International Journal of Automation and Computing 8(4), November 2011, 381-390
DOI: 10.1007/s11633-011-0595-z

Adaptive H_∞ Synchronization of Master-slave Systems with Mixed Time-varying Delays and Nonlinear Perturbations: An LMI Approach

Hamid Reza Karimi
Department of Engineering, Faculty of Engineering and Science, University of Agder, N-4898 Grimstad, Norway

Abstract: This paper proposes an adaptive synchronization problem for the master and slave structure of linear systems with nonlinear perturbations and mixed time-varying delays comprising different discrete and distributed time delays. Using an appropriate Lyapunov-Krasovskii functional, some delay-dependent sufficient conditions and an adaptation law including the master-slave parameters are established for designing a delayed synchronization law in terms of linear matrix inequalities(LMIs). The time-varying controller guarantees the H_∞ synchronization of the two coupled master and slave systems regardless of their initial states. Particularly, it is shown that the synchronization speed can be controlled by adjusting the updated gain of the synchronization signal. Two numerical examples are given to demonstrate the effectiveness of the method.

Keywords: Adaptive synchronization, master-slave systems, delay, H_∞ performance, nonlinear perturbations.

1 Introduction

Synchronization is a basic motion in nature that has been studied for a long time, ever since the discovery of Christian Huygens in 1665 on the synchronization of two pendulum clocks. The results of chaos synchronization are utilized in biology, chemistry, secret communication and cryptography, nonlinear oscillation synchronization, and some other nonlinear fields. The original idea involving synchronization of two identical chaotic systems with different initial conditions was introduced by Pecora and Carroll[1], and the method was realized in electronic circuits. The methods for synchronization of the chaotic systems have been widely studied in recent years, and many different methods have been applied theoretically and experimentally to the synchronize chaotic systems, such as feedback control[2−5], adaptive control[6−8], backstepping[9], and sliding mode control[10]. Recently, the theory of incremental input-to-state stability concerned with the problem of synchronization in a complex dynamical network of identical nodes using chaotic nodes as a typical platform was studied by Cai and Chen[11].

On the other hand, delay differential systems are playing an increasingly important role in many disciplines like economic, mathematics, science, and engineering. For instance, in economic systems, delays appear in a natural way since decisions and effects are separated by some time interval. The presence of a delay in a system could be the result from some essential simplification of the corresponding process model. The delay effects problem on the stability of systems including delays in the state and/or input is a problem of recurring interest since the presence of delay may induce complex behaviors (oscillation, instability, and bad performances) for the schemes[12−15]. Some recent views and improved methods pertaining to the problems of determining robust stability criteria and robust control design of uncertain time-delay systems have been reported, and the examples could be seen in [16] and the references cited therein. When dealing with time-varying delays and the reduction of the level of design conservatism, one has to select an appropriate Lyapunov-Krasovskii functional (LKF) with moderate number of terms[17].

In the past few decades, increasing attention has been devoted to the problem of robust delay-independent stability or delay-dependent stability and stabilization via different approaches (for example, model transformation techniques[18, 19], the improved bounding techniques[20−22], and the properly chosen LKFs[23−28] for a number of different neutral systems with delayed state and/or input, parameter uncertainties, and nonlinear perturbations[17, 29−33]). Recently, the control synthesis problem for a class of linear systems with time-varying delays and actuator saturation is investigated in [34]. Furthermore, the stability analysis and stabilization problems for a class of discrete-time Markov jump linear systems with partially known transition probabilities and time-varying delays are studied in [35].

On the synchronization problems of systems with time-delays and nonlinear perturbation terms, we see that there have been some research works (for instance, see [3, 36–40] and the references cited therein). In [41], the adaptive decentralized synchronization of master-slave large-scale time-varying delayed systems with unknown signal propagation delays was investigated based on the Lyapunov stability theorem. In [42], the adaptive complete synchronization of chaotic and hyper chaotic systems with fully unknown parameters is studied based on the Lyapunov stability theorem. An adaptive scheme for the stabilization and synchronization of chaotic Lur′e systems with time-varying de-

Manuscript received July 10, 2010; revised November 17, 2010

lay was proposed based on the invariant principle of functional differential equations in [43]. In [6], based on the invariant principle of functional differential equations, an analytical and rigorous adaptive feedback scheme is proposed for the synchronization of almost all kinds of coupled identical neural networks with time-varying delay, which can be chaotic, periodic, etc. In [38], the synchronization problem is studied for a class of stochastic complex networks with time delays. By utilizing Lyapunov functional form based on the idea of "delay fractioning", the stochastic analysis techniques and the properties of Kronecker product are employed to establish delay-dependent synchronization criteria that guarantee the globally asymptotically mean-square synchronization of the addressed delayed networks with stochastic disturbances. Therefore, the development of synchronization methods for master-slave systems with time-varying delays using delay-dependent adaptive synchronization is important and has not been fully investigated in the past, and it remains to be an important and challenging problem. The fact motivates the present study.

In this paper, we make contribution to the further development of the adaptive synchronization problem for a class of master-slave systems with nonlinear perturbations and mixed time-delays comprising different discrete and distributed time delays. Some sufficient conditions and an adaptation law that include the master-slave parameters are obtained by using the LKFs method and linear matrix inequality (LMI) techniques. Then, the H_∞ synchronization law is developed based on the available information on the size of the discrete and distributed delays so as to guarantee that the controlled slave system can be synchronized with the master system regardless of their initial states. Particularly, the synchronization speed can be controlled by adjusting the updated gain of the synchronization signal. All the developed results are expressed in terms of convex optimization over LMIs and tested on two representative examples to demonstrate the feasibility and applicability of the proposed synchronization approach. The main contribution of the paper is three folds: 1) For the problem addressed, the assumption that the discrete and distributed delays appear in the system is different from the existing results; 2) An LKF-based method is provided to derive a new form of the delay-dependent bounded real lemma (BRL) for the system under consideration; 3) An adaptive delay-dependent time-varying synchronization controller with an adaptation law can be obtained by solving LMIs.

This paper is organized as follows. In Section 2, the model of master-slave systems with both time-varying discrete and distributed delays and nonlinear perturbations is described. In Section 3, the discrete-delay-dependent distributed-delay-dependent adaptive synchronization is derived based on LMIs. In Section 4, a numerical example is given to verify our results. Finally, in Section 5, a conclusion is drawn.

2 Problem description

A model of master and slave systems with mixed time-varying delays and nonlinear perturbations in the form of

$$\begin{cases} \dot{x}_m(t) = A_1\, x_m(t) + A_2\, x_m(t-h(t))+ \\ \qquad A_3 \displaystyle\int_{t-\tau(t)}^{t} x_m(s)\,\mathrm{d}s + N_1 f_1(t, x_m(t))+ \\ \qquad N_2\, f_2(t, x_m(t-h(t))) \\ x_m(t) = \phi(t), \qquad t\in[-\kappa,\ 0] \\ z_m(t) = C_1 x_m(t) + C_2 x_m(t-h(t)) + C_3 \displaystyle\int_{t-\tau(t)}^{t} x_m(s)\mathrm{d}s \end{cases} \tag{1}$$

