H_∞ Filter Design for a Class of Nonlinear Neutral Systems with Time-Varying Delays

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Abstract—In this note, the problem of H_∞ filtering for a class of nonlinear neutral systems with delayed states and outputs is investigated. By introducing a descriptor technique, using Lyapunov-Krasovskii functional and a suitable change of variables, new required sufficient conditions are established in terms of delay-dependent linear matrix inequalities (LMIs) for the existence of the desired H_∞ filters. The explicit expression of the filters is derived to satisfy both asymptotic stability and a prescribed level of disturbance attenuation for all admissible known nonlinear functions. A numerically example is provided to show the proposed design approach.

Index Terms—Neutral systems; H_∞ filtering; nonlinearity; LMI; time-delay.

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I. INTRODUCTION

Delay (or memory) systems represent a class of infinite-dimensional systems [1, 2] largely used to describe propagation and transport phenomena or population dynamics [3, 4]. Delay differential systems are assuming an increasingly important role in many disciplines like economic, mathematics, science, and engineering. For instance, in economic systems, delays appear in a natural way since decisions and effects are separated by some time interval. The presence of a delay in a system may be the result of some essential simplification of the corresponding process model. The delay effects problem on the (closed-loop) stability of (linear) systems including delays in the state and/or input is a problem of recurring interest since the delay presence may induce complex behaviors (oscillation, instability, bad performances) for the (closed-loop) schemes [2, 5].

Neutral delay systems constitute a more general class than those of the retarded type. It is important to point out that the highest order derivative of a retarded differential equation does not contain any delayed variables. When such a term does appear, then we have a differential equation of neutral type. Stability of these systems proves to be a more complex issue because the system involves the derivative of the delayed state. Especially, in the past few decades increased attention has been devoted to the problem of robust delay-independent stability or delay-dependent stability and stabilization via different approaches for linear neutral systems with delayed state and/or input and parameter uncertainties (see for instance [2, 6, 7]). Among the past results on neutral delay systems, the LMI approach is an efficient method to solve many control problems such as stability analysis and stabilization [8-13], H_∞ control problems [14-20] and guaranteed-cost (observer-based) control design [21-25].

On the other hand, the state estimation problem has been one of the fundamental issues in the control area and there have been many works following those of Kalman filter or H_2 optimal estimators (in the stochastic framework) and Luenberger filter (in the deterministic framework) [26]. Nevertheless there has been an increasing interest in the robust H_∞ filtering, which is concerned with the design of an estimator ensuring that the L_2-induced gain from the noise signal to the estimation error is less than a prescribed level, in the past years [27-31]. Compared with the conventional Kalman filtering, the H_∞ filter technique has several advantages. First, the noise sources in the H_∞ filtering setting are arbitrary signals with bounded energy or average power, and no exact statistics are required to be known [32]. Second, the H_∞ filter has been shown to be much more robust to parameter uncertainty in a control system. These advantages render the H_∞ filtering approach very appropriate to some practical applications. When parameter uncertainty arises in a system model, the robust H_∞ filtering problem has been studied, and a great number of results on this topic have been reported (see the references [33, 34]). In the case when parameter uncertainty and time delays appear simultaneously in a system model, the robust H_∞ filtering problem was dealt with in [35] via LMI approach, respectively. The corresponding results for uncertain discrete delay systems can be found in [36]. However, it is noted that the H_∞ filtering of nonlinear neutral systems has not been been fully investigated in the past and remains to be important and challenging. This motivates the present study.

In this paper, we are concerned to develop a new delay-dependent stability criterion for H_∞ filtering problem of nonlinear neutral systems with known nonlinear functions which satisfy the Lipschitz conditions. The main merit of the proposed method is the fact that it provides a convex problem with additional degree of freedom which lead to less conservative results. Our analysis is based on the Hamiltonian-Jacoby-Isaac (HJI) method. By introducing a descriptor technique, using Lyapunov-Krasovskii functional and a suitable change of variables, we establish new required sufficient conditions in terms of delay-dependent LMIs under which the desired H_∞ filters exist, and derive the explicit expression of these filters to satisfy both asymptotic stability and H_∞ performance. A desired filter can be constructed through a convex optimization problem, which can be solved by using standard numerical algorithms. Finally, a numerical example is given to illustrate the proposed design method.

Notations. The superscript $'T'$ stands for matrix transposition; $\Re^n$ denotes the n-dimensional Euclidean space; $\Re^{n\times m}$ is the set of all real m by n matrices. $\|.\|$ refers to the Euclidean vector norm or the induced matrix 2-norm. $col\{\cdots\}$ and $sym(A)$ represent, respectively, a column vector and the matrix $A+A^T$. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote, respectively, the smallest and largest eigenvalue of the square matrix A. The notation $P>0$ means that P is real symmetric and positive definite; the symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix. In addition, $L_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$. Matrices, if the dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

II. PROBLEM DESCRIPTION

We consider a class of nonlinear neutral systems with delayed states and outputs represented by

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1\,x(t-h(t)) + A_2\,\dot{x}(t-d(t)) + E_1\,f(x(t)) + E_2\,f(x(t-h(t))) + B_1\,w(t) \\ x(t) = \varphi(t) \qquad t \in \left[-\max\{h_1, d_1\},\, 0\right] \\ z(t) = C_1\,x(t) \\ y(t) = C_2\,x(t) + g(t, x(t)) \end{cases} \tag{1}$$

where $x(t)\in\Re^n$, $w(t)\in L_2^s[0,\infty)$, $z(t)\in\Re^z$ and $y(t)\in\Re^p$ are corresponded to state vector, disturbance input, estimated output and measured output. The time-varying function $\varphi(t)$ is continuous vector valued initial function and the parameters $h(t)$ and $d(t)$ are time-varying delays satisfying

$$0\le h(t)\le h_1, \qquad \dot{h}(t)\le h_2$$

$$0\le d(t)\le d_1, \qquad \dot{d}(t)\le d_2 <1.$$

Assumption 1:

1) The nonlinear function $f:\Re^n \to \Re^n$ is continuous and satisfies $f(0)=0$ and the Lipschitz condition, i.e.,$\|f(x_0)-f(y_0)\| \le \|U_1(x_0-y_0)\|$ for all $x_0, y_0 \in \Re^n$ and U_1 is a known matrix.

2) The nonlinear function $g:\Re \times \Re^n \to \Re^p$ is continuous and satisfies the Lipschitz condition, i.e., $\|g(t,x_0)-g(t,y_0)\| \le \|U_2(x_0-y_0)\|$ for all $x_0, y_0 \in \Re^n$ and U_2 is a known matrix.

