < 1, \quad (37)$$

which ensure the convergence of $E[V_{\sigma(k)}(\overline{x}(k),\xi(k))]$.

In this case,

$$\begin{aligned} &E\left[V_{\sigma(k)}(\overline{x}(k),\xi(k))\right] \\ &< E\left[\left(\lambda^{1+(1/T_a)\cdot(\ln\mu/\ln\lambda)}\right)^{(k-k_0)}\right] \\ &\quad\times E\left[V_{\sigma(k_0)}(\overline{x}(k_0),\xi(k_0))\right] \\ &= \lambda^{(k-k_0)[1+(1/E(T_a))\cdot(\ln\mu/\ln\lambda)]}E\left[V_{\sigma(k_0)}(\overline{x}(k_0),\xi(k_0))\right] \\ &\leq (\lambda^\rho)^{(k-k_0)}E\left[V_{\sigma(k_0)}(\overline{x}(k_0),\xi(k_0))\right]. \end{aligned} \quad (38)$$

Furthermore

$$\begin{aligned} E\left[a\|\overline{x}(k)\|^2\right] &\leq E\left[V_{\sigma(k)}(\overline{x}(k),\xi(k))\right] \\ &< (\lambda^\rho)^{(k-k_0)}E\left[b\|\overline{x}(k_0)\|^2\right]. \end{aligned} \quad (39)$$

Then

$$E\left[\|\overline{x}(k)\|^2\right] \leq \frac{b}{a}(\lambda^\rho)^{(k-k_0)}E\left[\|\overline{x}(k_0)\|^2\right]. \quad (40)$$

Therefore, by Definition 3, system (15) is mean square exponentially stable. □

Remark 7. From Lemma 2, we know that system (1) is also mean square exponentially stable.

3.2. H_∞ Performance Analysis and Controller Design. In this subsection, we are in the position to prove the main result. The H_∞ controller design method is given in the following theorem.

Theorem 8. *For given scalars λ_1, λ_2, γ, and $\mu \geq 1$, if there exist positive definite matrices P_i, X_i, S, Q, and matrix $\overline{K} = [K \;\; 0]$ of appropriate dimensions, $i, j = 1, 2, 3, \ldots, 2^v$, such that* (22)–(24) *and the following inequalities:*

$$\begin{bmatrix} -\lambda_1 P_i & 0 & 0 & \Gamma_{14} & \Gamma_{15} & \overline{C}^T & U^T \\ * & -I & 0 & \sqrt{\pi_{11}}I & \sqrt{\pi_{12}}I & 0 & 0 \\ * & * & -\gamma^2 I & \sqrt{\pi_{11}}\overline{H}^T & \sqrt{\pi_{12}}\overline{H}^T & 0 & 0 \\ * & * & * & -X_j & 0 & 0 & 0 \\ * & * & * & * & -S & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad (41)$$

where $\Gamma_{14} = \sqrt{\pi_{11}}\widetilde{A}_i^T + \sqrt{\pi_{11}}\overline{K}^T\widetilde{B}_i^T$, $\Gamma_{15} = \sqrt{\pi_{12}}\widetilde{A}_i^T + \sqrt{\pi_{12}}\overline{K}^T\widetilde{B}_i^T$,

$$\begin{bmatrix} -\lambda_2 Q & 0 & 0 & \sqrt{\pi_{21}}\overline{A}_2^T & \sqrt{\pi_{22}}\overline{A}_2^T & \overline{C}^T & U^T \\ * & -I & 0 & \sqrt{\pi_{21}}I & \sqrt{\pi_{22}}I & 0 & 0 \\ * & * & -\gamma^2 I & \sqrt{\pi_{21}}\overline{H}^T & \sqrt{\pi_{22}}\overline{H}^T & 0 & 0 \\ * & * & * & -X_j & 0 & 0 & 0 \\ * & * & * & * & -S & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \quad (42)$$

$$X_i P_i = I, \qquad SQ = I, \quad i, j = 1, 2, 3, \ldots, 2^v$$

hold, then system (15) *with the controller gain matrix* $\overline{K}$ *has robustly mean square exponential stability with* H_∞ *disturbance attenuation level* γ.