and

$$\begin{cases} \dot{x}_s(t) = A_1\, x_s(t) + A_2\, x_s(t-h(t)) + A_3 \displaystyle\int_{t-\tau(t)}^{t} x_s(s)\mathrm{d}s+ \\ \qquad N_1 f_1(t, x_s(t)) + N_2\, f_2(t, x_s(t-h(t)))+ \\ \qquad Bu(t) + Dw(t) \\ x_s(t) = \varphi(t), \qquad t\in[-\kappa,\ 0] \\ z_s(t) = C_1 x_s(t) + C_2 x_s(t-h(t)) + C_3 \displaystyle\int_{t-\tau(t)}^{t} x_s(s)\,\mathrm{d}s \end{cases} \tag{2}$$

is considered, where $\kappa = \max\{h_M, \tau_m\}$, $x_m(t)$, $x_s(t)$ are the $n\times 1$ state vector of the master and slave systems, respectively, and $u(t)\in\mathbf{R}^r$ is a control signal for synchronizing the master and slave systems. $w(t)\in\mathbf{R}^p$ is a disturbance term belonging to $L_2\,[0,\ \infty)$. $Z_m(t)$, $Z_s(t)$ are the output vector of the master and slave systems, respectively. The time-varying vector-valued initial functions $\phi(t)$ and $\varphi(t)$ are continuously differentiable functionals, and $f_i(\cdot,\cdot)$ are also time-varying vector-valued functions. The time-varying delays are satisfying

$$0 < h(t) \leqslant h_M, \qquad \dot{h}(t) \leqslant h_D \tag{3a}$$

$$0 < \tau(t) \leqslant \tau_M, \qquad \dot{\tau}(t) \leqslant \tau_D. \tag{3b}$$

Assumption 1. The functions $f_i : \mathbf{R}^+ \times \mathbf{R}^n \rightarrow \mathbf{R}^{n'}$ are continuous and satisfy $f_i(t,0) = 0$ and the Lipschitz conditions, i.e.,

$$\|f_i(t,x_0) - f_i(t,y_0)\| \leqslant \|\Gamma_i(x_0 - y_0)\|$$

for all t and for all x_0, $y_0\in\mathbf{R}^n$ such that Γ_i are some known matrices.

Remark 1. The model (1)–(2) can describe a large amount of well-known dynamical systems with time delays, such as the delayed Logistic model, the chaotic models with time delays, and the artificial neural network model with discrete time delays. In real applications, these coupled systems can be regarded as interacting dynamical elements in the entire system, such as physical particles, biological neurons, ecological populations, genic oscillations, and even automatic machines and robots. A feasible coupling design for successful synchronization leads us to fully command the intrinsic mechanism regulating the evolution of real systems, to fabricate emulate systems, and even to remotely control the machines and nodes in networks with large scales (see [1, 2, 9, 15, 37]).

Remark 2. Nonlinearities that satisfy Assumption 1 are not uncommon. Examples include trigonometric nonlinearities occurring in robotic applications, nonlinear softening spring models frequently used in mechanical systems, etc. Many nonlinearities satisfy Assumption 1 locally, such as nonlinearities that are square or cubic in nature. Moreover, when these nonlinearities occur in physical systems, they usually have saturation levels, making them globally Lipschitz nonlinearities[44].

Assumption 2. The full state variables $x_s(t)$ and $x_m(t)$ are available for the measurement.

Definition 1. The H_∞ performance measure of the master and slave systems (1)–(2) is defined as

$$J_\infty=\int_0^\infty [(z_m(t)-z_s(t))^{\mathrm{T}}(z_m(t)-z_s(t))-\gamma^2 w^{\mathrm{T}}(t)\,w(t)]\mathrm{d}t \tag{4}$$

where the positive scalar γ is given.

Now, it is required to synchronize the slave system with the master system at the same time. The synchronization error of the master and slave systems (1)–(2) is defined as $e(t) = x_m(t) - x_s(t)$, then the error dynamics between (1)–(2), namely, synchronization error system could be expressed by

$$\begin{aligned}\dot e(t) = {} & A_1\, e(t) + A_2\, e(t-h(t)) + A_3 \int_{t-\tau(t)}^{t} e(s)\ \mathrm{d}s+\\ & N_1 \hat f_1(t,e(t)) + N_2 \hat f_2(t,e(t-h(t)))-\\ & B\,u(t) - D\,w(t)\end{aligned} \tag{5}$$

where

$$\hat f_1(t,e(t)) = f_1(t,x_m(t)) - f_1(t,x_m(t)-e(t))$$

and

$$\begin{aligned}\hat f_2(t,e(t-h(t))) = {} & f_2(t,x_m(t-h(t)))-\\ & f_2(t,x_m(t-h(t))-e(t-h(t))).\end{aligned}$$

From Assumption 1, the corresponding uncertainty set is denoted by

$$\Xi_i(e(t)) = \{\hat f_i(t,\, e(t)) : \|\hat f_i(t,e(t))\| \leqslant \|\Gamma_i e\|\}. \tag{6}$$

The problem addressed in this paper is formulated as follows: for given master-slave systems (1)–(2) with both discrete and distributed time delays, we find out a delay-dependent adaptive synchronization control $u(t)$ for the slave system (2) so that the state of the slave system can follow that of a master model, i.e., $\lim_{t\to\infty}\ e(t) = 0$.

Before end of this section, we recall the following lemmas, which will be used in the proof of our main results in the next section.

Lemma 1[22] (Jensen′s inequality). For a given positive-definite matrix $P \in \mathbf{R}^{n\times n}$ and two scalars $b > a \geqslant 0$ for any vector $x(t) \in \mathbf{R}^n$, we have

$$\begin{aligned}&\textstyle\int_{t-b}^{t-a} x^{\mathrm{T}}(\omega)Px(\omega)\ \mathrm{d}\omega \geqslant\\ &\tfrac{1}{b-a}\Big(\textstyle\int_{t-b}^{t-a} x(\omega)\ \mathrm{d}\omega\Big)^{\mathrm{T}} P\Big(\textstyle\int_{t-b}^{t-a} x(\omega)\ \mathrm{d}\omega\Big).\end{aligned}$$

Lemma 2 (Barbalat lemma[45]). If $w : \mathbf{R}\to\mathbf{R}$ is a uniformly continuous function for $t \geqslant 0$ and if the limit of the integral

$$\lim_{t\to\infty}\int_0^t |w(\lambda)|\ \mathrm{d}\lambda$$

is finite and exists, then $\lim\limits_{t\to\infty}\ w(t) = 0$.

3 Main results

In this section, we propose sufficient conditions for the solvability of the adaptive synchronization problem of the master-slave systems (1)–(2) using the Lyapunov method and define the following LKF:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + q\,\rho(t)^2 \tag{7}$$

with $\rho(0) = 0$ and

$$V_1(t) = e(t)^{\mathrm{T}}\, P\, e(t),$$

$$V_2(t) = \int_{t-h(t)}^{t} e(\xi)^{\mathrm{T}} S e(\xi)\mathrm{d}\xi + \int_{t-h(t)}^{t}\int_{\xi}^{t} e(s)^{\mathrm{T}} R e(s)\mathrm{d}s\mathrm{d}\xi,$$

$$\begin{aligned}V_3(t) = {} & \int_{t-\tau(t)}^{t} [\int_s^t e(\theta)^{\mathrm{T}}\mathrm{d}\theta] U_1 [\int_s^t e(\theta)\mathrm{d}\theta]\mathrm{d}s+\\ & \int_0^{\tau(t)}\int_{t-s}^{t} (\theta - t + s)\, e(\theta)^{\mathrm{T}} U_1 e(\theta)\mathrm{d}\theta\mathrm{d}s\end{aligned}$$

where $\rho(t) \in \mathbf{R}$ denotes the adaptation errors that will be defined later.

