A Computational Method to Optimal Control of a Wind Turbine System using Wavelets

Hamid Reza Karimi

Abstract— **This paper deals with a computational optimization approach to the problem of state-feedback control design for a wind turbine system. The first step of the study is to develop a reduced order model for the system by considering the most important physical phenomena of aerodynamics and structural dynamics. Moreover, the behavior of the system can be influenced by the coupled dynamics between the tower motions and the blade pitch and turbine speed which can cause instabilities in the control loops in the worst case. By using a suitable wavelet funcation, called Haar functions, a recursive computational procedure is established for finding the system dynamics approximately by solving only algebraic equations instead of solving the Riccati differential. Simulation results are given to illustrate the usefulness of the proposed control methodology.**

I. INTRODUCTION

Since the early 1990s wind power has enjoyed a renewed interest, particularly in the European Union where the annual growth rate is about 20%. This growth is attributed to wind power's inherent attribute of generating carbon-emission-free electricity. In order to sustain such growth, wind turbine performance must continue to be improved [1]-[2].

Recently, linear controllers have been extensively used for power regulation through the control of blade pitch angle. However, the performance of these linear controllers is limited by the highly nonlinear characteristics of wind turbine. Advanced control is one research area where such improvement can be achieved. Typical power regulation control schemes use blade pitch angle as the only controller input. Generator torque is sometimes controlled according to the method employed for the below-rated wind speed conditions, known as the indirect control in torque technique. Most controllers hold the generator torque constant at its nominal value, making the controller monovariable in pitch only [3]-[5].

On the other hand, many papers analyze numerical methods for finding an efficient algorithm to calculate an active control using the feedback loop approach which is based on the instantaneous knowledge of the system states. Specifically, in the field of dynamic systems and control, orthogonal functions-based techniques of analysis, identification and control have received considerable attention in the recent years. This is evident from the vast amount of literature published over the last two decades [6]. The various systems of orthogonal functions may be classified into two categories: (1) piecewise constant basis functions such as Haar functions (HFs) [7]-[8], block pulse functions [9] and Walsh functions [10], and (2) orthogonal polynomials such as Legendre, Laguerre, Chebyshev, Jacobi, Hermite along with sine-cosine functions [11]-[12]. It is noting that the main characteristic of the piecewise constant basis functions is that these problems are reduced to those of solving a system of algebraic equations for the solution of problems described by differential equations. Thus, the solution, identification and optimisation procedure are either greatly reduced or much simplified accordingly [13]-[15]. However, the problems considered so far for orthogonal functions-based solutions include response analysis, optimal control, parameter estimation, model reduction, controller design, and state estimation. They have been applied to linear time-invariant and time-varying systems, nonlinear and distributed parameter systems, which include scaled systems, stiff systems, delay systems, singular systems and multivariable systems [16].

In the sequel, we present a computational algorithm to calculate the optimal control signals for a wind turbine system using Haar functions. The first step of the study is to present a mathematical model of the structure by considering the most important physical phenomena of aerodynamics and structural dynamics. Moreover, the behaviour of the system can be influenced by the coupled dynamics between the tower motions and the blade pitch and turbine speed which can cause instabilities in the control loops in the worst case. we apply the HFs to the finite-time optimal control problem of the system under consideration. The properties of HFs, Haar integral operational and Haar product operational matrices are utilized to provide a systematic computational framework to find the optimal trajectory and finite-time optimal control of the system approximately with respect to a quadratic cost function by solving only the linear algebraic equations instead of solving the differential equations. One of the main advantages is solving linear algebraic equations instead of solving nonlinear Riccati equation to optimize the control problem of the system. Numerical results are presented to illustrate the applicability of the technique.

1.1. Notations.

$(.): r\times s$ matrix $(.)$ with dimension $r\times s$;

I_r identity matrix with dimension $r\times r$;

0_r zero matrix with dimension $r\times r$;

$0_{r\times s}$ zero matrix with dimension $r\times s$;

$\otimes$ Kronecker product;

H.R. Karimi is with the Department of Engineering, Faculty of Engineering and Science, University of Agder, N-4898 Grimstad, Norway (e-mail: hamid.r.karimi@uia.no).