In this paper, the author's attention will be focused on the design of an $n-$th order delay-dependent H_∞ filter with the following state-space equations

$$\begin{cases} \dot{\hat{x}}(t) = F\,\hat{x}(t) + F_1\,\hat{x}(t-h(t)) + F_2\,\dot{\hat{x}}(t-d(t)) + F_3\,f(\hat{x}(t)) + F_4\,f(\hat{x}(t-h(t))) \\ \qquad\qquad + G\,(y(t) - C_2\,\hat{x}(t) - g(t,\hat{x}(t))) \\ \hat{x}(t) = 0 \qquad t \in \left[-\max\{h,d\},\, 0\right] \\ \hat{z}(t) = G_1\,\hat{x}(t) \end{cases} \tag{2}$$

where the state-space matrices F, F_1, F_2, F_3, F_4, G and G_1 of the appropriate dimensions are the filter design objectives to be determined. In the absence of $w(t)$, it is required that

$$\|x(t)-\hat{x}(t)\|_2 \to 0 \quad as \quad t \to \infty$$

where $\hat{x}(t) \in \Re^n$ and $\hat{z}(t)$ are the estimation of $x(t)$ and of $z(t)$, respectively, and $e(t) = x(t) - \hat{x}(t)$ is the estimation error. Then, the error dynamics between (1) and (2) can be expressed by

$$\begin{aligned} \dot{e}(t) &= (A-F)\,\hat{x}(t) + (A_1 - F_1)\,\hat{x}(t-h(t)) + (A_2 - F_2)\,\dot{\hat{x}}(t-d(t)) + (F - GC_2)e(t) \\ &\quad + F_1 e(t-h(t)) + F_2\dot{e}(t) - G\,\psi(t,e(t)) + (E_1 - F_3)\,f(x(t)) + (E_2 - F_4)\,f(x(t-h(t))) \\ &\quad + F_3\,\phi(e(t)) + F_4\,\phi(e(t-h(t))) + B_1 w(t) \end{aligned} \tag{3}$$

where $\phi(e(t)) := f(x(t)) - f(x(t)-e(t))$ and $\psi(t,e(t)) := g(t,x(t)) - g(t,x(t)-e(t))$. Now, we obtain the following state-space model, namely filtering error system:

$$
\begin{cases}
\dot{X}(t) = \hat{A}\,X(t) + \hat{A}_1\ X(t-h(t)) + \hat{A}_2\ \dot{X}(t-d(t)) + \hat{G}\,\psi(t,e(t)) + \hat{E}_1\ f(x(t)) + \hat{E}_2\ f(x(t-h(t))) \\
\qquad + \hat{E}_3\ \phi(e(t)) + \hat{E}_4\phi(e(t-h(t))) + \hat{B}\,w(t) \\
X(t) = \begin{bmatrix}\varphi(t)^T & \varphi(t)^T\end{bmatrix}^T \qquad t \in [-\max\{h_1, d_1\},\ 0] \\
z(t) - \hat{z}(t) = \hat{C}_1\ X(t)
\end{cases} \tag{4}
$$

where $X(t) = col\{x(t), e(t)\}$, $\hat{A} = \begin{bmatrix} A & 0 \\ A-F & F-GC_2 \end{bmatrix}$, $\hat{A}_1 := \begin{bmatrix} A_1 & 0 \\ A_1 - F_1 & F_1 \end{bmatrix}$, $\hat{A}_2 := \begin{bmatrix} A_2 & 0 \\ A_2 - F_2 & F_2 \end{bmatrix}$, $\hat{B} := \begin{bmatrix} B_1 \\ B_1 \end{bmatrix}$,

$\hat{G} := \begin{bmatrix} 0 \\ -G \end{bmatrix}$, $\hat{E}_1 := \begin{bmatrix} E_1 \\ E_1 - F_3 \end{bmatrix}$, $\hat{E}_2 := \begin{bmatrix} E_2 \\ E_2 - F_4 \end{bmatrix}$, $\hat{E}_3 := \begin{bmatrix} 0 \\ F_3 \end{bmatrix}$, $\hat{E}_4 := \begin{bmatrix} 0 \\ F_4 \end{bmatrix}$ and $\hat{C}_1 := \begin{bmatrix} C_1 - G_1 & G_1 \end{bmatrix}$.

Let

$$
s(\alpha,\beta) = \begin{cases} \dfrac{f(\alpha) - f(\beta)}{\alpha - \beta} & \alpha \neq \beta \\ \delta & \alpha = \beta \end{cases}, \quad \alpha, \beta \in \Re \tag{5}
$$

By Assumption 1, it is easy to see

$$
\begin{aligned}
\phi(e(t)) - \phi(e(t-h(t))) &= s(t)\,(e(t) - e(t-h(t))) \\
&= s(t) \int_{t-h(t)}^{t} \dot{e}(s)\ ds
\end{aligned} \tag{6}
$$

Therefore, from the Leibniz-Newton formula, i.e., $x(t) - x(t-h) = \int_{t-h}^{t} \dot{x}(s)\ ds$, the filtering error system (4) can be represented in a descriptor model form as

$$
\begin{cases}
\dot{X}(t) = \eta(t) \\
\eta(t) = (\hat{A} + \hat{A}_1)X(t) + \hat{A}_2\ \eta(t-d) + \hat{G}\,\psi(t,e(t)) + \hat{E}_1\ f(x(t)) + \hat{E}_2\ f(x(t-h(t))) \\
\qquad + \hat{E}_3\ \phi(e(t)) - (\hat{A}_1 + \hat{E}_4 J\ s(t)) \displaystyle\int_{t-h(t)}^{t} \eta(s)\ ds + \hat{B}\,w(t)
\end{cases} \tag{7}
$$

Definition 1:

1. The delay-dependent H_∞ filter of the type (2) is said to achieve asymptotic stability in the Lyapunov sense for $w(t) = 0$ if the augmented system (4) is asymptotically stable for all admissible nonlinear functions $f(x(t))$ and $g(t, x(t))$.

2. The delay-dependent H_∞ filter of the type (2) is said to guarantee robust disturbance attenuation if under zero initial condition

$$\sup_{\|w\|_2 \neq 0} \frac{\|z(t) - \hat{z}(t)\|_2}{\|w(t)\|_2} \leq \gamma \tag{8}$$

holds for all bounded energy disturbances and a prescribed positive value γ.

The filtering problem we address here is as follows: *Given a prescribed level of disturbance attenuation* $\gamma > 0$*, find the delay-dependent* H_∞ *filter (2) in the sense of Definition 1.*

Before ending this section, we recall a well-known lemma, which will be used in the proof our main results.

Lemma 1 ([7]): For any arbitrary column vectors $a(t)$, $b(t)$, matrices $\Phi(t)$, H, U and W the following inequality holds:

$$-2\int_{t-r}^{t} a(s)^T \Phi(s) b(s)\, ds \leq \int_{t-r}^{t} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix}^T \begin{bmatrix} H & U - \Phi(s) \\ * & W \end{bmatrix} \begin{bmatrix} a(s) \\ b(s) \end{bmatrix} ds$$

where $\begin{bmatrix} H & U \\ * & W \end{bmatrix} \geq 0$.