It is noted that the first three terms in LKF (7) will provide a delay-dependent BRL for the system under consideration, and the last term in (7) is chosen to find an adaptation law deriving the synchronization controller.

Now, in order to establish the H_∞ performance measure for the system (1)-(2), we assume zero initial condition and then have $V(t)|_{t=0} = 0$. Considering the index J_∞ in (4), along with the solution of (5) for any nonzero $w(t)$, then the inequality

$$\begin{aligned}J_\infty \leqslant {} & \\ & \int_0^\infty [(z_m(t)-z_s(t))^{\mathrm{T}}(z_m(t)-z_s(t))-\gamma^2 w^{\mathrm{T}}(t)w(t)]\mathrm{d}t-\\ & V(x(t))|_{t=0} + V(x(t))|_{t=\infty} \leqslant\\ & \int_0^\infty H[u(t), w(t), \hat f_1(t,e(t)), \hat f_2(t,e(t-h(t)))]\mathrm{d}t\end{aligned} \tag{8}$$

holds, where the function

$$\begin{aligned}&H[u(t),\, w(t),\, \hat f_1(t,e(t)),\, \hat f_2(t,e(t-h(t)))] =\\ &(z_m(t)-z_s(t))^{\mathrm{T}}(z_m(t)-z_s(t)) - \gamma^2 w^{\mathrm{T}}(t)\,w(t) + \dot V(t)\end{aligned}$$

is called a Hamiltonian function. It is well known that a sufficient condition for achieving robust disturbance attenuation, i.e., $J_\infty < 0$, is that the inequality

$$\begin{aligned}&H[u(t),\, w(t),\, \hat f_1(t,\, e(t)),\, \hat f_2(t,e(t-h(t)))] < 0,\\ &\forall\, w \in L^2,\ \hat f_i(t,e(t)) \in \Xi_i(e(t)),\ \ i = 1, 2\end{aligned} \tag{9}$$

results in $V(t)$, which is strictly radial unbounded.

Theorem 1. Based on Assumptions 1–2, the master-slave systems (1)–(2) with the different discrete and distributed time-varying delays can be synchronized if there exist the scalar $\gamma > 0$, matrices L_1, L_2, L_3, and positive-definite matrices X, $\tilde S$, $\tilde R$, $\tilde U_1$ such that the following LMI holds:

$$\tilde{\Pi}=\begin{bmatrix}\tilde{\Pi}_{11} & A_2X-BL_2 & 0 & A_3X-BL_3 & N_1 & N_2 & -D & XC_1^{\rm T} & X\Gamma_1^{\rm T} & 0\\ * & -(1-h_D)\tilde{S} & 0 & 0 & 0 & 0 & 0 & XC_2^{\rm T} & 0 & X\Gamma_2^{\rm T}\\ * & * & -(1-h_D)\tilde{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & -(1-\tau_D)\tilde{U}_1 & 0 & 0 & 0 & XC_3^{\rm T} & 0 & 0\\ * & * & * & * & -I & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & -I & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & -\gamma^2 I & 0 & 0 & 0\\ * & * & * & * & * & * & * & -I & 0 & 0\\ * & * & * & * & * & * & * & * & -I & 0\\ * & * & * & * & * & * & * & * & * & -I\end{bmatrix}<0 \quad (10)$$

where $\tilde{\Pi}_{11}=A_1X+XA_1^{\rm T}-BL_1-L_1^{\rm T}B^{\rm T}+\tilde{S}+h_M\tilde{R}+\tau_M^2\,\tilde{U}_1$.

Then, the adaptive synchronization controller is given by

$$u(t)=K_1\,e(t)+K_2\,e(t-h(t))+K_3\int_{t-\tau(t)}^{t}e(s)\;{\rm d}s-\rho(t)\sin(\Lambda_x) \quad (11)$$

with the adaptation law

$$\dot{\rho}(t)=q^{-1}\left\|\Lambda_x\right\|,\quad \rho(0)=\rho_0 \quad (12)$$

where $\Lambda_x=B^{\rm T}X^{-1}e(t)$, $K_i=L_iX^{-1}$, $i=1,2,3$, and the positive constants q and ρ_0 are specified by the designer.

Proof. We will prove the theorem by showing that the control law (11) will guarantee the inequality of (10).

By using the Jensen's inequality, and the properties of the time delays, derivatives of $V_i(t)$, $i=1,2,3$, are given, respectively, by

$$\dot{V}_1(t)=2\dot{e}(t)^{\rm T}P\,e(t) \quad (13a)$$

$$\begin{aligned}\dot{V}_2(t)=\;&e(t)^{\rm T}(S+h(t)R)\,e(t)-(1-\dot{h}(t))e(t-h(t))^{\rm T}\times\\ &Se(t-h(t))-(1-\dot{h}(t))\int_{t-h(t)}^{t}e(s)^{\rm T}Re(s){\rm d}s\leqslant\\ &e(t)^{\rm T}(S+h_MR)\,e(t)-(1-h_D)e(t-h(t))^{\rm T}\times\\ &Se(t-h(t))-(1-h_D)\times\\ &\left(\int_{t-h(t)}^{t}e(s)^{\rm T}\;{\rm d}s\right)R\left(\int_{t-h(t)}^{t}e(s)\;{\rm d}s\right)\end{aligned} \quad (13b)$$

$$\begin{aligned}\dot{V}_3(t)=&-(1-\dot{\tau}(t))\left[\int_{t-h(t)}^{t}e(\theta)^{\rm T}{\rm d}\theta\right]U_1\left[\int_{t-h(t)}^{t}e(\theta)^{\rm T}{\rm d}\theta\right]+\\ &2\int_{t-h(t)}^{t}(\theta-t+\tau(t))e(t)^{\rm T}U_1e(\theta){\rm d}\theta+\\ &\int_{0}^{\tau(t)}se(t)^{\rm T}U_1e(t){\rm d}s-\int_{0}^{\tau(t)}\int_{t-s}^{t}e(\theta){\rm d}\theta{\rm d}s\leqslant\\ &\int_{t-h(t)}^{t}(\theta-t+\tau(t))[e(t)^{\rm T}U_1e(t)+e(\theta)^{\rm T}U_1e(\theta)]{\rm d}\theta-\\ &(1-\dot{\tau}(t))\left[\int_{t-\tau(t)}^{t}e(\theta)^{\rm T}{\rm d}\theta\right]U_1\left[\int_{t-\tau(t)}^{t}e(\theta){\rm d}\theta\right]+\\ &\int_{0}^{\tau(t)}se(t)^{\rm T}U_1e(t){\rm d}s-\\ &\int_{t-\tau(t)}^{t}(\theta-t+\tau(t))e(\theta)^{\rm T}U_1e(\theta){\rm d}\theta=\tau_M^2e(t)^{\rm T}U_1e(t)-\\ &(1-\tau_D)\left[\int_{t-h(t)}^{t}{\rm d}\theta\right]U_1\left[\int_{t-h(t)}^{t}e(\theta){\rm d}\theta\right].\end{aligned} \quad (13c)$$

From (6), we have

$$-\hat{f}_1(t,e(t))^{\rm T}\hat{f}_1(t,e(t))+e(t)^{\rm T}\Gamma_1^{\rm T}\Gamma_1\,e(t)\geqslant 0 \quad (14a)$$

$$\begin{aligned}&-\hat{f}_2(t,e(t-h(t)))^{\rm T}\hat{f}_2(t,e(t-h(t)))+\\ &e(t-h(t))^{\rm T}\Gamma_2^{\rm T}\Gamma_2\,e(t-h(t))\geqslant 0.\end{aligned} \quad (14b)$$