$\text{vec}(X)$ the vector obtained by putting matrix X into one column;

$\text{tr}(A)$ trace of matrix A.

II. Haar Functions

The oldest and most basic of the wavelet systems is named Haar wavelet, whose functions are given by

$$\psi_0(t) = 1, \qquad t \in [0, 1),$$
$$\psi_1(t) = \begin{cases} 1, & \text{for} \quad t \in [0, \tfrac{1}{2}), \\ -1, & \text{for} \quad t \in [\tfrac{1}{2}, 1), \end{cases} \tag{1}$$

where $\phi(t) = \psi_0(t)$ and $\psi_i(t) = \psi_1(2^j t - k)$ for a given $i \geq 1$ with $i = 2^j + k$ where $j \geq 0$ and $0 \leq k < 2^j$. $\phi(.)$ is sometimes called the 'father wavelet' and $\psi(.)$, the 'mother wavelet' [8].

The finite series representation of any square integrable function $y(t)$ in terms of HFs in the interval $[0,1)$, namely $\hat{y}(t)$, is given by

$$\hat{y}(t) = \sum_{i=0}^{m-1} a_i\, \psi_i(t) \; := a^T\, \Psi_m(t) \tag{2}$$

where $a := [a_0 \; a_1 \; \cdots \; a_{m-1}]^T$ and $\Psi_m(t) := [\psi_0(t) \; \psi_1(t) \cdots \psi_{m-1}(t)]^T$ for $m = 2^j$ and the Haar coefficients a_i are given by

$$a_i = 2^j \int_0^1 y(t)\, \psi_i(t)\; dt. \tag{3}$$

The integration of the vector $\Psi_m(t)$ can be approximated by

$$\int_0^t \Psi_m(r)\, dr = P_m\, \Psi_m(t) \tag{4}$$

where the matrix P_m represents the integral operator matrix for piecewise constant basis functions on the interval $[0,1)$ at the resolution m. For HFs, the square matrix P_m satisfies the following recursive formula [17]:

$$P_m = \frac{1}{2m} \begin{bmatrix} 2m P_{\frac{m}{2}} & -H_{\frac{m}{2}} \\ H^{-1}_{\frac{m}{2}} & 0_{\frac{m}{2}} \end{bmatrix} \tag{5}$$

with $P_1 = \dfrac{1}{2}$ and $H_m^{-1} = \dfrac{1}{m} H_m^T \; \text{diag}(r)$ where the vector r is represented by $r := (1,1,2,2,4,4,4,4,\cdots,\underbrace{(\tfrac{m}{2}),(\tfrac{m}{2}),\cdots,(\tfrac{m}{2})}_{(\frac{m}{2})\,\text{elements}})^T$ for $m > 2$ and the matrix H_m for $\dfrac{i}{m} \leq t_i < \dfrac{i+1}{m}$ is defined as

$$H_m = [\Psi_m(t_0), \Psi_m(t_1), \cdots, \Psi_m(t_{m-1})]. \tag{6}$$

On the other hand, the product of two vectors $\Psi_m(t)$ is also evaluated as

$$R_m(t) := \Psi_m(t)\Psi_m^T(t) \tag{7}$$

where $R_m(t)$ satisfies the following recursive formula [17]

$$R_m(t) = \frac{1}{2m} \begin{bmatrix} R_{\frac{m}{2}}(t) & H_{\frac{m}{2}}\; \text{diag}(\Psi_b(t)) \\ (H_{\frac{m}{2}}\; \text{diag}(\Psi_b(t)))^T & \text{diag}(H^{-1}_{\frac{m}{2}} \Psi_a(t)) \end{bmatrix} \tag{8}$$

with $R_1(t) = \psi_0(t)\psi_0^T(t)$ and

$$\begin{aligned} \Psi_a(t) &:= \left[\psi_0(t), \psi_1(t), \cdots, \psi_{\frac{m}{2}-1}(t)\right]^T = \Psi_{\frac{m}{2}}(t) \\ \Psi_b(t) &:= \left[\psi_{\frac{m}{2}}(t), \psi_{\frac{m}{2}+1}(t), \cdots, \psi_{m-1}(t)\right]^T. \end{aligned} \tag{9}$$