Proof. It is easy to obtain that (20) and (21) can be deduced from (41) and (42), respectively. Then from Theorem 6, it can be verified that closed-loop system (15) is mean square exponentially stable with $w(k) = 0$.

For the nonzero $w(k)$, using the same Lyapunov function candidates as in Theorem 6, the following relations can be obtained:

$$\begin{aligned} &\Delta V_1 [\overline{x}(k+1)] \\ &\quad = E[V_1(\overline{x}(k+1), \xi(k+1))] - \lambda_1 V_1(\overline{x}(k), \xi(k)) \\ &\quad \le \left(\sum_{i=1}^{2^v} \xi_i(k) A_i \overline{x}(k) + \overline{f}(x,k) + \overline{H}w(k)\right)^T \\ &\quad\quad \times \left(\pi_{11}\sum_{j=1}^{2^v}\xi_j(k+1) P_j + \pi_{12}Q\right) \\ &\quad\quad \cdot \left(\sum_{i=1}^{2^v} \xi_i(k) A_i \overline{x}(k) + \overline{f}(x,k) + \overline{H}w(k)\right) \\ &\quad\quad - \lambda_1 \overline{x}^T(k)\left(\sum_{i=1}^{2^v}\xi_i(k) P_i\right)\overline{x}(k) \\ &\quad\quad + \overline{x}^T(k) U^T U \overline{x}(k) - \overline{f}^T(x,k)\overline{f}(x,k), \end{aligned} \quad (43)$$

$$\begin{aligned} &\Delta V_2 [\overline{x}(k+1)] \\ &\quad = E[V_2(\overline{x}(k+1), \xi(k+1))] - \lambda_2 V_2(\overline{x}(k), \xi(k)) \\ &\quad \le \left(\overline{A_2}\overline{x}(k) + \overline{f}(x,k) + \overline{H}w(k)\right)^T \\ &\quad\quad \times \left(\pi_{21}\sum_{j=1}^{2^v}\xi_j(k+1) P_j + \pi_{22}Q\right) \\ &\quad\quad \cdot \left(\overline{A_2}\overline{x}(k) + \overline{f}(x,k) + \overline{H}w(k)\right) \\ &\quad\quad - \lambda_2 \overline{x}^T(k) Q \overline{x}(k) + \overline{x}^T(k) U^T U \overline{x}(k) \\ &\quad\quad - \overline{f}^T(x,k)\overline{f}(x,k). \end{aligned}$$

From inequalities (43), we have

$$\begin{aligned} &\Delta V_1 [\overline{x}(k+1)] + z^T(k) z(k) - \gamma^2 w^T(k) w(k) \\ &\quad \le \sum_{i=1}^{2^v}\xi_i(k)\sum_{j=1}^{2^v}\xi_j(k+1)\eta^T(k)\Xi_1\eta(k), \end{aligned} \quad (44)$$

$$\begin{aligned} &\Delta V_2 [\overline{x}(k+1)] + z^T(k) z(k) - \gamma^2 w^T(k) w(k) \\ &\quad \le \sum_{j=1}^{2^v}\xi_j(k+1)\ \eta^T(k)\Xi_2\eta(k), \end{aligned} \quad (45)$$