Substituting $u(t)$ by

$$\begin{aligned}u(t)=\;&K_1\,e(t)+K_2\,e(t-h(t))+K_3\int_{t-\tau(t)}^{t}e(s)\;{\rm d}s-\\ &\rho(t)\frac{B^{\rm T}P\,e(t)}{\|B^{\rm T}P\,e(t)\|}\end{aligned} \quad (15)$$

for all $\left\|B^{\rm T}P\,e(t)\right\|\neq 0$, and from (13) and adding the left sides of equations (14) to $\dot{V}(t)$, we get

$$\begin{aligned}&H[u(t),w(t),\hat{f}_1(t,e(t)),\hat{f}_2(t,e(t-h(t)))]=\\ &(C_1e(t)+C_2e(t-h(t))+C_3\int_{t-\tau(t)}^{t}e(s)\;{\rm d}s)^{\rm T}(C_1e(t)+\\ &C_2e(t-h(t))+C_3\int_{t-\tau(t)}^{t}e(s)\;{\rm d}s)-\gamma^2w(t)^{\rm T}w(t)+\\ &\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)+2q\,\rho(t)\dot{\rho}(t)\leqslant\\ &e(t)^{\rm T}\,(S+h_MR+\tau_M^2\,U_1+C_1^{\rm T}C_1+\Gamma_1^{\rm T}\Gamma_1)\,e(t)+\\ &2\dot{e}(t)^{\rm T}P\,e(t)+e(t-h(t))^{\rm T}(-(1-h_D)S+\Gamma_2^{\rm T}\Gamma_2+\\ &C_2^{\rm T}C_2)\,e(t-h(t))+2q\,\rho(t)\dot{\rho}(t)+\\ &2e(t)^{\rm T}C_1^{\rm T}C_2e(t-h(t))+2e(t)^{\rm T}C_1^{\rm T}C_3\int_{t-\tau(t)}^{t}e(s){\rm d}s+\\ &2e(t-h(t))^{\rm T}C_2^{\rm T}C_3\int_{t-\tau(t)}^{t}e(s){\rm d}s-\hat{f}_1(t,e(t))^{\rm T}\hat{f}_1(t,e(t))-\\ &\hat{f}_2(t,e(t-h(t)))^{\rm T}\hat{f}_2(t,e(t-h(t)))-\gamma^2w(t)^{\rm T}w(t)-\end{aligned}$$

$$(1-h_D)(\int_{t-h(t)}^{t} e(s)^{\rm T}{\rm d}s)R(\int_{t-h(t)}^{t} e(s){\rm d}s)+$$
$$(\int_{t-\tau(t)}^{t} e(s)^{\rm T}{\rm d}s)(-(1-\tau_D)U_1+C_3^{\rm T}C_3)\times$$
$$(\int_{t-\tau(t)}^{t} e(s){\rm d}s). \tag{16}$$

By using some matrix norm operations, the above inequality can be rewritten as follows:

$$H[u(t),\, w(t),\, \dot{f}_1(t,\, e(t)),\, \dot{f}_2(t,\, e(t-h(t)))] \leqslant$$
$$\chi(t)^{\rm T}\Pi\chi(t)+2\rho(t)e(t)^{\rm T}PB\frac{B^{\rm T}Pe(t)}{\|B^{\rm T}Pe(t)\|}+2q\rho(t)\dot{\rho}(t) \tag{17}$$

where

$$\chi(t)=\Big[e(t),\, e(t-h(t)),\, \int_{t-h(t)}^{t} e(s)\,{\rm d}s,\, \int_{t-\tau(t)}^{t} e(s)\,{\rm d}s,$$
$$\hat{f}_1(t,e(t)), \hat{f}_2(t,e(t-h(t))), w(t)\Big]$$

and

$$\Pi=\begin{bmatrix} \Pi_{11} & P(A_2-BK_2)+C_1^{\rm T}C_2 & 0 & P(A_3-BK_3)+C_1^{\rm T}C_3 & PN_1 & PN_2 & -PD \\ * & -(1-h_D)S+\Gamma_2^{\rm T}\Gamma_2+C_2^{\rm T}C_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & -(1-h_D)R & 0 & 0 & 0 & 0 \\ * & * & * & -(1-\tau_D)U_1+C_3^{\rm T}C_3 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} \tag{18}$$

where $\Pi_{11}=P(A_1-BK_1)+(A_1-BK_1)^{\rm T}P+S+h_MR+\tau_M^2U_1+C_1^{\rm T}C_1+\Gamma_1^{\rm T}\Gamma_1$. Substituting the relation (12) in (17), the inequality

$$H[u(t),w(t),\hat{f}_1(t,e(t)),\hat{f}_2(t,e(t-h(t)))] \leqslant \chi(t)^{\rm T}\Pi\chi(t) \tag{19}$$

is obtained. Let

$$\zeta={\rm diag}\,\{X,X,X,X,I,I,I\}$$

with $X=P^{-1}$. By premultiplying ζ and postmultiplying $\zeta^{\rm T}$ to the matrix inequality $\Pi<0$ and considering $\tilde{S}=XSX$, $\tilde{R}=XRX$, $\tilde{U}_1=XU_1X$, and $L_i=K_iX$, $i=1,2,3$, the LMI (10) is got by applying Schur complement lemma. Moreover, the condition $\Pi<0$ implies that either $J_\infty<0$ or

$$\dot{V}(t)\leqslant -e(t)^{\rm T}C_1^{\rm T}C_1e(t)\leqslant 0 \tag{20}$$

for $w(t)=0$. Then, we have $V(t)<V(0)$. It can be easily seen that (20) holds for all $\|B^{\rm T}P\,e(t)\|=0$ (with $u(t)=0$ and $\rho(t)=\rho_0$) as well. Integrating (20) from 0 to t, it yields

$$V(0)\geqslant V(t)+\int_0^t e(s)^{\rm T}C_1^{\rm T}C_1e(s){\rm d}s\geqslant\int_0^t e(s)^{\rm T}C_1^{\rm T}C_1e(s){\rm d}s. \tag{21}$$

Since the term $V(0)$ is positive and finite, the following limit exists and is finite:

$$\lim_{t\to\infty}\int_0^t e(s)^{\rm T}C_1^{\rm T}C_1e(s)\;{\rm d}s=\lim_{t\to\infty}\int_0^t \left|e(s)^{\rm T}C_1^{\rm T}C_1e(s)\right|\;{\rm d}s. \tag{22}$$

Thus, according to Barbalat lemma, we obtain

$$\lim_{t\to\infty}e(t)^{\rm T}C_1^{\rm T}C_1e(t)=0. \tag{23}$$

Then, the synchronization of master-slave systems with mixed time-delays and nonlinear perturbations is achieved under the neutral-delay-dependent adaptive synchronization law (11). Moreover, it is clear that the Lyapunov function (7) results in

$$V(t)\geqslant\lambda_{\min}(P)\;|e(t)|^2+\lambda_{\min}(S)\int_{t-h(t)}^{t}|e(\xi)|^{\rm T}\;{\rm d}\xi \tag{24}$$

and one gets

$$V(0)\leqslant\Delta\;\|\zeta\|^2 \tag{25}$$

where

$$\Delta=\lambda_{\max}(P)+h_M\lambda_{\max}(S)+\frac{1}{2}h_M^2\,\lambda_{\max}(R)+$$
$$\frac{1}{3}\tau_M^3\lambda_{\max}(U_1)+\frac{1}{6}\tau_M^3\lambda_{\max}(U_1).$$