III. H_∞ FILTER DESIGN

In this section, both the asymptotic stability and H_∞ performance of the filtering error system is investigated such a sufficient stability condition is derived for the existence of the filter (2). The approach employed here is to develop a criterion for the existence of such filter based on the LMI approach combined with the Lyapunov method. In the literature, extensions of the quadratic Lyapunov functions to the quadratic Lyapunov-Krasovskii functionals have been proposed for time-delayed systems (see for instance the references [2, 6, 7, 23, 25] and the references therein).

We choose a Lyapunov-Krasovskii functional candidate for the nonlinear neutral system (1) as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{9}$$

where

$$V_1(t) = X(t)^T P_1 X(t) = \begin{bmatrix} X(t) \\ \eta(t) \end{bmatrix}^T T\,P \begin{bmatrix} X(t) \\ \eta(t) \end{bmatrix}$$

$$V_2(t) = \int_{t-h(t)}^{t} X(s)^T\, Q_1\, X(s)\; ds + \int_{t-d(t)}^{t} \eta(s)^T\, Q_2\, \eta(s)\; ds$$

$$V_3(t) = \int_{t-h_1}^{t} \int_{s}^{t} \eta(\theta)^T (Q_3 + Q_4)\, \eta(\theta) \quad d\theta\, ds$$

with

$$P := \begin{bmatrix} P_1 & 0 \\ P_3 & P_2 \end{bmatrix}, P_1 = P_1^T > 0\,,\; T := \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}. \tag{10}$$

In the following, we state our main results in terms of LMIs on the delay-dependent H_∞ filter design for the nonlinear neutral system (1) based on Lyapunov stability theory.

Theorem 1: Consider system (1) and let the matrices U_1, U_2 and the scalars $h_1, d_1 > 0, d_2 < 1, h_2$ and $\gamma > 0$ be given scalars. If there exist the matrices $P_{11}, P_{12}, P_{22}, G_1, H, U, W_1, \cdots, W_6, M_1, \cdots, M_9$, the positive definite matrices $P_1, Q_1, \cdots, Q_4$ and the scalar ε, satisfying the following LMIs

$$\left[\begin{array}{ccccc} [1,1] & -U - M_1 + \begin{bmatrix} \varepsilon\Sigma_2 \\ \Sigma_2 \end{bmatrix} + J^T M_2^T & \begin{bmatrix} \varepsilon\Sigma_3 \\ \Sigma_3 \end{bmatrix} + J^T M_3^T & \begin{bmatrix} \varepsilon\Sigma_4 \\ \Sigma_4 \end{bmatrix} + J^T M_4^T & \begin{bmatrix} \varepsilon\Sigma_5 \\ \Sigma_5 \end{bmatrix} + J^T M_5^T \\ * & [2,2] & -M_3^T & -M_4^T & -M_5^T \\ * & * & -(1-d_2)Q_2 & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{array}\right.$$

$$
\begin{bmatrix}
\begin{bmatrix} \varepsilon(\Sigma_6-\Sigma_7) \\ \Sigma_6-\Sigma_7 \end{bmatrix}+J^T M_6^T & \begin{bmatrix} \varepsilon\Sigma_7 \\ \Sigma_7 \end{bmatrix}+J^T M_7^T & \begin{bmatrix} \varepsilon\Sigma_8 \\ \Sigma_8 \end{bmatrix}+J^T M_8^T & \begin{bmatrix} \varepsilon\Sigma_9 \\ \Sigma_9 \end{bmatrix}+J^T M_9^T & J^T\hat{C}_1^T \\
-M_6^T & -M_7^T & -M_8^T & -M_9^T & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-I & 0 & 0 & 0 & 0 \\
* & -I & 0 & 0 & 0 \\
* & * & -I & 0 & 0 \\
* & * & * & -\gamma^2 I & 0 \\
* & * & * & * & -I
\end{bmatrix} < 0
\tag{11a}
$$

$$
\begin{bmatrix} H & U \\ * & Q_3 \end{bmatrix} \geq 0 \tag{11b}
$$

where

$$
[1,1] = sym\left\{\begin{bmatrix} \varepsilon(\Sigma_1+\Sigma_2) & P_1-\varepsilon\begin{bmatrix} P_{11}^T & P_{22}^T \\ P_{12}^T & P_{22}^T \end{bmatrix} \\ \Sigma_1+\Sigma_2 & -\begin{bmatrix} P_{11}^T & P_{22}^T \\ P_{12}^T & P_{22}^T \end{bmatrix} \end{bmatrix}\right\} - sym\left\{\begin{bmatrix} \varepsilon\Sigma_2 \\ \Sigma_2 \end{bmatrix}J-(U+M_1)J\right\}+h_1 H
$$

$$
+\begin{bmatrix} Q_1+J^T U_1^T U_1 J & 0 \\ 0 & Q_2+h_1(Q_3+Q_4)+\hat{J}^T(U_1^T U_1+U_2^T U_2)\hat{J} \end{bmatrix},
$$

$$
[2,2] = -(1-h_2)Q_1 - sym\{M_2\} + J^T U_1^T U_1 J + \hat{J}^T U_1^T U_1 \hat{J},
$$

$$
\Sigma_1 := \begin{bmatrix} (P_{11}^T+P_{22}^T)A-W_1 & W_1-W_6C_2 \\ (P_{11}^T+P_{22}^T)A-W_1 & W_1-W_6C_2 \end{bmatrix},\ \Sigma_2 := \begin{bmatrix} (P_{11}^T+P_{22}^T)A_1-W_2 & W_2 \\ (P_{11}^T+P_{22}^T)A_1-W_2 & W_2 \end{bmatrix},
$$

$$
\Sigma_3 := \begin{bmatrix} (P_{11}^T+P_{22}^T)A_2-W_3 & W_3 \\ (P_{11}^T+P_{22}^T)A_2-W_3 & W_3 \end{bmatrix}, \Sigma_4 := \begin{bmatrix} (P_{11}^T+P_{22}^T)E_1 \\ (P_{12}^T+P_{22}^T)E_1 \end{bmatrix}-\Sigma_6,\ \Sigma_5 := \begin{bmatrix} (P_{11}^T+P_{22}^T)E_2 \\ (P_{12}^T+P_{22}^T)E_2 \end{bmatrix}-\Sigma_7,
$$

$$
\Sigma_6 := \begin{bmatrix} W_4 \\ W_4 \end{bmatrix},\ \Sigma_7 := \begin{bmatrix} W_5 \\ W_5 \end{bmatrix},\ \Sigma_8 := -\begin{bmatrix} W_6 \\ W_6 \end{bmatrix},\ \Sigma_9 := \begin{bmatrix} (P_{11}^T+P_{22}^T)B_1 \\ (P_{12}^T+P_{22}^T)B_1 \end{bmatrix}.
$$

with $J := [I,\ 0]$ and $\hat{J} := [0,\ I]$, then there exists a delay-dependent H_∞ filter of the type (2) which achieve the asymptotic stability and H_∞ performance, simultaneously, in the sense of Definition 1. Moreover, the state-space matrices of the filter are given by

$$\begin{bmatrix} F & F_1 & F_2 & F_3 & F_4 & G \end{bmatrix} := (P_{22}^T)^{-1} \begin{bmatrix} W_1 & W_2 & W_3 & W_4 & W_5 & W_6 \end{bmatrix} \text{ and } G_1 \text{ from LMIs (11).} \tag{12}$$