where

$$\begin{aligned} \eta(k) &= \left[\overline{x}^T(k)\ \ \overline{f}^T(x,k)\ \ w^T(k)\right]^T, \\ \Xi_1 &= \begin{bmatrix} \psi_1 & A_i^T(\pi_{11}P_j + \pi_{12}Q) & A_i^T(\pi_{11}P_j + \pi_{12}Q)\overline{H} \\ * & \pi_{11}P_j + \pi_{12}Q - I & (\pi_{11}P_j + \pi_{12}Q)\overline{H} \\ * & * & \varphi_1 \end{bmatrix}, \\ \Xi_2 &= \begin{bmatrix} \psi_2 & \overline{A}_2^T(\pi_{21}P_j + \pi_{22}Q) & \overline{A}_2^T(\pi_{21}P_j + \pi_{22}Q)\overline{H} \\ * & \pi_{21}P_j + \pi_{22}Q - I & (\pi_{21}P_j + \pi_{22}Q)\overline{H} \\ * & * & \varphi_2 \end{bmatrix}, \\ \psi_1 &= A_i^T(\pi_{11}P_j + \pi_{12}Q)A_i - \lambda_1 P_i + \overline{C}^T\overline{C} + U^T U, \\ \psi_2 &= \overline{A}_2^T(\pi_{21}P_j + \pi_{22}Q)\overline{A_2} - \lambda_2 Q + \overline{C}^T\overline{C} + U^T U, \\ \varphi_1 &= \overline{H}^T(\pi_{11}P_j + \pi_{12}Q)\overline{H} - \gamma^2 I, \\ \varphi_2 &= \overline{H}^T(\pi_{21}P_j + \pi_{22}Q)\overline{H} - \gamma^2 I. \end{aligned} \quad (46)$$

In terms of the Schur complement, we obtain

$$\begin{bmatrix} -\lambda_1 P_i + \overline{C}^T\overline{C} + \overline{U}^T\overline{U} & 0 & 0 & \sqrt{\pi_{11}}A_i^T & \sqrt{\pi_{12}}A_i^T \\ * & -I & 0 & \sqrt{\pi_{11}}I & \sqrt{\pi_{12}}I \\ * & * & -\gamma^2 I & \sqrt{\pi_{11}}\overline{H}^T & \sqrt{\pi_{12}}\overline{H}^T \\ * & * & * & -P_j^{-1} & 0 \\ * & * & * & * & -Q^{-1} \end{bmatrix} < 0, \quad (47)$$

where $A_i = \widetilde{A}_i + \widetilde{B}_i\overline{K}$,

$$\begin{bmatrix} -\lambda_2 Q + \overline{C}^T\overline{C} + \overline{U}^T\overline{U} & 0 & 0 & \sqrt{\pi_{21}}\overline{A}_2^T & \sqrt{\pi_{22}}\overline{A}_2^T \\ * & -I & 0 & \sqrt{\pi_{21}}I & \sqrt{\pi_{22}}I \\ * & * & -\gamma^2 I & \sqrt{\pi_{21}}\overline{H}^T & \sqrt{\pi_{22}}\overline{H}^T \\ * & * & * & -P_j^{-1} & 0 \\ * & * & * & * & -Q^{-1} \end{bmatrix} < 0. \quad (48)$$

In light of Lemma 1, if equalities (47) and (48) hold, then combining (44) and (45), we have that

$$\begin{aligned} E[V_1(\overline{x}(k+1), \xi(k+1))] &< \lambda_1 V_1(\overline{x}(k), \xi(k)) - J(k), \\ E[V_2(\overline{x}(k+1), \xi(k+1))] &< \lambda_2 V_2(\overline{x}(k), \xi(k)) - J(k), \end{aligned} \quad (49)$$

where $J(k) = z^T(k)z(k) - \gamma^2 w^T(k)w(k)$.

Combining (22) and (49), it can be seen that

$$\begin{aligned}
&E\left[V_{\sigma(k)}\left(\bar{x}(k),\xi(k)\right)\right] \\
&< E\left[\mu\lambda_{\sigma(k_q)}^{k-k_q}V_{\sigma(k_{q-1})}\left(k_{q-1}\right)-\sum_{s=k_q}^{k-1}\lambda_{\sigma(k_q)}^{k-s-1}J(s)\right] \\
&< E\left\{\mu\lambda_{\sigma(k_q)}^{k-k_q}\left[\lambda_{k_{q-1}}^{k_q-k_{q-1}}V_{\sigma(k_{q-1})}\left(k_{q-1}\right)-\sum_{s=k_{q-1}}^{k_q-1}\lambda_{\sigma(k_{q-1})}^{k_q-s-1}J(s)\right]\right. \\
&\qquad\left.-\sum_{s=k_q}^{k-1}\lambda_{\sigma(k_q)}^{k-s-1}J(s)\right\} \\
&\vdots \\
&< E\left[\mu^{N_\sigma[k_0,k)}\lambda_1^{T^1[k_0,k)}\lambda_2^{T^2[k_0,k)}V_{\sigma(k_0)}\left(k_0\right)\right. \\
&\qquad\left.-\sum_{s=k_0}^{k-1}\mu^{N_\sigma[s,k)}\lambda_1^{T^1[s,k-1)}\lambda_2^{T^2[s,k-1)}J(s)\right].
\end{aligned} \tag{50}$$