Therefore, we have

$$|e(t)|^2+\varpi\int_0^t|e(t)|^2\;{\rm d}s\leqslant\frac{1}{\lambda_{\min}(P)}\,[\Delta\;\|\zeta\|^2+q\,\rho_0^2] \tag{26}$$

where $\varpi=\lambda_{\min}(C_1^{\rm T}C_1)/\lambda_{\min}(P)$. The substitution

$$Z(t)=\int_0^t|e(s)|^2\;{\rm d}s \tag{27}$$

reduces (26) to the following first-order differential inequality:

$$\dot{Z}(t)+\varpi\,Z(t)\leqslant\frac{1}{\lambda_{\min}(P)}\,[\Delta\;\|\zeta\|^2+q\,\rho_0^2] \tag{28}$$

where $Z(0)=0$. Solving the inequality (28), it gives

$$Z(t)\leqslant\frac{1-{\rm e}^{-\varpi\,t}}{\varpi\,\lambda_{\min}(P)}\,[\Delta\;\|\zeta\|^2+q\,\rho_0^2] \tag{29}$$

by transforming back to the variable $|e(t)|^2$ in (24), we find out

$$|e(t)|^2\leqslant|e(0)|^2+\frac{1}{\lambda_{\min}(P)}\,{\rm e}^{-\varpi\,t}[\Delta\;\|\zeta\|^2+q\,\rho_0^2] \tag{30}$$

or

$$|e(t)|\leqslant|e(0)|+{\rm e}^{-\varpi\frac{t}{2}}\left[\sqrt{\frac{\Delta}{\lambda_{\min}(P)}}\;\|\zeta\|+\sqrt{\frac{q}{\lambda_{\min}(P)}}\;|\rho_0|\right] \tag{31}$$

which indicates that the difference operator of the synchronization error system $e(t)$ is globally exponentially bounded with an exponential decay rate $\varpi/2$, which depends on the matrices C_1 and P. Therefore, synchronization speed can be controlled by adjusting positive constants q and ρ_0. Furthermore, from (21), it is observed that $V(t)$ in (7) is bounded since $V(0)$ is finite. This implies that $e(t)$ and $\rho(t)$ are bounded for all $t > 0$. Moreover, the state $x_m(t)$ of the master model is always bounded, and then, it is concluded that the state $x_s(t)$ is also bounded. □

Corollary 1. The following master and slave systems without time delays are considered:

$$\begin{cases} \dot{x}_m(t) = A_1\, x_m(t) + N_1 f_1(t, x_m(t)) \\ z_m(t) = C_1 x_m(t) \end{cases} \tag{32}$$

and

$$\begin{cases} \dot{x}_s(t) = A_1\, x_s(t) + N_1 f_1(t, x_s(t)) + B\, u(t) + D\, w(t), \\ z_s(t) = C_1 x_s(t). \end{cases} \tag{33}$$

Under Assumptions 1–2 and for a given scalar $\gamma > 0$, the master-slave systems (32)–(33) can be synchronized when the adaptive synchronization controller is given by

$$u(t) = K_1\, e(t) - \rho(t) \sin(\Lambda_x) \tag{34}$$

with the adaptation law

$$\dot{\rho}(t) = q^{-1} \|\Lambda_x\|, \quad \rho(0) = \rho_0 \tag{35}$$

where $K_1 = L_1 X^{-1}$ and the positive constants q and ρ_0 are specified by the designer, and the matrix L_1 and the positive-definite matrix X are solutions of the following LMI:

$$\begin{bmatrix} \vec{\Pi}_{11} & N_1 & -D & XC_1^{\rm T} & X\Gamma_1^{\rm T} \\ * & -I & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{36}$$

with $\vec{\Pi}_{11} = A_1 X + X A_1^{\rm T} - BL_1 - L_1^{\rm T} B^{\rm T}$.

Proof. Similar to the proof of Theorem 1, we can easily derive the result. Its proof is straightforward and hence omitted. □

Remark 3. The results presented in Theorem 1 are dependent on the upper bounds of the time-varying discrete and the distributed delays and the upper bounds of their derivative as well. These results give a less conservative design than the available delay-independent results in [41]. Therefore, the treatment in the present paper is more general.

Remark 4. It is worth noting that the number of variables to be determined in the LMI (10) is $n\,(2\,n + 3\,r + 2)$. Therefore, for large dimensions "n" and/or "r" in the LMI (10), there are some efficient algorithms proposed in the literature to improve LMI relaxations of increasing dimensions in order to reduce the computational complexities (see related work [476, 47]). It is also observed that the LMI (10) is linear in the set of matrices L_1, L_2, L_3, X, $\tilde{S}$, $\tilde{R}$, and $\tilde{U}_1$, and the scalar γ^2. It implies that the scalar γ^2 can be included as one of the optimization variables in LMI (10) in order to obtain the minimum disturbance attenuation level. Then, the suboptimal solution for the adaptive synchronization problem can be found by solving the following convex optimization problem:

$$\begin{gathered} \text{Min} \quad \lambda \\ \text{s. t. LMI (10) with } \lambda = \gamma^2. \end{gathered}$$

4 Simulation results

In this section, we will verify the proposed methodology by giving two illustrative examples. We solved LMI (10) by using Matlab LMI control toolbox, which implements state-of-the-art interior-point algorithms and is significantly faster than classical convex optimization algorithms. The example is given as follows.

Example 1. Consider the master-slave systems (1)–(2) with the following state-space matrices for an aircraft model

$$A_1 = \begin{bmatrix} -0.2 & 0.01 \\ -0.1 & -0.5 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.1 & 0.2 \\ 0.01 & -0.1 \end{bmatrix}, A_3 = \begin{bmatrix} -0.3 & -0.1 \\ 0.1 & -0.15 \end{bmatrix}$$

$$B = D = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, N_1 = N_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C_1 = [\ 1 \quad 1\ ], \quad C_2 = C_3 = [\ 1 \quad 1\ ]$$

with

$$f_1(t, x(t)) = f_2(t, x(t)) = \begin{bmatrix} 0.5(|x_1(t)+1| - |x_1(t)-1|) \\ 0.5(|x_2(t)+1| - |x_2(t)-1|) \end{bmatrix}. \tag{37}$$

The delays $h(t) = \tau(t) = (1 - {\rm e}^{-t})/(1 - {\rm e}^{-t})(1 + {\rm e}^{-t})(1 + {\rm e}^{-t})$ satisfy $0 \leqslant h(t) = \tau(t) \leqslant 1$ and $\dot{h}(t) = \dot{\tau}(t) \leqslant 0.5$, where $x_m(t) = [x_{1m}(t),\ x_{2m}(t)]^{\rm T}$, $x_s(t) = [x_{1s}(t),\ x_{2s}(t)]^{\rm T}$.