Proof: Differentiating $V_1(t)$ in t along the trajectory of the filtering error system (4) we obtain

$$\begin{aligned}
\dot{V}_1(t) &= 2X(t)^T P_1 \dot{X}(t) = 2\begin{bmatrix} X(t) \\ \eta(t) \end{bmatrix}^T P^T \begin{bmatrix} \dot{X}(t) \\ 0 \end{bmatrix} \\
&= 2\begin{bmatrix} X(t) \\ \eta(t) \end{bmatrix}^T P^T \begin{bmatrix} \eta(t) \\ \left(\begin{array}{l} -\eta(t) + (\hat{A} + \hat{A}_1)X(t) + \hat{A}_2\, \eta(t-d(t)) + \hat{G}\psi(t,e(t)) + \hat{E}_1\, f(x(t)) \\ + \hat{E}_2\, f(x(t-h(t))) + \hat{E}_3\, \phi(e(t)) - (\hat{A}_1 + \hat{E}_4 J\, s(t)) \int_{t-h(t)}^{t} \eta(s)\, ds + \hat{B}\, w(t) \end{array} \right) \end{bmatrix} \\
&= 2\begin{bmatrix} X(t) \\ \eta(t) \end{bmatrix}^T P^T \left(\begin{array}{l} \bar{A}\begin{bmatrix} X(t) \\ \eta(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \hat{A}_2 \end{bmatrix}\eta(t-d(t)) + \begin{bmatrix} 0 \\ \hat{G} \end{bmatrix}\psi(t,e(t)) + \begin{bmatrix} 0 \\ \hat{E}_1 \end{bmatrix} f(x(t)) + \begin{bmatrix} 0 \\ \hat{E}_2 \end{bmatrix} \\ \times f(x(t-h(t))) + \begin{bmatrix} 0 \\ \hat{E}_3 \end{bmatrix}\phi(e(t)) - \begin{bmatrix} 0 \\ \hat{A}_1 + \hat{E}_4 J\, s(t) \end{bmatrix} \int_{t-h(t)}^{t} \eta(s)\, ds + \begin{bmatrix} 0 \\ \hat{B} \end{bmatrix} w(t) \end{array} \right)
\end{aligned} \tag{13}$$

and time derivative of the second and third terms of $V(t)$ are, respectively, as

$$\begin{aligned}
\dot{V}_2(t) &= X(t)^T Q_1\, X(t) - (1-\dot{h}(t))X(t-h(t))^T Q_1\, X(t-h(t)) \\
&\quad + \eta(t)^T Q_2\, \eta(t) - (1-\dot{d}(t))\eta(t-d(t))^T Q_2\, \eta(t-d(t)) \\
&\le X(t)^T Q_1\, X(t) - (1-h_2)X(t-h(t))^T Q_1\, X(t-h(t)) \\
&\quad + \eta(t)^T Q_2\, \eta(t) - (1-d_2)\eta(t-d(t))^T Q_2\, \eta(t-d(t))
\end{aligned} \tag{14}$$

and

$$\begin{aligned}
\dot{V}_3(t) &= h_1\, \eta(t)^T (Q_3 + Q_4)\eta(t) - \int_{t-h_1}^{t} \eta(s)^T (Q_3 + Q_4)\eta(s) \quad ds \\
&= h_1\, \eta(t)^T (Q_3 + Q_4)\eta(t) - \int_{t-h_1}^{t} \eta(s)^T Q_3\, \eta(s) \quad ds - \int_{t-h(t)}^{t} \eta(s)^T Q_4\, \eta(s) \quad ds - \int_{t-h_1}^{t-h(t)} \eta(s)^T Q_4\, \eta(s) \quad ds
\end{aligned} \tag{15}$$

Construct a HJI function in the form of

$$J[X(t),w(t)] = \frac{d}{dt}V(t) + (z(t) - \hat{z}(t))^T (z(t) - \hat{z}(t)) - \gamma^2 w(t)^T w(t), \tag{16}$$

where derivative of $V(t)$ is evaluated along the trajectory of the filtering error system (4). It is well known that a *sufficient condition* for achieving robust disturbance attenuation is that the inequality $J[X(t),w(t)] < 0$ for every $w(t) \in L_2^s[0,\infty)$ results in a function $V(t)$, which is strictly radially unbounded (see for instance the reference [37]).

From (13)–(16) we obtain

$$\begin{aligned}
J[X(t),w(t)] = {} & 2\overline{\eta}(t)^T P^T \Bigg(\overline{A}\,\overline{\eta}(t) + \begin{bmatrix} 0 \\ \hat{A}_2 \end{bmatrix} \eta(t-d(t)) + \begin{bmatrix} 0 \\ \hat{G} \end{bmatrix} \psi(t,e(t)) + \begin{bmatrix} 0 \\ \hat{E}_1 \end{bmatrix} f(x(t)) + \begin{bmatrix} 0 \\ \hat{E}_2 \end{bmatrix} \\
& \times f(x(t-h(t))) + \begin{bmatrix} 0 \\ \hat{E}_3 \end{bmatrix} \phi(e(t)) - \begin{bmatrix} 0 \\ \hat{A}_1 + \hat{E}_4 J\, s(t) \end{bmatrix} \int_{t-h(t)}^{t} \eta(s)\; ds + \begin{bmatrix} 0 \\ \hat{B} \end{bmatrix} w(t) \Bigg) \\
& + X(t)^T (Q_1 + \hat{C}_1^T \hat{C}_1) X(t) - (1-h_2) X(t-h(t))^T Q_1 X(t-h(t)) + \eta(t)^T \\
& \times (Q_2 + h_1 (Q_3 + Q_4)) \eta(t) - (1-d_2) \eta(t-d(t))^T Q_2\, \eta(t-d(t)) - \int_{t-h_1}^{t} \eta(s)^T Q_3\, \eta(s) \quad ds \\
& - \int_{t-h(t)}^{t} \eta(s)^T Q_4\, \eta(s) \quad ds - \int_{t-h_1}^{t-h(t)} \eta(s)^T Q_4\, \eta(s) \quad ds - \gamma^2 w(t)^T w(t).
\end{aligned} \tag{17}$$