III. System Description

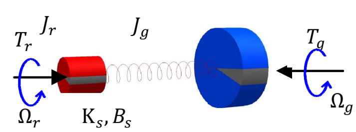
A mechanical system of arbitrary complexity can be described by the equation of motion

$$M\,\ddot{q} + C\,\dot{q} + K\,q = Q(\dot{q}, q, t, u) \tag{10}$$

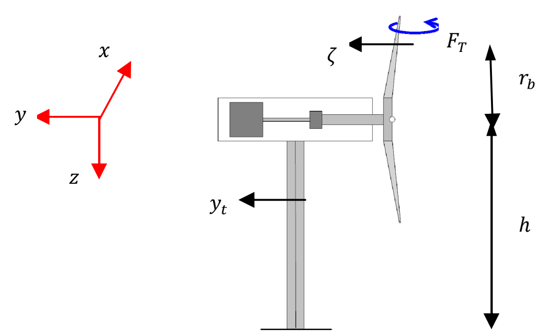
where M, C and K are the mass, damping and stiffness matrices and Q is the vector of forces acting on the system. For mechanical structures having few degrees of freedom, the Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial E_d}{\partial \dot{q}_i} + \frac{\partial E_p}{\partial q_i} = Q_i \tag{11}$$

offers a systematic procedure to derive mathematical models. E_k, E_d and E_p denote the kinetic. Dissipated and potential energy, respectively. Besides, q_i is the generalized coordinate and Q_i stands for the generalised force.



(a) Drive-train



(b) Wind turbine structure

Fig. 1. Schematic diagram of the mechanical subsystem.

Figure 1 shows a schematic diagram of the mechanical model with the parameters defined in Table 1. This model has three degrees of freedom, namely the torsion of the drive-train, the axial tower bending and the flapping. Fig.

1a illustrates the drive-train, which is modeled as two rigid bodies linked by a flexible shaft. According to [4], in Fig. 1b, it is assumed that the blades move in unison and support the same forces.

Table 1.
Parameters for the mechanical subsystem model

Symbol	Description
$\mathbf{m_t}$	Mass of the tower and nacelle
$\mathbf{m_b}$	Mass of each blade
$\mathbf{J_r}$	Inertia of the rotor
$\mathbf{J_g}$	Inertia of the generator
$\mathbf{K_t}$	Stiffness of the tower
$\mathbf{K_b}$	Stiffness of each blade
$\mathbf{K_s}$	Stiffness of the transmission
$\mathbf{B_t}$	Damping of the tower
$\mathbf{B_b}$	Damping of the blade
$\mathbf{B_s}$	Damping of the transmission
$\mathbf{N}$	Number of blades

For the model of the system under consideration, the generalized coordinates can be adopted as $q = [y_t \quad \zeta \quad \theta_r \quad \theta_g]^T$, where y_t is the axial displacement of the nacelle, ζ is the angular displacement out of the plane of rotation and θ_r and θ_g are the angular positions of the rotor and generator, respectively. Substitution of the following energy terms

$$E_k = \frac{m_t}{2}\dot{y_t}^2 + \frac{N}{2} m_b(\dot{y}_t + r_b\dot{\zeta})^2 + \frac{J_r}{2}\Omega_r^2 + \frac{J_g}{2}\Omega_g^2$$
$$E_d = \frac{B_t}{2}\dot{y_t}^2 + \frac{N}{2} B_b(r_b\dot{\zeta})^2 + \frac{B_s}{2}\left(\Omega_r - \Omega_g\right)^2$$
$$E_p = \frac{K_t}{2} {y_t}^2 + \frac{N}{2} K_b(r_b\zeta)^2 + \frac{K_s}{2}\left(\theta_r - \theta_g\right)^2 \qquad (12)$$

into the Lagrange's equation (11) yields the equation of motion (10) with matrices