It is required to design the synchronization signal (11) with the adaptive law (12) such that the trajectories of the slave subsystem and master subsystem (1)–(2) can be synchronized. To this end, in light of Theorem 1, we solved the LMI (10) for $\gamma = 0.8$ and obtained

$$X = \begin{bmatrix} 0.0042 & 0.0076 \\ 0.0076 & 0.0364 \end{bmatrix}.$$

For simulation purposes, we set values of the designed parameters as $q = 10$, $\rho_0 = 1$ with the following initial conditions:

$$x_m(t) = [1,\ -1]^{\rm T}, \qquad t \in [-1,\ 0]$$

$$x_s(t) = 0, \qquad t \in [-1,\ 0]$$

and an exogenous disturbance input is set as

$$w(t) = \frac{1}{1 + \sqrt{t}}, \quad t \geqslant 0.$$

Now, by applying the synchronization signal (11) with the adaptive law (12) and the parameters above, the temporal evolution of each variable of the master-slave systems $x_{1m}(t)$, $x_{2m}(t)$, $x_{1s}(t)$, $x_{2s}(t)$ with the related synchronization errors, i.e., $e(t) = x_s(t) - x_m(t)$, are shown in Figs. 1 and 2.It is indicated that the synchronization errors $e_1(t) = x_{s1}(t) - x_{m1}(t)$ and $e_2(t) = x_{s2}(t) - x_{m2}(t)$ converge to zero. Moreover, the adaptation parameter $\rho(t)$ is depicted in Fig. 3.

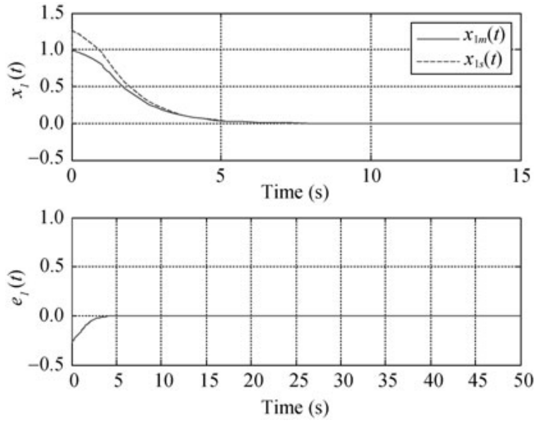

Fig. 1 Time responses of the first state of the master-slave systems and the related synchronization error

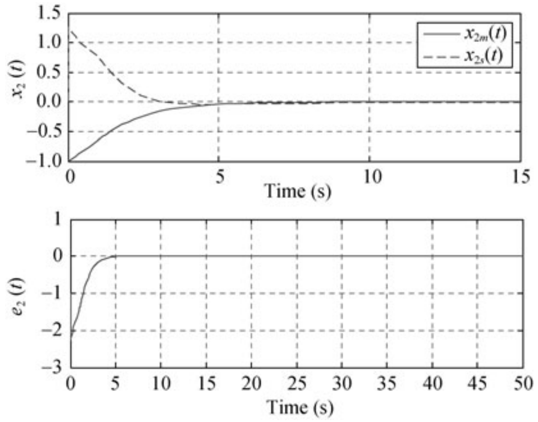

Fig. 2 Time responses of the second state of the master-slave systems and the related synchronization error

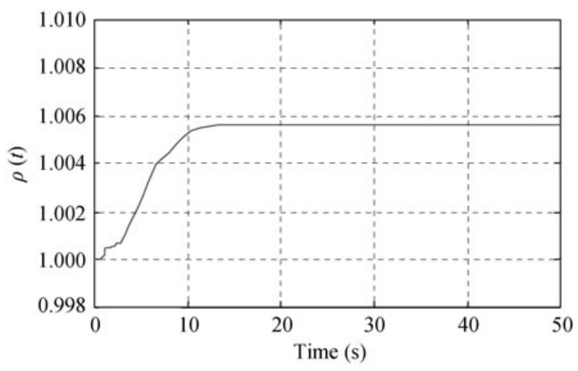

Fig. 3 Time response of the adaptation parameter

Example 2. Consider a master-slave chaotic neural network as follows[40]:

$$\begin{cases} \dot{x}_m(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_m(t) + \begin{bmatrix} 1+\frac{\pi}{4} & 20 \\ 0.1 & 1+\frac{\pi}{4} \end{bmatrix} f_1(t, x_m(t)) + \\ \qquad \begin{bmatrix} -\frac{1.3\pi\sqrt{2}}{4} & 0.1 \\ 0.1 & -\frac{1.3\pi\sqrt{2}}{4} \end{bmatrix} f_2(t, x_m(t-h)) \\ x_m(t) = \begin{bmatrix} 1 & -1 \end{bmatrix}^{\rm T}, \qquad t \in [-h,\ 0] \\ z_m(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_m(t) \end{cases} \tag{38}$$

and

$$\begin{cases} \dot{x}_s(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_s(t) + \begin{bmatrix} 1+\frac{\pi}{4} & 20 \\ 0.1 & 1+\frac{\pi}{4} \end{bmatrix} f_1(t, x_s(t)) + \\ \qquad \begin{bmatrix} -\frac{1.3\pi\sqrt{2}}{4} & 0.1 \\ 0.1 & -\frac{1.3\pi\sqrt{2}}{4} \end{bmatrix} f_2(t, x_s(t-h)) + \\ \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} w(t) \\ x_s(t) = \begin{bmatrix} -0.1 & 0.3 \end{bmatrix}^{\rm T}, \qquad t \in [-h,\ 0] \\ z_s(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x_s(t) \end{cases}$$

with $f_i(t, x(t))$ given in (37), $h = 1\,\rm s$, and $w(t)$ is a Gaussian noise with mean 0 and variance 1. With the above parameters, the delayed neural network systems (38) and (39) exhibit their chaotic behaviors, as shown in Fig. 4. By applying the conditions in Theorem 1 with constant parameters $q = 10$, $\rho_0 = 1$, and the disturbance attenuation $\gamma = 0.55$, we can obtain the following controller by using Matlab LMI control toolbox:

$$\begin{aligned} u(t) = 10^4 \begin{bmatrix} 5.3901 & 5.3908 \end{bmatrix} \times \\ e(t) + \begin{bmatrix} 10.1412 & 10.1164 \end{bmatrix} e(t-1) - \\ \rho(t) \frac{10^{-4} \begin{bmatrix} 0.1855 & 0.1855 \end{bmatrix} e(t)}{\left\| 10^{-4} \begin{bmatrix} 0.1855 & 0.1855 \end{bmatrix} e(t) \right\|} \end{aligned}$$

where

$$\dot{\rho}(t) = q^{-1} \left\| 10^{-4} \begin{bmatrix} 0.1855 & 0.1855 \end{bmatrix} e(t) \right\|, \quad \rho(0) = \rho_0.$$

The synchronization error between the master system and the slave system is shown in Figs. 5 and 6.

5 Conclusions

In this paper, an adaptive H_∞ synchronization problem was proposed for the master and slave structure of linear systems with nonlinear perturbations and mixed time-varying delays comprising different discrete and distributed time delays. Using an appropriate LKF, some delay-dependent sufficient conditions and an adaptation law that include the master-slave parameters were established for designing a delayed synchronization law in terms of linear matrix inequalities. The controller guarantees the H_∞ synchronization of the two coupled master and slave systems regardless of their initial states. Particularly, it was shown that the synchronization speed can be controlled by adjusting the updated gain of the synchronization signal. Two numerical examples were given to illustrate the effectiveness of the method.