where $\overline{\eta}(t) := col\{X(t), \eta(t)\}$ and $\overline{A} = \begin{bmatrix} 0 & I \\ \hat{A} + \hat{A}_1 & -I \end{bmatrix}$. By Lemma 1 and (11b), it is clear that

$$\begin{aligned}
-2\overline{\eta}(t)^T P^T \begin{bmatrix} 0 \\ \hat{A}_1 + \hat{E}_4 J\, s(t) \end{bmatrix} \int_{t-h(t)}^{t} \eta(s)\; ds \le {} & \int_{t-h(t)}^{t} \begin{bmatrix} \overline{\eta}(t) \\ \eta(s) \end{bmatrix}^T \begin{bmatrix} H & U - P^T \begin{bmatrix} 0 \\ \hat{A}_1 + \hat{E}_4 J\, s(t) \end{bmatrix} \\ * & Q_3 \end{bmatrix} \begin{bmatrix} \overline{\eta}(t) \\ \eta(s) \end{bmatrix} ds \\
\le {} & \int_{t-h_1}^{t} \eta(s)^T Q_3\, \eta(s)\, ds + h_1 \overline{\eta}(t)^T H \overline{\eta}(t) + 2\overline{\eta}(t)^T (U - P^T \begin{bmatrix} 0 \\ \hat{A}_1 \end{bmatrix})(X(t) - X(t-h(t))) \\
& - 2\overline{\eta}(t)^T P^T \begin{bmatrix} 0 \\ \hat{E}_4 \end{bmatrix} (\phi(e(t)) - \phi(e(t-h(t))))
\end{aligned} \tag{18}$$

Using Assumption 1, we have

$$0 \le -f(x(t))^T f(x(t)) + x(t)^T U_1^T U_1 x(t) \tag{19a}$$

$$0 \le -f(x(t-h(t)))^T f(x(t-h(t))) + x(t-h(t))^T U_1^T U_1 x(t-h(t)) \tag{19b}$$

$$0 \le -\phi(e(t))^T \phi(e(t)) + e(t)^T U_1^T U_1 e(t) \tag{19c}$$

$$0 \le -\phi(e(t-h(t)))^T \phi(e(t-h(t))) + e(t-h(t))^T U_1^T U_1 e(t-h(t)) \tag{19d}$$

and

$$0 \le -\psi(t,e(t))^T \psi(t,e(t)) + e(t)^T U_2^T U_2 e(t) \tag{19e}$$

Moreover, from the Leibniz-Newton formula, the following equation holds for any matrix M with an appropriate dimension

$$2\upsilon(t)^T M(X(t) - X(t-h(t)) - \int_{t-h(t)}^{t} \eta(s)\ ds) = 0 \tag{20}$$

where $M := col\{M_1, M_2, \cdots, M_9\}$ and

$\vartheta(t) := col\{\overline{\eta}(t), X(t-h(t)), \eta(t-d(t)),\ f(x(t)), f(x(t-h(t))), \phi(x(t)), \phi(x(t-h(t))), \psi(t,e(t)), w(t)\}$.

By adding the right- and the left- hand sides of (19) and (20), respectively, to (17) and using the inequality (18), it follows that

$$\begin{aligned} J[X(t),w(t)] \le \vartheta(t)^T (\Pi + h_1 M Q_4^{-1} M^T) \vartheta(t) - \int_{t-h_1}^{t-h(t)} \eta(s)^T Q_4 \eta(s) \quad ds \\ - \int_{t-h(t)}^{t} (\vartheta(t)^T M + \eta(s)^T Q_4) Q_4^{-1} (\vartheta(t)^T M + \eta(s)^T Q_4)^T \quad ds \end{aligned} \tag{21}$$

where the matrix Π is given by

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & P^T \begin{bmatrix} 0 \\ \hat{A}_2 \end{bmatrix} + J^T M_3^T & P^T \begin{bmatrix} 0 \\ \hat{E}_1 \end{bmatrix} + J^T M_4^T & P^T \begin{bmatrix} 0 \\ \hat{E}_2 \end{bmatrix} + J^T M_5^T \\ * & \Pi_{22} & -M_3^T & -M_4^T & -M_5^T \\ * & * & -(1-d_2)Q_2 & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix}$$

$$\left.\begin{array}{cccc}
P^T\begin{bmatrix}0\\ \hat{E}_3-\hat{E}_4\end{bmatrix}+J^TM_6^T & P^T\begin{bmatrix}0\\ \hat{E}_4\end{bmatrix}+J^TM_7^T & P^T\begin{bmatrix}0\\ \hat{G}\end{bmatrix}+J^TM_8^T & P^T\begin{bmatrix}0\\ \hat{B}\end{bmatrix}+J^TM_9^T \\
-M_6^T & -M_7^T & -M_8^T & -M_9^T \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
-I & 0 & 0 & 0 \\
* & -I & 0 & 0 \\
* & * & -I & 0 \\
* & * & * & -\gamma^2 I
\end{array}\right]$$

with

$$\Pi_{11} = sym\{P^T\overline{A}\} - sym\{P^T\begin{bmatrix}0\\ \hat{A}_1\end{bmatrix}J-(U+M_1)J\}+h_1H$$
$$+\begin{bmatrix}Q_1+\hat{C}_1^T\hat{C}_1+J^TU_1^TU_1J & 0\\ 0 & Q_2+h_1(Q_3+Q_4)+\hat{J}^T(U_1^TU_1+U_2^TU_2)\hat{J}\end{bmatrix},$$

$$\Pi_{12} = -U-M_1+P^T\begin{bmatrix}0\\ \hat{A}_1\end{bmatrix}+J^TM_2^T,$$

$$\Pi_{22} = -(1-h_2)Q_1 - sym\{M_2\}+J^TU_1^TU_1J+\hat{J}^TU_1^TU_1\hat{J}.$$

Thus, if the inequality

$$\Pi + h_1MQ_4^{-1}M^T < 0 \tag{22}$$

holds, it follows from $J[X(t),w(t)]\big|_{w(t)\equiv 0}\le 0$ that $\frac{d}{dt}V(t)\le 0$ or $V(t)\le V(0)$. Then, from (9), it can be deduced

$$V(0) = X(0)^T P_1 X(0) + \int_{-h(0)}^{0} X(s)^T Q_1 X(s)\ ds + \int_{-d(0)}^{0}\eta(s)^T Q_2\,\eta(s)\ ds$$
$$+\int_{-h_1}^{0}\int_{s}^{0}\eta(\theta)^T(Q_3+Q_4)\eta(\theta)\quad d\theta\, ds$$

$$\le \lambda_{\max}(P_1)\|\varphi\|_2^2 + \lambda_{\max}(Q_1)\int_{-h(0)}^{0} X(s)^T X(s)\; ds + \lambda_{\max}(Q_2)\int_{-d(0)}^{0} \eta(s)^T \eta(s)\; ds$$

$$+\lambda_{\max}(Q_3+Q_4)\int_{-h_1}^{0}\int_{s}^{0} \eta(\theta)^T \eta(\theta)\quad d\theta\, ds$$

$$\le \sigma_1\|\varphi\|_2^2 + \sigma_2\|\eta\|_2^2$$

where $\sigma_1 := \lambda_{\max}(P_1) + h_1\lambda_{\max}(Q_1)$ and $\sigma_2 := (d_1\lambda_{\max}(Q_2) + 0.5\, h_1^2\lambda_{\max}(Q_3+Q_4))$. Then, we have:

$$\lambda_{\min}(P_1)\|\varphi\|_2^2 \le V(t) \le \sigma_1\|\varphi\|_2^2 + \sigma_2\|\eta\|_2^2 .$$