$$M = \begin{bmatrix} m_t + N m_b & N m_b r_b & 0 & 0 \\ N m_b r_b & N m_b {r_b}^2 & 0 & 0 \\ 0 & 0 & J_r & 0 \\ 0 & 0 & 0 & J_g \end{bmatrix},$$
$$C = \begin{bmatrix} B_t & 0 & 0 & 0 \\ 0 & B_b {r_b}^2 & 0 & 0 \\ 0 & 0 & B_s & -B_s \\ 0 & 0 & -B_s & B_s \end{bmatrix},$$
$$K = \begin{bmatrix} K_t & 0 & 0 & 0 \\ 0 & K_b {r_b}^2 & 0 & 0 \\ 0 & 0 & K_s & -K_s \\ 0 & 0 & -K_s & K_s \end{bmatrix},$$
$$Q(\dot{q}, q, t, u) = [N F_T \quad N F_T r_b \quad T_r \quad -T_g]^T$$

where $T_g = B_g(\Omega_g - \Omega_z)$ is a linear spproximation of the steady-state torque-speed characteristic, where Ω_z is the zero-torque speed and B_g is the slope of the real curve of the torque characteristic of the induction generator at $\Omega_s = p\pi f_s/n_s$ (f_s is the frequency at the generator terminals, p is the number of poles of the machine and n_s is the gear ratio). The input-output map of the aerodynamic subsystem is described as follows

$$\begin{bmatrix} F_T \\ T_r \end{bmatrix} = \begin{bmatrix} \frac{\rho\pi R^2}{2} C_T(\frac{\Omega_r R}{V_e}, \beta)V_e^2 \\ \frac{\rho\pi R^3}{2} C_Q(\frac{\Omega_r R}{V_e}, \beta)V_e^2 \end{bmatrix}$$

where C_T, C_Q are non-dimensional thrust and torque coefficients, respectively. $V_e = V - \dot{y}_b$ is the wind speed relative to the rotor and $\dot{y}_b = \dot{y}_t + r_b\dot{\zeta}$ denotes the axial displacement of the blades caused by flapping and tower bending. Finally, after some manipulation, the following state-space matrices describe the linearised model of the wind turbine system

$$\dot{x} = Ax + Bu$$
$$= \begin{bmatrix} 0 & L & 0 \\ -M^{-1}K & -M^{-1}(C + C_a + C_g) & M^{-1}\tilde{Q}_1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} x + \begin{bmatrix} 0 \\ -M^{-1}\tilde{Q}_2 \\ \begin{bmatrix} 0 & \frac{1}{\tau} & 0 \end{bmatrix} \end{bmatrix} u \qquad (13)$$

where $x = [y_t, \zeta, \theta_s, \dot{y}_t, \dot{\zeta}, \ \Omega_r, \Omega_g, \ \beta]^T$, $u = [\hat{V}, \ \beta_d, \Omega_z]^T$, where $\theta_s = \theta_r - \theta_g$ is the torsion angle with $\tilde{Q}_1 = \left[N\, k_{T,\beta} \quad N\, r_b\, k_{r,\beta} \quad k_{r,\beta} \quad 0\right]^T$, $\tilde{Q}_2 = \begin{bmatrix} N\, k_{T,V} & 0 & 0 \\ N\, r_b\, k_{r,V} & 0 & 0 \\ k_{r,V} & 0 & 0 \\ 0 & 0 & B_g \end{bmatrix}$, $C_a + C_g = \begin{bmatrix} N\, k_{T,V} & N\, r_b\, k_{T,V} & N\, B_T & 0 \\ N\, r_b\, k_{T,V} & N\, {r_b}^2\, k_{T,V} & N r_b B_T & 0 \\ k_{r,V} & r_b k_{r,V} & B_r & 0 \\ 0 & 0 & 0 & B_g \end{bmatrix}$, $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$, and $B_r = -\left.\frac{\partial T_r}{\partial \Omega_r}\right|_{(\overline{\Omega}, \overline{\beta}, \overline{V})}$, $B_T = -\left.\frac{\partial F_T}{\partial \Omega_r}\right|_{(\overline{\Omega}, \overline{\beta}, \overline{V})}$, $k_{T,V} = \left.\frac{\partial F_T}{\partial V}\right|_{(\overline{\Omega}, \overline{\beta}, \overline{V})}$, $k_{T,\beta} = \left.\frac{\partial F_T}{\partial \beta}\right|_{(\overline{\Omega}, \overline{\beta}, \overline{V})}$, $k_{r,V} = \left.\frac{\partial T_r}{\partial V}\right|_{(\overline{\Omega}, \overline{\beta}, \overline{V})}$, $k_{r,\beta} = \left.\frac{\partial T_r}{\partial \beta}\right|_{(\overline{\Omega}, \overline{\beta}, \overline{V})}$, where $\overline{\Omega}$, $\overline{\beta}$, $\overline{V}$ denote the values of rotational speed, pitch angle and wind speed at the operationg point, respectively.