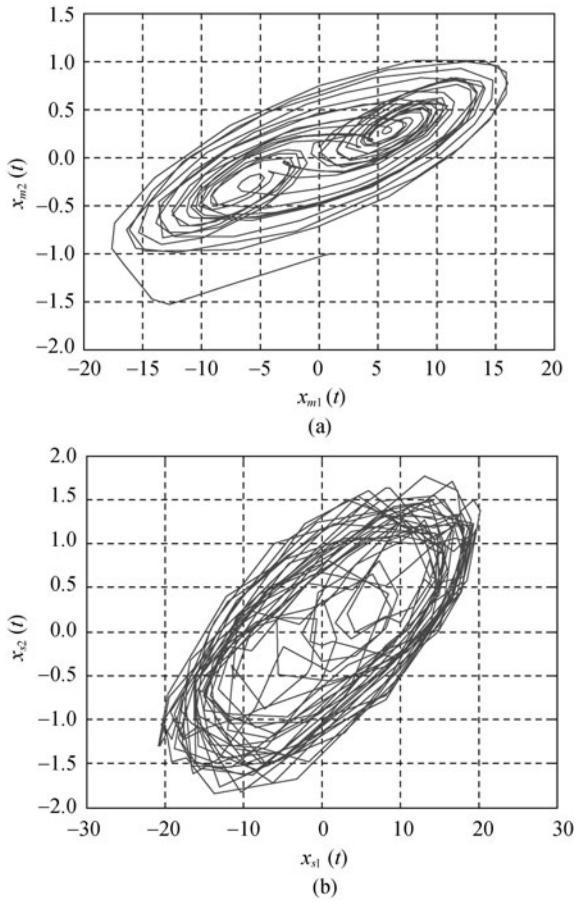

Fig. 4 (a) Plot of $x_{m1} - x_{s1}$; (b) Plot of $x_{m2} - x_{s2}$

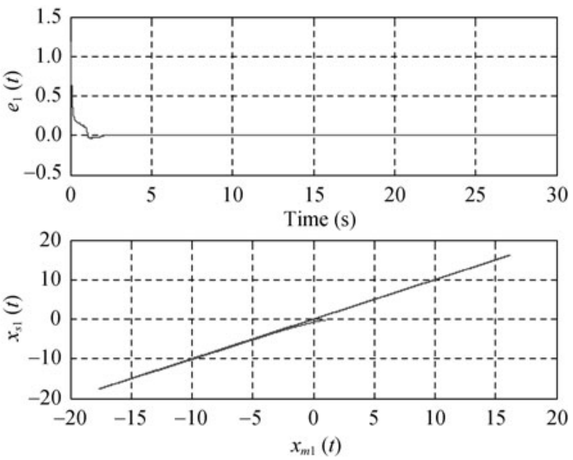

Fig. 5 The synchronization error $x_{m1}(t) - x_{s1}(t)$

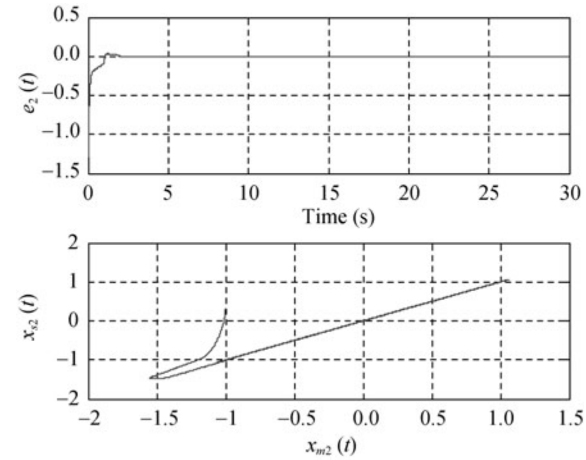

Fig. 6 The synchronization error $x_{m2}(t) - x_{s2}(t)$

References

[1] L. M. Pecora, T. L. Carroll. Synchronization in chaotic systems. *Physical Review Letters*, vol. 64, no. 8, pp. 821–824, 1990.

[2] H. Gao, J. Lam, G. Chen. New criteria for synchronization stability of general complex dynamical networks with coupling delays. *Physics Letters A*, vol. 360, no. 2, pp. 263–273, 2006.

[3] H. R. Karimi, P. Maass. Delay-range-dependent exponential H_∞ synchronization of a class of delayed neural networks. *Chaos, Solitons & Fractals*, vol. 41, no. 3, pp. 1125–1135, 2009.

[4] G. Wen, Q. G. Wang, C. Lin, X. Han, G. Li. Synthesis for robust synchronization of chaotic systems under output feedback control with multiple random delays. *Chaos, Solitons & Fractals*, vol. 29, no. 5, pp. 1142–1146, 2006.

[5] H. Lu, C. Leeuwen. Synchronization of chaotic neural networks via output or state coupling. *Chaos, Solitons & Fractals*, vol. 30, no. 1, pp. 166–176, 2006.

[6] J. Cao, J. Lu. Adaptive synchronization of neural networks with or without time-varying delay. *Chaos*, vol. 16, no. 1, pp. 313–318, 2006.

[7] T. L. Liao, S. H. Tsai. Adaptive synchronization of chaotic systems and its application to secure communication. *Chaos, Solitons & Fractals*, vol. 11, no. 9, pp. 1387–1396, 2000.

[8] A. L. Fradkov, H. Nijmeijer, A. Markov. Adaptive observer-based synchronization for communications. *International Journal of Bifurcation and Chaos*, vol. 10, no. 12, pp. 2807–2813, 2000.

[9] J. H. Park. Synchronization of Genesio chaotic system via backstepping approach. *Chaos, Solitons & Fractals*, vol. 27, no. 5, pp. 1369–1375, 2006.

[10] J. J. Yan, M. L. Hung, T. Y. Chiang, Y. S. Yang. Robust synchronization of chaotic systems via adaptive sliding mode control. *Physics Letters A*, vol. 356, no. 3, pp. 220–225, 2006.

[11] C. Cai, G. Chen. Synchronization of complex dynamical networks by the incremental ISS approach. *Physica A: Statistical and Theoretical Physics*, vol. 371, no. 2, pp. 754–766, 2006.

[12] J. K. Hale, S. M. V. Lunel. *Introduction to Functional Differential Equations*, New York, USA: Springer Verlag, 1993.

[13] Z. Wang, H. Shu, Y. Liu, D. W. C. Ho, X. Liu. Robust stability analysis of generalized neural networks with discrete and distributed time delays. *Chaos, Solitons & Fractals*, vol. 30, no. 4, pp. 886–896, 2006.

[14] Z. Wang, Y. Liu, X. Liu. On global asymptotic stability of neural networks with discrete and distributed delays. *Physics Letters A*, vol. 345, no. 4–6, pp. 299–308, 2005.

[15] H. Gao, T. Chen, J. Lam. A new delay system approach to network-based control. *Automatica*, vol. 44, no. 1, pp. 39–52, 2008.

[16] Y. Wang, L. Xie, C. E. Souza. Robust control of a class of uncertain nonlinear systems. *Systems and Control Letters*, vol. 19, no. 2, pp. 139–149, 1992.

[17] H. R. Karimi, H. Gao. LMI-based delay-dependent mixed H_2/H_∞ control of second-order neutral systems with time-varying state and input delays. *ISA Transactions*, vol. 47, no. 3, pp. 311–324, 2008.

[18] E. Fridman. New Lyapunov-Krasovskii functionals for stability of linear retarded and neutral type systems. *Systems & Control Letters*, vol. 43, no. 4, pp. 309–319, 2001.

[19] Q. L. Han, L. Yu. Robust stability of linear neutral systems with nonlinear parameter perturbations. *IEE Proceedings of Control Theory and Applications*, vol. 151, no. 5, pp. 539–546, 2004.

[20] S. Mou, H. Gao, J. Lam, W. Qiang. A new criterion of delay-dependent asymptotic stability for Hopfield neural networks with time delay. *IEEE Transactions on Neural Networks*, vol. 19, no. 3, pp. 532–535, 2008.