Therefore, we conclude that the filtering error system (4) is asymptotically stable. Notice that the matrix inequality (22) includes multiplication of filter matrices and Lyapunov matrices which are unknown and occur in nonlinear fashion. Hence, the inequality (22) cannot be considered an LMI problem. In the literature, more attention has been paid to the problems having this nature, which called bilinear matrix inequality (BMI) problems [38]. In the following, it is shown that, by considering $P_3 = \varepsilon P_2$ where

$$P_2 = \begin{bmatrix} P_{11} & P_{12} \\ P_{22} & P_{22} \end{bmatrix}, \tag{23}$$

and introducing change of variables

$$\begin{bmatrix} W_1 & W_2 & W_3 & W_4 & W_5 & W_6 \end{bmatrix} := P_{22}^T \begin{bmatrix} F & F_1 & F_2 & F_3 & F_4 & G \end{bmatrix} \tag{24}$$

the matrix inequality (22) is converted into LMI (11a) and can be solved via convex optimization algorithms. It is also easy to see that the inequality (22) implies $\Pi_{11} < 0$. Hence by Proposition 4.2 in the reference [15], the matrix P is nonsingular. Then, according to the structure of the matrix P in (10), the matrix P_2 (or P_{22}) is also nonsingular. This completes the proof. ■

Remark 1: It is worth noting that in the case when $x(t) \in \Re^n$, $w(t) \in \Re^s$, $z(t) \in \Re^z$ and $y(t) \in \Re^p$, the number of the variables to be determined in the LMIs (11) is $0.5\,n(17n + 2p + 2z + 5) + 5$. It is also observed that the LMIs (11) are linear in the set of matrices $P_{11}, P_{12}, P_{22}, G_1, H, U, W_1, \cdots, W_6, M_1, \cdots, M_9$,

$P_1, Q_1, \cdots, Q_4$ and the scalars ε, γ^2. This implies that the scalar γ^2 can be included as one of the optimization variables in LMIs (11) to obtain the minimum disturbance attenuation level. Then, the optimal solution to the delay-dependent H_∞ filtering can be found by solving the following convex optimization problem

$$\begin{aligned} &Min \quad \lambda \\ &subject\ \ to\ \ (11)\ \ with\ \ \lambda := \gamma^2. \end{aligned} \tag{25}$$

IV. EXAMPLE

In this section, we will verify the proposed methodology by giving an illustrative example. We solved LMIs (13) by using Matlab LMI Control Toolbox [39], which implements state-of-the-art interior-point algorithms and is significantly faster than classical convex optimization algorithms [40]. The example is given below.

Consider the system (1) with the following matrices

$$A = \begin{bmatrix} -1 & 0.5 \\ 0.3 & -2 \end{bmatrix};\ A_1 = \begin{bmatrix} -0.5 & 0.1 \\ 0.1 & -0.6 \end{bmatrix};\ A_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \end{bmatrix};\ B_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix};\ E_1 = E_2 = I_2;$$

$$C_1 = \begin{bmatrix} 1 & 1 \end{bmatrix};\ C_2 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix};\ f(x(t)) = g(t, x(t)) = 0.5(|x(t)+1| - |x(t)-1|).$$

The delays $h(t) = d(t) = (1 - e^{-t}) / (1 + e^{-t})$ are time varying and satisfy $0 \le h(t) = d(t) \le 1$ and $\dot{h}(t) = \dot{d}(t) \le 0.5$. For simulation purpose, a uniformly distributed random signal, shown in Figure 1, with minimum and maximum -1 and 1, respectively, as the disturbance is imposed on the system. With the above parameters, the filtering error system (4) exhibits the chaotic behaviours such the state trajectories of the system with initial condition $x(0) = (0,\ 0)$ is depicted in Figure 2.

By solving the LMIs (11) in Theorem 1 with the disturbance attenuation $\gamma = 0.2$ we get the following state-space matrices of the delay-dependent H_∞ filter (2):

$$F = \begin{bmatrix} -2.8807 & 1.1770 \\ 1.0575 & -4.9106 \end{bmatrix},\ F_1 = \begin{bmatrix} -0.3991 & 0.2557 \\ 0.2297 & -0.7907 \end{bmatrix},\ F_2 = \begin{bmatrix} -0.0835 & -0.1410 \\ 0.0209 & -0.1002 \end{bmatrix},\ F_3 = \begin{bmatrix} 1.5747 & -0.4885 \\ -0.3693 & 2.7097 \end{bmatrix},$$

$$F_4 = \begin{bmatrix} 1.1810 & -0.3664 \\ -0.2770 & 2.0323 \end{bmatrix},\ G = \begin{bmatrix} -0.0226 \\ -0.0662 \end{bmatrix},\ G_1 = \begin{bmatrix} 0.5414 & 0.4628 \end{bmatrix}.$$

For initial conditions $x(0) = (-1, 1)$, the simulation results are shown in Figures 3 and 4. The trajectories of the estimation error are plotted in Figure 3. Finally, to observe the H_∞ performance, curve of the function $\|z(t) - \hat{z}(t)\|_2 / \|w(t)\|_2$ is depicted in Figure 4 which shows that the H_∞ constraint in (8) is satisfied as well.

V. CONCLUSION

The problem of delay-dependent H_∞ filtering was proposed for a class of nonlinear neutral systems with delayed states and outputs. New required sufficient conditions were established in terms of delay-dependent LMIs for the existence of the desired robust H_∞ filters. The explicit expression of the robust H_∞ filters was derived to satisfy both asymptotic stability and a prescribed level of disturbance attenuation for all admissible known nonlinear functions. A numerically example was presented to illustrate the effectiveness of the designed filter.

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REFERENCES

[1] Malek-Zavarei, M., and Jamshidi, M., '*Time-delay systems: Analysis, optimisation and application.*' Amsterdam, The Netherlands: North-Holland, 1987.

[2] Niculescu S. I., ‘*Delay Effects on Stability: A Robust Control Approach’* (Berlin: Springer), 2001.