IV. SYSTEM EQUATIONS

The problem of solving the differential equations of the system (13) in terms of the input control and exogenous disturbance is investigated using HFs and an appropriate algebraic equation is developed.

Based on definition of HFs on the time interval $[0, 1]$, we need to rescale the finite time interval $[0, T_f]$ into $[0, 1]$

by considering $t = T_f\ \sigma$; normalizing the system Eq. (13) with the time scale would be as follows

$$A\,\dot{x}(\sigma) + T_f\, B\, x(\sigma) = T_f D\, u(\sigma) \tag{14}$$

Now by integrating the system above in an interval $[0, \sigma]$, we obtain

$$A\,(x(\sigma) - x_0) + T_f\, B\int_0^{\sigma} x(\tau)d\tau = T_f\, D\int_0^{\sigma} u(\tau)\,d\tau \tag{15}$$

By using the HF expansion (2), we express in the following the solution $x(\sigma)$, input force $u(\sigma)$ in terms of HFs

$$\begin{aligned} x(\sigma) &= X\ \Psi_m(\sigma) \\ u(\sigma) &= U\ \Psi_m(\sigma) \end{aligned} \tag{16}$$

where $X: 8\times m$, $U: 3\times m$ denote the wavelet coefficients of $x(\sigma)$, $u(\sigma)$, respectively. The initial conditions of $x(0)$ and $\dot{x}(0)$ are represented by $x(0) = X_0\ \Psi_m(\sigma)$ and $\dot{x}(0) = \overline{X}_0\ \Psi_m(\sigma)$, where $X_0: 8\times m$ and $\overline{X}_0: 8\times m$ are defined as

$$X_0 := [x(0)\ \underbrace{0_{2\times 1}\ \ \ldots\ \ 0_{2\times 1}}_{(m-1)}] \tag{17}$$

Therefore, using the HF expansions (16), the relation (15) becomes

$$A(X - X_0)\Psi_m(\sigma) + T_f B X\int_0^{\sigma}\Psi_m(\tau)d\tau = T_f D U\int_0^{\sigma}\Psi_m(\tau)d\tau. \tag{18}$$

or equivalently, we have

$$A(X - X_0) + T_f\, B X P_m = T_f\, D U P_m. \tag{19}$$

For calculating the matrix X, we apply the operator $vec(.)$ to Eq. (19) and according to the property of the Kronecker product, the following algebraic relation is obtained

$$\begin{aligned} (I_m \otimes A)(vec(X) - vec(X_0)) + T_f\, (P_m^T \otimes B)\, vec(X) \\ = T_f\, (P_m^T \otimes D)\, vec(U) \end{aligned} \tag{20}$$

Solving Eq. (20) for $vec(X)$ leads to

$$vec(X) = K_1 vec(U) + K_2\ vec(X_0) \tag{21}$$

where

$$K_1 = T_f\ \ (T_f\, (P_m^T \otimes B) + I_m \otimes A)^{-1}(P_m^T \otimes D)$$

$$K_2 = (T_f\, (P_m^T \otimes B) + I_m \otimes A)^{-1}(I_m \otimes A)$$

Consequently, from (21) and the properties of the Kronecker product, the solution of the system (13) is approximately