[21] S. Mou, H. Gao, W. Qiang, K. Chen. New delay-dependent exponential stability for neural networks with time delay. *IEEE Transactions on Systems, Man, and Cybernetics–Part B: Cybernetics*, vol. 38, no. 2, pp. 571–576, 2008.

[22] P. Park. A delay-dependent stability criterion for systems with uncertain time-invariant delays. *IEEE Transactions on Automatic Control*, vol. 44, no. 4, pp. 876–877, 1999.

[23] Y. He, Q. G. Wang, C. Lin, M. Wu. Delay-range-dependent stability for systems with time-varying delay. *Automatica*, vol. 43, no. 2, pp. 371–376, 2007.

[24] Y. S. Moon, P. Park, W. H. Kwon, Y. S. Lee. Delay-dependent robust stabilization of uncertain state-delayed systems. *International Journal of Control*, vol. 74, no. 14, pp. 1447–1455, 2001.

[25] J. H. Park, O. Kwon. On new stability criterion for delay differential systems of neutral type. *Applied Mathematics and Computation*, vol. 162, no. 2, pp. 627–637, 2005.

[26] X. Sun, Q. L. Zhang, C. Y. Yang, Z. Su, Y. Y. Shao. An improved approach to delay-dependent robust stabilization for uncertain singular time-delay systems. *International Journal of Automation and Computing*, vol. 7, no. 2, pp. 205–212, 2010.

[27] Y. Y. Wu, T. Li, Y. Q. Wu. Improved exponential stability criteria for recurrent neural networks with time-varying discrete and distributed delays. *International Journal of Automation and Computing*, vol. 7, no. 2, pp. 199–204, 2010.

[28] G. D. Zong, J. Liu. New delay-dependent global asymptotic stability condition for hopfield neural networks with time-varying delays. *International Journal of Automation and Computing*, vol. 6, no. 4, pp. 415–419, 2009.

[29] H. R. Karimi. Observer-based mixed H_2/H_∞ control design for linear systems with time-varying delays: An LMI approach. *International Journal of Control, Automation, and Systems*, vol. 6, no. 1, pp. 1–14, 2008.

[30] Z. Wang, Y. Liu, L. Yu, X. Liu. Exponential stability of delayed recurrent neural networks with Markovian jumping parameters. *Physics Letters A*, vol. 356, no. 4–5, pp. 346–352, 2006.

[31] X. Jiang, Q. L. Han. Delay-dependent robust stability for uncertain linear systems with interval time-varying delay. *Automatica*, vol. 42, no. 6, pp. 1059–1065, 2006.

[32] J. Lam, H. Gao, C. Wang. H_∞ model reduction of linear systems with distributed delay. *IEE Proceedings–Control Theory and Applications*, vol. 152, no. 6, pp. 662–674, 2005.

[33] J. Zhang, P. Shi, J. Qiu. Robust stability criteria for uncertain neutral system with time delay and nonlinear uncertainties. *Chaos, Solitons & Fractals*, vol. 38, no. 1, pp. 160–167, 2008.

[34] L. Zhang, E. K. Boukas, A. Haidar. Delay-range-dependent control synthesis for time-delay systems with actuator saturation. *Automatica*, vol. 44, no. 10, pp. 2691–2695, 2008.

[35] L. Zhang, E. K. Boukas, J. LAM. Analysis and synthesis of Markov jump linear systems with time-varying delays and partially known transition probabilities. *IEEE Transactions on Automatic Control*, vol. 53, no. 10, pp. 2458–2464, 2008.

[36] H. R. Karimi, H. Gao. New delay-dependent exponential H_∞ synchronization for uncertain neural networks with mixed time delays. *IEEE Transactions on Systems, Man, Cybernetics–Part B: Cybernetics*, vol. 40, no. 1, pp. 173–185, 2010.

[37] Y. Sun. J. Cao, Z. Wang. Exponential synchronization of stochastic perturbed chaotic delayed neural networks. *Neurocomputing*, vol. 70, no. 13–15, pp. 2477–2485, 2007.

[38] Y. Wang, Z. Wang, J. Liang. A delay fractioning approach to global synchronization of delayed complex networks with stochastic disturbances. *Physics Letters A*, vol. 372, no. 39, pp. 6066–6073, 2008.

[39] L. Wang, J. Cao. Global robust point dissipativity of interval neural networks with mixed time-varying delays. *Nonlinear Dynamics*, vol. 55, no. 1–2, pp. 169–178, 2009.

[40] L. Sheng, H. Z. Yang. H_∞ synchronization of chaotic systems via delayed feedback control. *International Journal of Automation and Computing*, vol. 7, no. 2, pp. 230–235, 2010.

[41] J. J. Yan, W. D. Chang, M. L. Hung. An adaptive decentralized synchronization of master-slave large-scale systems with unknown signal propagation delays. *Chaos, Solitons & Fractals*, vol. 29, no. 2, pp. 506–513, 2006.

[42] J. Lu, J. Cao. Adaptive complete synchronization of two identical or different chaotic (hyperchaotic) systems with fully unknown parameters. *Chaos*, vol. 15, no. 4, 2005. doi: 10.1063/1.2089207.

[43] J. Lu, J. Cao, D. W. C. Ho. Adaptive stabilization and synchronization for chaotic Lur′e systems with time-varying delay. *IEEE Transactions on Circuits and Systems*, vol. 55, no. 5, pp. 1347–1356, 2008.

[44] A. M. Pertew, H. J. Marquez, Q. Zhao. H_∞ synthesis of unknown input observers for non-linear Lipschitz systems. *International Journal of Control*, vol. 78, no. 15, pp. 1155–1165, 2005.

[45] V. M. Popov. *Hyperstability of Control System*, Berlin, Germany: Springer-Verlag, 1973.

[46] S. Kim, M. Kojima, H. Waki. Generalized lagrangian duals and sums of squares relaxations of sparse polynomial optimization problems. *SIAM Journal on Optimization*, vol. 15, no. 3, pp. 697–719, 2005.

[47] J. B. Lasserre. Global optimization with polynomials and the problems of moments. *SIAM Journal on Optimization*, vol. 11, no. 3, pp. 796–817, 2001.

Hamid Reza Karimi is a professor in control systems at the Faculty of Engineering and Science of the University of Agder in Norway. He is a senior member of IEEE and serves as a chairman of the IEEE chapter on control systems at IEEE Norway section. He is also serving as an editorial board member for some international journals, such as *Mechatronics, Journal of the Franklin Institute, Neurocomputing, Mathematical Problems in Engineering, International Journal of Control, Automation and Systems, Journal of Innovative Computing Information and Control-Express Letters, International Journal of Control Theory and Applications,* etc. He is a member of IEEE Technical Committee on Systems with Uncertainty, IFAC Technical Committee on Robust Control and IFAC Technical Committee on Automotive Control. He is the recipient of the Juan de la Cierva Research Award in 2008, Alexander-von-Humboldt-Stiftung Research Fellowship in 2006, German Academic Exchange Service (DAAD) Research Fellowship in 2003, National Presidency Prize for Distinguished Ph. D. student of Electrical Engineering in 2005, National Students Book Agency′s Award for Distinguished Research Thesis in 2007, etc.

His research interests include nonlinear systems, networked control systems, robust control/filter design, time-delay systems, wavelets and vibration control of flexible structures with an emphasis on applications in engineering.

E-mail: hamid.r.karimi@uia.no