[3] MacDonald, N., ‘Biological delay systems: linear stability theory.’ Cambridge University Press, Cambridge, 1989.

[4] Kuang, Y., ‘Delay differential equations with applications in population dynamics.’ Acadelic Press, Boston, 1993.

[5] Krasovskii, N.N., ‘*Stability of motion.’* Stanford, CA: Stanford University Press, 1963.

[6] Han, C.L. and Yu, L., ‘Robust stability of linear neutral systems with nonlinear parameter perturbations.’ *IEE Proc. Control Theory Appl.*, 151(5), 539-546, 2004.

[7] Park P., ‘A delay-dependent stability criterion for systems with uncertain time-invariant delays’ *IEEE Trans. Automatic Control*, vol. 44, pp. 876-877, 1999.

[8] Fridman, E., ‘New Lyapunov-Krasovskii functionals for stability of linear retarded and neutral type systems.’ *Systems & Control Letters*, 43, 309-319, 2001.

[9] Hu, G. D, and Hu, G. D., ‘Some simple stability criteria of neutral delay-differential systems.’ *Applied Mathematics and Computation*, 80, 257-271, 1996.

[10] Chen, W.H., and Zheng W.X., ‘Delay-dependent robust stabilization for uncertain neutral systems with distributed delays’ *Automatics*, 43, 95-104, 2007.

[11] Chen, J. D., Lien, C. H., Fan, K. K., and Chou, J. H., ‘Criteria for asymptotic stability of a class of neutral systems via a LMI approach.’ *IEE Proc. Control Theory and Applications*, 148, 442-447, 2001.

[12] Yue D., Won S., and Kwon O., ‘Delay dependent stability of neutral systems with time delay: an LMI approach’ *IEE Proc. Control Theory Appl.,* vol. 150, no. 1, pp. 23-27, 2003.

[13] Xu, S., Lam, J., and Yang, C., ‘Quadratic stability and stabilization of uncertain linear discrete-time systems with state delay.’ *Systems & Control Letters*, 43, 77–84, 2001.

[14] Chen, J. D., 'LMI-based robust H_∞ control of uncertain neutral systems with state and input Delays.' *Journal of Optimization Theory and Applications*, 126, 553-570, 2005.

[15] Fridman, E., and Shaked, U., 'Delay-dependent stability and H_∞ control: constant and time-varying delays.' *International Journal of Control*, 76, 48-60, 2003.

[16] Gao, H., and Wang, C., 'Comments and further results on 'A descriptor system approach to H_∞ control of linear time-delay systems.' *IEEE Transactions on Automatic control*, 48, 520-525, 2003.

[17] Xu, S., Lam, J., and Yang, C., 'H_∞ and positive-real control for linear neutral delay systems.' *IEEE Transactions on Automatic control*, 46, 1321-1326, 2001.

[18] Chen J.D., 'LMI approach to robust delay-dependent mixed H_2/H_∞ controller of uncertain neutral systems with discrete and distributed time-varying delays' *J. Optimization Theory and Applications*, 131(3), 383-403, 2006.

[19] Xu, S., Lam, J., and Yang, C., 'Robust H_∞ control for uncertain linear neutral delay systems.' *Optimal Control Applications and Methods*, 23, 113-123, 2002.

[20] Xu, S., Chu, Y., Lu, J., and Zou, Y., 'Exponential dynamic output feedback controller design for stochastic neutral systems with distributed delays.' *IEEE Transactions on Systems, Man, and Cybernetics A: Systems and Humans*, 36, 540-548, 2006.

[21] Chen, B., Lam, J., and Xu, S., 'Memory state feedback guaranteed cost control for neutral systems' *Int. J. Innovative Computing, Information and Control*, 2(2), 293-303, 2006.

[22] Karimi H.R., 'Robust mixed H_2/H_∞ delayed state-feedback control of neutral delay systems with time-varying delays' *Asian Journal of Control*, 10(5), 2008 (In Press).

[23] Lien, C.H., 'Guaranteed cost observer-based controls for a class of uncertain neutral time-delay systems.' *J. Optimization Theory and Applications*, 126(1), 137-156, 2005.

[24] Lien C.H., 'H_∞ observer-based control for a class of uncertain neutral time-delay systems via LMI optimization approach' *J. Optimization Theory and Applications*, 127(1), 129-144, 2005.

[25] Karimi H.R., 'Observer-based mixed H_2/H_∞ control design for linear systems with time-varying delays: An LMI approach' *Int. J. of Control, Automation, and Systems*, 6(1), 1-14, 2008.

[26] Anderson B.D.O. and Moore J.B., 'Optimal filtering' Englewood Cliffs, NJ: Prentice-Hall, 1979.

[27] Li H. and Yang C., 'Robust H_∞ filtering for uncertain linear neutral delay systems' *Proc. ACC*, pp. 2251-2255, 2006.

[28] Gao H. and Wang C., 'Delay-dependent robust H_∞ and $L_2 - L_\infty$ filtering for a class of uncertain nonlinear time-delay systems' *IEEE Trans. Automatic Control,* vol. 48, n0. 9, pp. 1661-1666, 2003.

[29] Gao H. and Wang C., 'A delay-dependent approach to robust H_∞ filtering for uncertain discrete-time state-delayed systems' *IEEE Trans. Signal Processing*, vol. 52, no. 6, pp. 1631-1640, 2004.

[30] Germoel J.C. and de Oliveira M.C., 'H_2 and H_∞ robust filtering for convex bounded uncertain systems' *IEEE Trans. Automatic Control*, vol. 46, no. 1, pp. 100-107, 2001.

[31] Guo L., Yang F., and Fang J., 'Multiobjective filtering for nonlinear time-delay systems with nonzero initial conditions based on convex optimization' *Circuits Systems Signal Processing*, vol. 25, no. 5, pp. 591–607, 2006.

[32] Nagpal, K.M., and Khargonekar, P.P., 'Filtering and smoothing in an H_∞ setting.' *IEEE Transactions on Automatic Control,* 36, 152–166, 1991.

[33] de Souza, C.E., Xie, L., and Wang, Y., 'H_∞ filtering for a class of uncertain nonlinear systems.' *Systems & Control Letter*, 20, 419–426, 1993.

[34] Jin, S.H., Park, J.B., 'Robust H_∞ filtering for polytopic uncertain systems via convex optimisation.' *IEE Proceedings–Control Theory Applications,* 148, 55–59, 2001.

[35] de Souza, C.E., Palhares, R.M., Peres, P.L.D., 'Robust H_∞ filter design for uncertain linear systems with multiple time-varying state delays.' *IEEE Transactions on Signal Processing,* 49, 569–576, 2001.

[36] Xu, S., 'Robust H_∞ filtering for a class of discrete-time uncertain nonlinear systems with state delay.' *IEEE Transactions on Circuits Systems I*, 49, 1853–1859, 2002.

