Local Capacity H_∞ Control for Production Networks of Autonomous Work Systems With Time-Varying Delays

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***Abstract*—This paper considers the problem of local capacity H_∞ control for a class of production networks of autonomous work systems with time-varying delays in the capacity changes. The system under consideration is modeled in a discrete-time singular form. Attention is focused on the design of a controller gain for the local capacity adjustments which maintains the work-in-progress (WIP) in each work system in the vicinity of planned levels and guarantees the asymptotic stability of the system and reduces the effect of the disturbance input on the controlled output to a prescribed level. In terms of a matrix inequality, a sufficient condition for the solvability of this problem is presented using an appropriate Lyapunov function, which depends on the size of the delay and is solved by existing convex optimization techniques. When this matrix inequality is feasible, the controller gain can be found by using LMI Toolbox Matlab. Finally, numerical results are provided to demonstrate the proposed approach.**

***Note to Practitioners*—Modern production networks become large due to new communication technologies and globalization processes. These networks are subject to many disturbances such as changes on market, transport congestions, communications delays, machine failures etc. Their complexity makes it difficult to control them in a centralized way. One possible solution is to introduce autonomous control, i.e., to allow some parts of a large network to make their own decisions based on local situation and available information. However, stability of the network and robustness with respect to external and internal disturbances and time delays in signals must be assured to guarantee a reasonable performance and vitality of the whole system. For this purpose, this paper proposes an approach for controller design for large scale autonomous work systems capable to cope with time delays and explains its implementation and advantages on a concrete example.**

***Index Terms*—Autonomous systems, delay, linear matrix inequality (LMI), production networks.**

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I. Introduction

PRODUCTION networks are emerging as a new type of cooperation between and within companies, requiring new techniques and methods for their operation and management [1], [2]. Coordination of resource used is a key challenge in achieving short delivery times and delivery time reliability. It is shown by Helo [3] that these networks can exhibit unfavorable dynamic behavior as individual organizations respond to variations in orders in the absence of sufficient communication and collaboration. The global control becomes difficult and vulnerable in case of large size and high complexity of production networks. This is due to permanent changes of, e.g., market requirements, order sizes and internal disturbances. These changes can destabilize the dynamics of a production network and lead to low performance and economic losses. A compromise is to allow some entities of a network, e.g., single machines or separate plants to make decisions by their own based on local situation and available information. Such entities are called autonomous work systems in this paper. A set of rules to make decisions for a single autonomous work system is called autonomous control. However, the structural and dynamic complexity of these emerging networks inhibit collection of the information necessary for centralized planning and control, and decentralized coordination must be provided by logistic processes with autonomous capabilities [4]. Furthermore, to develop and analyze autonomous control strategies dynamic models are required. To this end different modeling approaches are investigated regarding their abilities to describe an exemplary scenario—an autonomously controlled production network. A discrete-event simulation model is compared to a deterministic fluid model for a continuous product queue, both based on previous work by Scholz-Reiter *et al.* [5]. Here, the term continuous denotes the continuous material flow in comparison to the flow of discrete parts in the discrete-event simulation model. Recently, models and control strategies based on the idea of pheromones was developed in [6]. That is, the decision which path to choose through the production network is not made by a manager or operator, but by the individual part itself, based on the "experience" of other parts of the same type.

A production network with several autonomous work systems is depicted in Fig. 1. The behavior of such a network is affected by external and internal order flows, planning, internal disturbances, and the control laws used locally in the work systems to adjust resources for processing orders [7], [8]. In prior work, sharing of capacity information between work systems has been modelled along with the benefits of alternative control laws and