Since $V_{\sigma(k)} > 0$ and the zero-initial state assumption, it can be seen that

$$E\left[\sum_{s=k_0}^{k-1}\mu^{N_\sigma[s,k)}\lambda_1^{T^1[s,k-1)}\lambda_2^{T^2[s,k-1)}J(s)\right] < 0. \tag{51}$$

From (34), (51) can be written as

$$E\left[\sum_{s=k_0}^{k-1}\mu^{N_\sigma[s,k)}\lambda^{k-1-s}J(s)\right] < 0. \tag{52}$$

Multiplying both sides of inequality (52) by $-N_\sigma[0,k)$, we can obtain

$$\begin{aligned}
&E\left[\mu^{-N_\sigma[0,k)}\sum_{s=k_0}^{k-1}\mu^{N_\sigma[s,k)}\lambda^{(k-1-s)}z^T(s)z(s)\right] \\
&\quad< E\left[\mu^{-N_\sigma[0,k)}\sum_{s=k_0}^{k-1}\mu^{N_\sigma[s,k)}\lambda^{(k-1-s)}\gamma^2w^T(s)w(s)\right],
\end{aligned} \tag{53}$$

which is equivalent to

$$\begin{aligned}
&E\left[\sum_{s=k_0}^{k-1}\mu^{-N_\sigma[0,s)}\lambda^{(k-1-s)}z^T(s)z(s)\right] \\
&\quad< E\left[\sum_{s=k_0}^{k-1}\mu^{-N_\sigma[0,s)}\lambda^{(k-1-s)}\gamma^2w^T(s)w(s)\right].
\end{aligned} \tag{54}$$

Then, from Definition 4 and (24)

$$N_\sigma[0,s) \le \frac{s}{T_a} < s\cdot\max\left\{\pi_{12},\pi_{21}\right\} < s\cdot\left(-\frac{\ln\lambda}{\ln\mu}\right), \tag{55}$$

we have

$$\begin{aligned}
&E\left[\sum_{s=k_0}^{k-1}\mu^{-N_\sigma[0,s)}\lambda^{(k-1-s)}z^T(s)z(s)\right] \\
&\quad> E\left[\sum_{s=k_0}^{k-1}\mu^{s\cdot\ln\lambda/\ln\mu}\lambda^{(k-1-s)}z^T(s)z(s)\right] \\
&\quad= E\left[\sum_{s=k_0}^{k-1}\lambda^{(k-1)}z^T(s)z(s)\right], \\
&E\left[\sum_{s=k_0}^{k-1}\mu^{-N_\sigma[0,s)}\lambda^{(k-1-s)}\gamma^2w^T(s)w(s)\right] \\
&\quad< E\left[\sum_{s=k_0}^{k-1}\lambda^{(k-1-s)}\gamma^2w^T(s)w(s)\right].
\end{aligned} \tag{56}$$

Therefore

$$\begin{aligned}
&E\left[\sum_{s=k_0}^{k-1}\lambda^{(k-1)}z^T(s)z(s)\right] \\
&\quad< E\left[\sum_{s=k_0}^{k-1}\lambda^{(k-1-s)}\gamma^2w^T(s)w(s)\right],
\end{aligned} \tag{57}$$

which implies that

$$\begin{aligned}
&E\left[\sum_{s=k_0}^{\infty}z^T(s)z(s)\sum_{k=s+1}^{\infty}\lambda^{(k-1)}\right] \\
&\quad< E\left[\sum_{s=k_0}^{\infty}\gamma^2w^T(s)w(s)\sum_{k=s+1}^{\infty}\lambda^{(k-1-s)}\right].
\end{aligned} \tag{58}$$

Then

$$E\left[\sum_{s=k_0}^{\infty}\lambda^s z^T(s)z(s)\right] < E\left[\sum_{s=k_0}^{\infty}\gamma^2w^T(s)w(s)\right]. \tag{59}$$

By Definition 5, system (15) has an exponential H_∞ performance γ. This completes the proof. □

Remark 9. It should be pointed out that the conditions proposed in Theorem 8 are not standard LMIs. In this paper, it is suggested to use the cone complementarity linearization (CCL) algorithm to solve this problem [26]; a nonlinear constraint can be converted to a linear optimization problem with a rank constraint.