[37] Zhou K. and Khargonekar P.P., 'Robust stabilization of linear systems with norm-bounded time-varying uncertainty' *System Control Letters,* 10, 17-20, 1988.

[38] Safonov M.G., Goh K.C., and Ly J.H., 'Control system synthesis via bilinear matrix inequalities', *Proc. ACC.*, 45-49, 1994.

[39] Gahinet P., Nemirovsky A., Laub A.J. and Chilali M., '*LMI control Toolbox: For use with Matlab*' Natik, MA: The MATH Works, Inc, 1995.

[40] Boyd S., Ghaoui E. L., Feron E. and Balakrishnan V., '*Linear matrix inequalities in systems and control theory*' Studies in Applied Mathematics, SIAM, Philadelphia, Pennsylvania, Vol. 15, 1994.

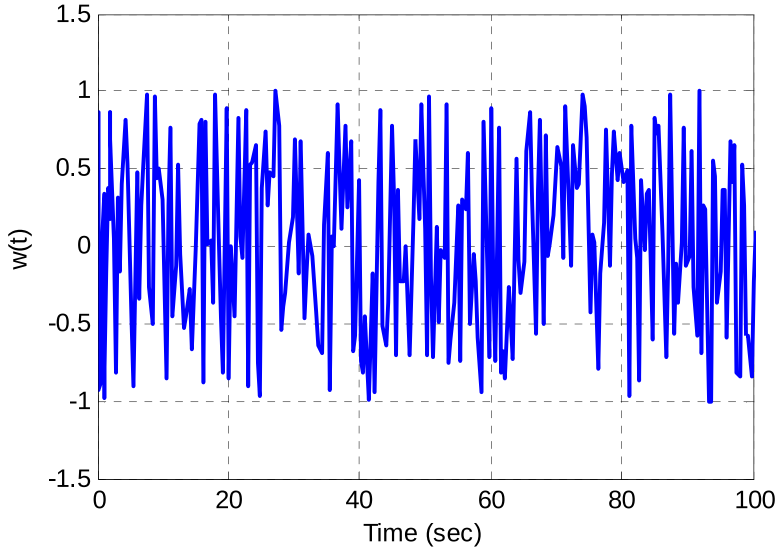

Fig. 1. The disturbance signal.

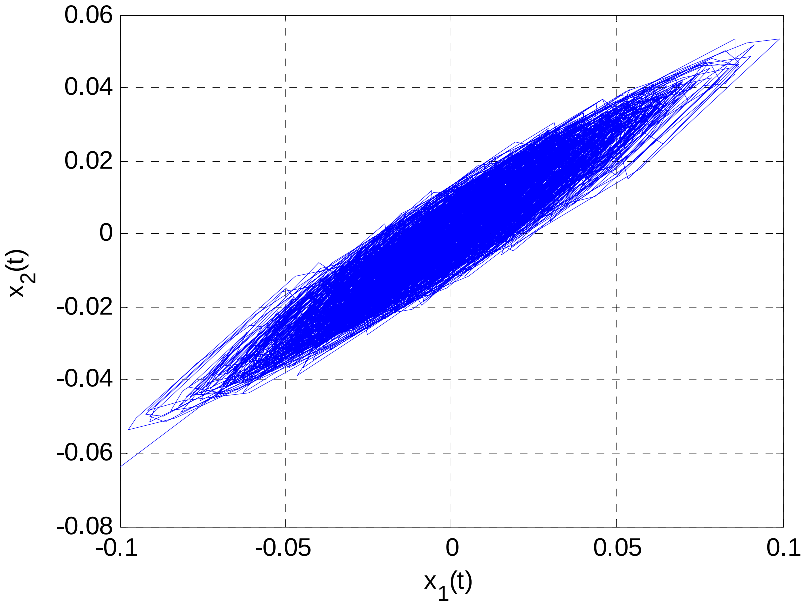

Fig. 2. The phase trajectories.

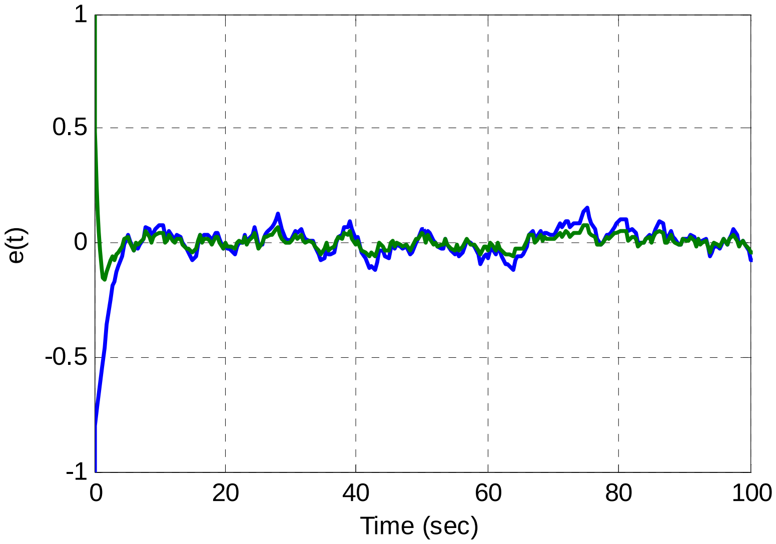

Fig. 3. Curves of estimation error signal.

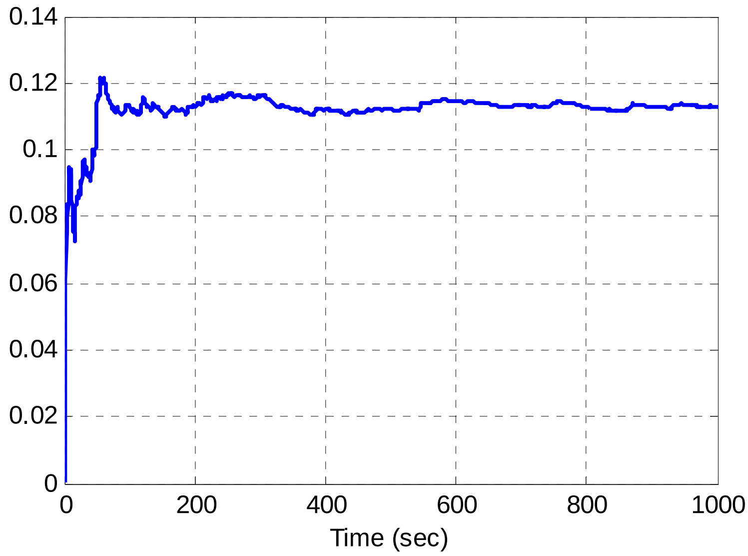

Fig. 4. Curve of function $\|z(t)-\hat{z}(t)\|_2 / \|w(t)\|_2$.