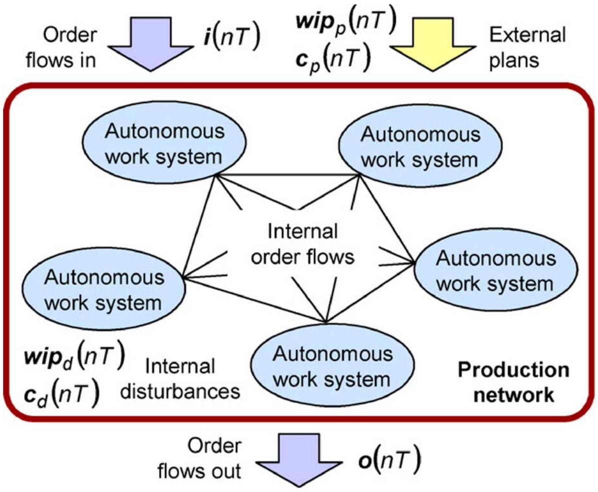

Fig. 1. Production network consisting of a group of autonomous work systems.

reducing delay in capacity changes [9]. Several authors have described both linear and nonlinear dynamical models for control of variables such as inventory levels and work-in-progress (WIP), including the use of pipeline flow concepts to represent lead times and production delays [2], [10]. Delivery reliability and delivery time have established themselves as equivalent buying criteria alongside product quality and price [1]. High delivery reliability and short delivery times for companies require for high schedule reliability and short throughput times in production. In order to manufacture economically under such conditions, it is necessary to minimize WIP levels in production and utilize operational resources in the best possible way [11].

Production Planning and Control (PPC) has become more challenging as manufacturing companies adapt to a fast changing market [12]. Current PPC methods often do not deal with unplanned orders and other types of turbulence in a satisfactory manner [13]. Assumptions such as infinite capacity and fixed lead time are often made, leading to a static view of the production system may not be valid because WIP affects lead time and performance, while capacity is finite and varies both according to plan and due to unplanned disturbances such as equipment breakdowns, worker illness, market changes, etc. Understanding the dynamic nature of production systems requires new approaches for the design of PPC based on company's logistics. The controllers implicitly interact to adjust capacity to eliminate backlog as the system maintains its planned WIP level [13]. A discrete closed-loop PPC model was developed and analyzed by Duffie and Falu [14] in which two discrete controllers, one for backlog and one for WIP, with different periods between adjustments of work input and capacity, respectively, were selected and evaluated using transfer function analysis and time-response simulation. A second architecture for continuous WIP control and discrete backlog control, with delay capacity adjustment, was developed and analyzed by Ratering and Duffie for cases of high and low WIP [15]. This analysis was facilitated by linearization of the logistic function using operating point analysis. A proportional backlog controller was designed and evaluated at the extreme cases of high and low WIP using control theoretic methods. Response times for elimination of backlog were found to be relatively slow due to the limitations of the control algorithms used. A closed-loop production planning and control concept has been employed with adaptive inventory control in decision support systems in a multiproduct medical supplies market [12]. State-space models have been used for switching between a library of optimal controllers to adjust WIP in serial production systems in the presence of machine failures [11], and switching of control policies in response to market strategies has been investigated by Deif and ElMaraghy [16]. A discrete state-space dynamic model was developed for production networks with an arbitrarily large number of work systems by Duffie *et al.* [4]. It is illustrated the use of this generic model to predict performance, and comparing the results with results obtained using discrete event simulation.