$$x(\sigma) = (\Psi_m^T(\sigma) \otimes I_2)\ vec(X). \tag{22}$$

V. CONTROL DESIGN

In this section, the control objective is to find the optimal state feedback control $u(t)$ with respect to a quadratic cost functional approximately such acts as the active force to the structure. The quadratic cost functional weights the states and their derivatives with respect to time in the cost function as follows:

$$J = \frac{1}{2}x^T(T_f)S_1\, x\,(T_f) + \frac{1}{2}\int_0^{T_f}(x^T(t)\,Q_1\, x(t) + u^T(t)\,R\, u(t))\, dt. \tag{23}$$

where $S_1: 8\times 8$, $Q_1: 8\times 8$ are positive-definite matrices and R is a positive scalar. Normalizing (23) with the time scale $t = T_f\ \ \sigma$ yields

$$J = \frac{1}{2}x^T(1)S_1\, x(1) + \frac{T_f}{2}\int_0^1(x^T(\sigma)Q_1\, x(\sigma) + u^T(\sigma)R\, u(\sigma))\, d\sigma. \tag{24}$$

By using the properties of the operator $tr(.)$, the cost function (24) is given by

$$J = \frac{1}{2}(\, tr(M_f X^T S_1 X)) + \frac{T_f}{2}(tr(M X^T Q_1 X) + tr(M\, U^T R U)) \tag{25}$$

where $M_f = \Psi_m(1)\Psi_m^T(1)$ and $M = \int_0^1 \Psi_m(\sigma)\Psi_m^T(\sigma)\ d\sigma$.

Using the properties of the Kronecker product, we can write (25) as

$$J = \frac{1}{2}(vec^T(X)\Pi_1\, vec(X) + vec^T(U)\Pi_2\ vec(U)) \tag{26}$$

where $\Pi_1 = M_f^T \otimes S_1 + T_f\,(M^T \otimes Q_1)$ and $\Pi_2 = T_f\,(M^T \otimes R)$. It is clear that the cost function of $J(.)$ is a function of $\frac{i}{m} \le \sigma_i < \frac{i+1}{m}$, then for finding the optimal control law, which minimizes the cost functional $J(.)$, the following necessary condition should be satisfied

$$\frac{\partial J}{\partial vec(U)} = 0. \tag{27}$$

By considering $vec(X)$, which is a function of $vec(U)$, and using the properties of derivatives of inner product of Kronecker product, we find

$$\begin{aligned} \frac{\partial J}{\partial vec(U)} = \frac{1}{2}(\frac{\partial\, vec^T(X)}{\partial\, vec(U)}\frac{\partial}{\partial\, vec(X)}(vec^T(X)\Pi_1\, vec(X)) \\ + \frac{\partial}{\partial\, vec(U)}(vec^T(U)\Pi_2\ vec(U))). \end{aligned} \tag{28}$$

Then the wavelet coefficients of the optimal control law will be in vector form as

$$vec(U) = -(\Pi_2 + \Pi_2^T)^{-1}K_1^T(\Pi_1 + \Pi_1^T)\ vec(X) \tag{29}$$

Consequently, the optimal vectors of $vec(X)$ and $vec(F)$ are found, respectively, in the following forms

$$vec\,(X) = (I_{nm} + K_1\,(\Pi_2 + \Pi_2^T)^{-1}K_1^T\,(\Pi_1 + \Pi_1^T))^{-1}K_2\begin{bmatrix} I_n \\ 0 \\ \vdots \\ 0 \end{bmatrix} x_0 \tag{30}$$

and

$$\mathrm{vec}(U) = -(\Pi_2+\Pi_2^T)^{-1} K_1^T (\Pi_1+\Pi_1^T)\ (I_{nm}+K_1(\Pi_2+\Pi_2^T)^{-1} \times K_1^T(\Pi_1+\Pi_1^T))^{-1} K_2 \begin{bmatrix} I_n \\ 0 \\ \vdots \\ 0 \end{bmatrix} x_0 \tag{31}$$

Finally, the Haar function-based optimal trajectories and optimal control are obtained approximately from Eq. (22) and $u(t)=\Psi_m^T(t)\ \mathrm{vec}(U)$.