Remark 10. In this paper, the mean square exponential H_∞ performance of the system (15) can be guaranteed, which means the noise attenuation performance is different when the decay degree of the system is different, and the decay degree has a close relation with the elements of the transition probabilities. Note that the scalar λ in the sequel symbolizes the decreasing rate of the Lyapunov function to be constructed for each subsystem from Theorem 6. Then, if $\lambda \rightarrow 1$, the evaluated performance index will approach the normal H_∞ performance for the whole time domain.

4. Numerical Example

In this section, we present an example to illustrate the effectiveness of the proposed approach. Consider the following system:

$$\dot{x}(t) = \begin{bmatrix} -1 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t) + \begin{bmatrix} 0.06x_1 \sin x_1 \\ 0.01x_2 \cos x_2 \end{bmatrix} + \begin{bmatrix} 0.05 \\ 0.01 \end{bmatrix} w(t),$$
$$z(t) = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix} x(t). \quad (60)$$

Let the sampling period be $T = 0.3$ s, and $0 \leq \tau_k \leq 0.1$ s. Assume that the transition probability matrix of stochastic switching signals is given as $P = \left[\begin{smallmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{smallmatrix}\right]$; the corresponding matrices are given by

$$\widetilde{A}_1 = \begin{bmatrix} 0.5488 & 0.2219 & 0 \\ 0 & 0.9704 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\widetilde{A}_2 = \begin{bmatrix} 0.5488 & 0.2219 & 0.1 \\ 0 & 0.9704 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\widetilde{A}_3 = \begin{bmatrix} 0.5488 & 0.2219 & -0.0980 \\ 0 & 0.9704 & 0.0098 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\widetilde{A}_4 = \begin{bmatrix} 0.5488 & 0.2219 & 0.0020 \\ 0 & 0.9704 & 0.0098 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\overline{A}_2 = \begin{bmatrix} 0.5488 & 0.2219 & 0.0037 \\ 0 & 0.9704 & 0.0296 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \widetilde{B}_1 = \begin{bmatrix} 0.0037 \\ 0.0296 \\ 1 \end{bmatrix},$$
$$\widetilde{B}_2 = \begin{bmatrix} -0.0963 \\ 0.0296 \\ 1 \end{bmatrix}, \qquad \widetilde{B}_3 = \begin{bmatrix} 0.1017 \\ 0.0198 \\ 1 \end{bmatrix},$$
$$\widetilde{B}_4 = \begin{bmatrix} 0.1017 \\ 0.0198 \\ 1 \end{bmatrix}, \qquad \overline{H} = \begin{bmatrix} 0.0116 \\ 0.0030 \\ 0 \end{bmatrix},$$
$$U = \operatorname{diag}\{0.2, 0.2, 0\}, \qquad \overline{C} = \begin{bmatrix} 0.1 & 0.5 & 0 \end{bmatrix}. \quad (61)$$

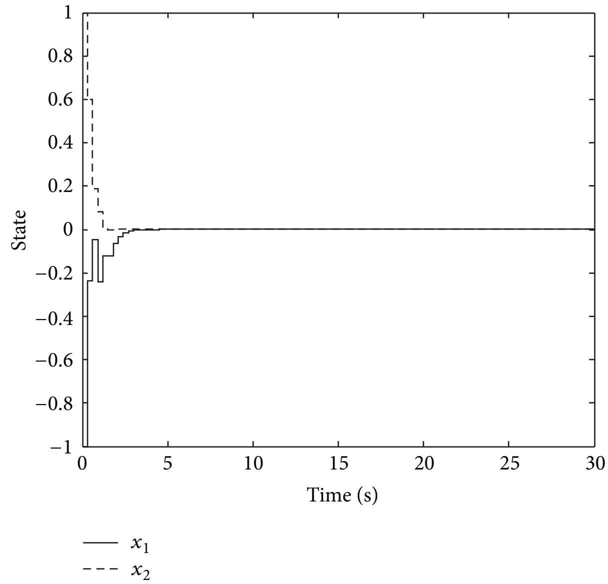

Figure 1: State trajectories of the closed-loop system.