On the other hand, delay (or memory) systems represent a class of infinite-dimensional systems largely used to describe propagation and transport phenomena or population dynamics [17]. Delay differential systems are assuming an increasingly important role in many disciplines like economic, mathematics, science, and engineering. For instance, in economic systems, delays appear in a natural way since decisions and effects are separated by some time interval. The presence of a delay in a system may be the result of some essential simplification of the corresponding process model. The delay effects problem on the stability of systems including delays in the state and/or input is a problem of recurring interest since the delay presence may induce complex and undesired behaviors (oscillation, instability, bad performance) for the schemes, see for instance [18] and the references therein. Thus, there has been increasing interest in the robust and/or H_∞ stabilization of uncertain time-delay systems in the last decades. For the continuous-time case, most results have been obtained based on the modified Riccati equation/inequality approach [19] and the linear matrix inequality (LMI) approach [18]. It should be pointed out that, the discrete-time systems with time-delay have received little attention compared with its continuous-time counterpart [20]–[22]. The main reason for this is that for precisely known discrete-time systems with constant time-delay, it is always possible to obtain an augmented system without delayed states. This approach, however, does not seem to be suitable for time-varying delay, delay-independent stability characterization, and for robust-system stabilization [23]. With regard to the stability analysis issue, Verriest and Ivanov in [24] studied the sufficient conditions for the asymptotic stability of the discrete-time state delayed systems by using an algebraic matrix inequality approach. Concerning the problem of designing control systems, Song and Kim in [25] have established the H_∞ control problem for linear discrete-time uncertain time-delay systems and a sufficient condition has been derived in terms of a Riccati-like matrix inequality. In the context of discrete time-delay systems, sufficient conditions for the solvability of the H_∞ control problem was obtained in [26] in terms of a modified Riccati equation. Recently, the problem of robust H_∞ control for a class of discrete systems with time-varying delays and time-varying norm-bounded parameter uncertainties was studied by Xu and Chen [27]. However, robust stability analysis of the production networks that includes time-delays in their local capacity adjustments is an important problem and, so far, very little attention has been paid for the investigation of this problem, see for instance [4].

In this paper, we contribute to the further development of a local capacity control design for a class of production networks of autonomous work systems with time-varying delays in the capacity changes. The system under consideration is modelled as a discrete-time singular form. An appropriate Lyapunov function is constructed in order to establish a delay-range-dependent sufficient condition in terms of a matrix inequality for finding a controller gain for the local capacity adjustments which maintains the WIP in each work system in the vicinity of planned levels and guarantees the asymptotic stability of the system and reduces the effect of the disturbance input on the controlled output to a prescribed level. When this matrix inequality is feasible, the controller gain can be found by using LMI Toolbox Matlab. Finally, numerical results are provided to demonstrate the proposed approach.

The notation used throughout the paper are fairly standard. The superscript 'T' stands for matrix transposition; $\Re^n$ denotes the n-dimensional Euclidean space; $\Re^{n\times m}$ is the set of all real m by n matrices. $\|.\|$ refers to the Euclidean vector norm or the induced matrix 2-norm. $col\{\cdots\}$ and $diag\{\cdots\}$ represent, respectively, a column vector and a block diagonal matrix and the operator $sym(A)$ represents $A+A^T$. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote, respectively, the smallest and largest eigenvalue of the square matrix A. The notation $P>0$ means that P is real symmetric and positive definite; the symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix. If $\Sigma \in \Re^{m\times n}$ and $rank(\Sigma)=r$, the orthogonal complement $\Sigma^\perp$ is defined as a possibly nonunique $n\times(n-r)$ matrix with rank $n-r$, such that $\Sigma\Sigma^\perp=0$.

II. Model of Autonomous Work Systems

In this section, we consider a linear discrete-time dynamic approach for modeling the flow of orders into, out of, and between work systems. The model chosen promotes straightforward calculation of fundamental dynamic properties such as characteristic times and damping. In this network, the work systems do not share information regarding the expected physical flow of orders between them.

Assume that there are N work systems in a production network, as shown in Fig. 1. Vector $\boldsymbol{i}(nT)$ specifies the rate at which orders are input to the N work systems from sources external to the production network, which is constant over time $nT \le t < (n+1)T$ where $n=0,1,2,\cdots$. The parameter T is a time period between capacity adjustments [for example, 1 shop-calendar day (scd)]. The total orders that have been input to the work systems up to time $(k+1)T$ then can be represented as the vector [4]

$$w_i\left((n+1)T\right) = w_i(nT) + T\left(i(nT) + R^T c_a(nT)\right) \quad \text{(1a)}$$

where vector $c_a(nT)$ is the rate at which orders are output from the N work systems during time $nT \le t < (n+1)T$ (the actual capacity of each work system) and R is a matrix in which element r_{jk} approximates the fraction of the flow out of work system j that flows into work system k.