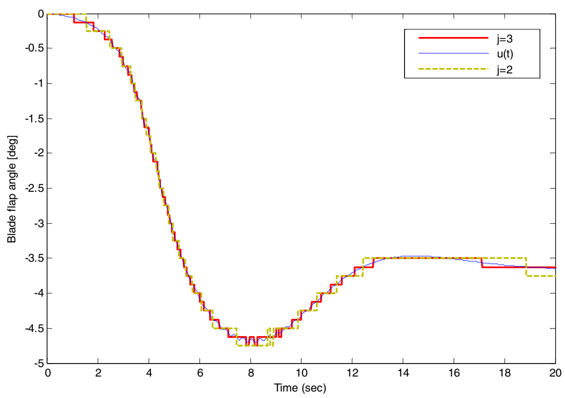


Fig. 2. Comparison of control input found by HFs at resolution levels $j=2,3$ and by analytic solution.

VI. Numerical Results

The simulation are carried out using the Matlab/Simulink software and the turbine model is the one provided in the SymDyn library [18].

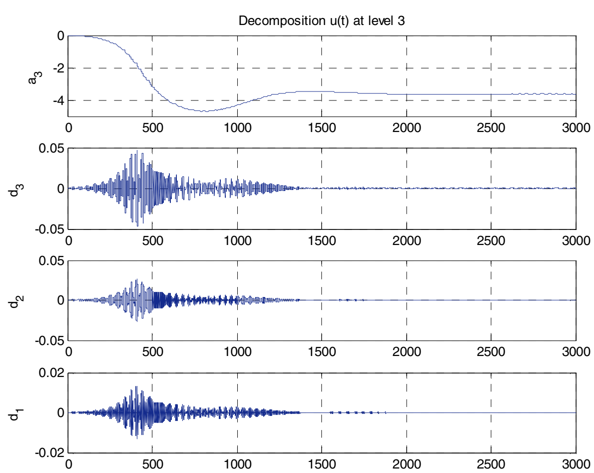


Fig. 3. Decomposition of the blade flap angle at level j = 3 in terms of approximation coefficient (a_3) and detail coefficients (d_1, d_2, d_3).

Figure 2 shows comparison of the blade flap angle of the wind turbine found by HFs at different resolution levels j=2 and 3 and the analytic solution found by solving the Riccati equation, respectively. Decomposition of the control input at level j = 3 in terms of approximation coefficient (a_3) and detail coefficients (d_1, d_2, d_3) are depicted in Figures 3 and 4. Figures show that the HFs can construct the wind turbine signals as well. It is clear that by increasing the resolution level j the accuracy of the approximation can be improved as well.

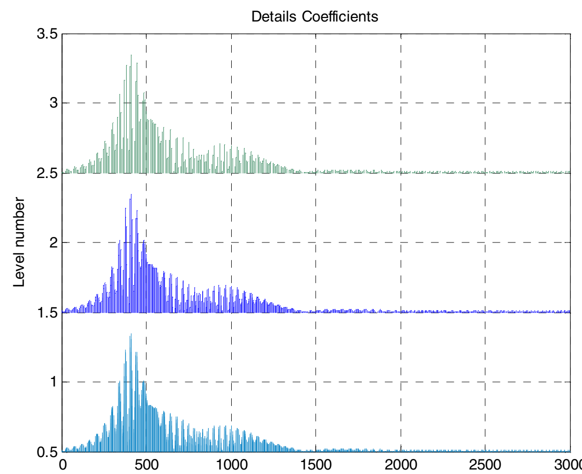


Fig. 4. Detail coefficients of the blade flap angle at level j = 3.

VII. Conclusion

This paper proposed a computational optimization approach to the problem of state-feedback control design for a wind turbine system. By using a suitable wavelet funcation, called Haar functions, a recursive computational procedure was established for finding the system dynamics approximately by solving only algebraic equations instead of solving the Riccati differential. Simulation results were given to illustrate the usefulness of the proposed control methodology.

Acknowledgment

This work has been (partially) funded by Norwegian Centre for Offshore Wind Energy (NORCOWE) under grant 193821/S60 from Research Council of Norway (RCN). NORCOWE is a consortium with partners from industry and science, hosted by Christian Michelsen Research.

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