For subsystem 2 without state feedback, $\overline{A}_2$ is an unstable matrix, and $\lambda_2 > 1$. By Theorem 6, we can get $0 < \lambda_1 < 1$. Take $\lambda_1 = 0.45$, $\lambda_2 = 2$; then $\lambda = 0.6533 < 1$, which satisfies the condition (16). It is assumed that $\gamma = 2$; solving LMIs (41) and (42) in Theorem 8, we get the following solutions:

$$X_1 = \begin{bmatrix} 2.3889 & -0.1809 & 0.9960 \\ -0.1809 & 0.0703 & -0.8737 \\ 0.9960 & -0.8737 & 40.8886 \end{bmatrix},$$
$$X_2 = \begin{bmatrix} 6.4614 & -0.1005 & -31.2945 \\ -0.1005 & 0.0663 & -1.2282 \\ -31.2945 & -1.2282 & 275.1873 \end{bmatrix},$$
$$X_3 = \begin{bmatrix} 7.8714 & -0.3703 & 43.1660 \\ -0.3703 & 0.0723 & -1.6709 \\ 43.1660 & -1.6709 & 343.6616 \end{bmatrix}, \quad (62)$$
$$X_4 = \begin{bmatrix} 2.4000 & -0.1227 & 4.0524 \\ -0.1227 & 0.2325 & -11.7220 \\ 4.0524 & -11.7220 & 145.1491 \end{bmatrix},$$
$$S = \begin{bmatrix} 9.7289 & -2.4859 & 9.0787 \\ -2.4859 & 0.7924 & -4.3508 \\ 9.0787 & -4.3508 & 85.3273 \end{bmatrix}.$$

Then the controller gain can be obtained:

$$K = \begin{bmatrix} -0.8405 & -14.2332 \end{bmatrix}. \quad (63)$$

The state trajectories of the NCS and the corresponding switching signal are shown in Figures 1 and 2, respectively, where the initial condition $x_0 = [-1 \ \ 1]^T$ and $w(k) = 0.05\exp(-0.01k)$.

From simulation results, it can be seen that the NCS is robustly mean square exponentially stable and the H_∞ disturbance attenuation level $\gamma = 2$.

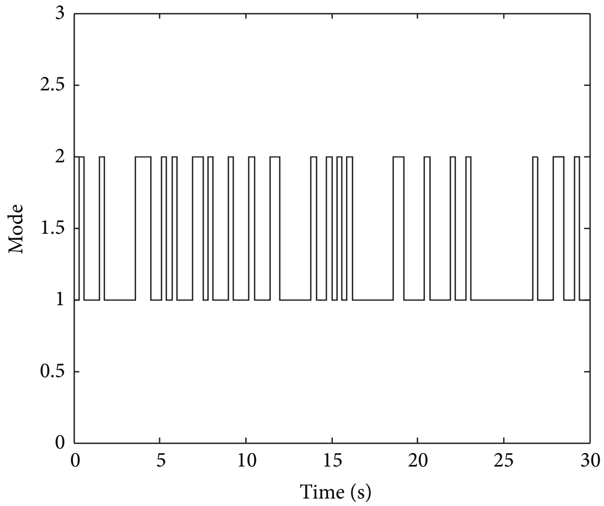

Figure 2: The stochastic switching signal.

5. Conclusions

In this paper, a discrete-time switched system with two subsystems has been presented to model the NCS with time delay and packet dropout. A new approach by using the average dwell time method is proposed to study the robust stabilization and H_∞ control of the addressed NCS. Finally, a numerical example has been given to demonstrate the effectiveness of the proposed method.

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