The total number of orders that are completed by the work systems up to time $nT \le t < (n+1)T$ can be represented by the vector

$$w_o\left((n+1)T\right) = w_o(nT) + Tc_a(nT) \quad \text{(1b)}$$

while the rate at which orders are output from the network during time $nT \le t < (n+1)T$ is

$$o(nT) = R_o c_a(nT) \quad \text{(1c)}$$

where R_o is a diagonal matrix in which nonzero diagonal element $R_{o_{ii}}$ represents the fraction of orders flowing out of work system i that flow out of the network. R_o is assumed to be constant, and

$$R_{o_{ii}} + \sum_{\substack{j=1\\ j\neq i}}^{N} R_{o_{ij}} = 1. \quad \text{(1d)}$$

The WIP in the work systems is

$$wip_a(nT) = w_i(nT) - w_o(nT) + w_d(nT) \quad \text{(1e)}$$

where $w_d(nT)$ represents local work disturbance, such as rush order, that affect the work system. Furthermore, the actual capacity of each work system depends on three components as follows:

$$c_a(nT) = c_p(nT) + c_m\left((n-d(n))\,T\right) - c_d(nT) \quad \text{(1f)}$$

where $c_d(nT)$ represents local capacity disturbances such as equipment failures and $c_p(nT)$ denotes planned capacities of the work systems. Also, $c_m(nT)$ represents local capacity adjustments to maintain the WIP in each work system in the vicinity of the planned levels $wip_p(nT)$ using straightforward proportionality k_c. In other words, $c_m(nT)$ can be described in the form of

$$c_m(nT) = k_c\left(wip_a(nT) - wip_p(nT)\right). \quad \text{(1g)}$$

While each work system could have a different value of the control parameter k_c, here it is assumed to be the same throughout the network. Furthermore, the actual capacity may be less than the full capacity due to capacity disturbances $c_d(nT)$ such as operator illness, work system starvation due to insufficient WIP, etc. It is assumed that a time-varying delay $d(n)T$ exists in the capacity changes $c_m(nT)$ for logistic reasons such as operator work rules and satisfies

$$d_1 \le d(n) \le d_2 \quad \text{(2)}$$

where the known parameters d_1 and d_2 are lower and upper bound of the time-varying delay $d(n)$, respectively, and the planned capacity and WIP are also assumed to be known and delay free in advance.

In (1a)–(1g), the general case of omnidirectional order-flow structures is assumed in which the flow of orders into a given work system can be a function of the flow of orders out of that same work system. The simplest case is when some of the orders that leave a work system reenter the same work system. Furthermore, information used in calculating full capacity, such as expected order flows into the network and capacity plans for the work systems, is assumed to be available $d(n)T$ time periods in advance regardless of whether it is the result of external planning or derived from information shared within the network.

Remark 1: It is clear from (1a)–(1g) that the fundamental dynamic properties of the network are a function of order-flow structure. With the objective of establishing and maintaining consistent and desirable fundamental dynamic properties, we may consider a network in which each work system shares expected capacity information with all other work systems in the network, allowing individual work systems to locally compensate for physical order-flow coupling.

Remark 2: Capacity adjustments can be large, and there can be relatively large differences between successive capacity adjustments. Such adjustments must be acceptable in application. Delay in capacity adjustment has been included to represent the inability to make instantaneous adjustments. For this case, $d(n)$ is called an interval-like or range-like time-varying delay [27]. It is also noted that this kind of time delay describes the real situation in many practical engineering systems. For example, in the field of networked control systems, the network transmission induced delays (either from the sensor to the controller or from the controller to the plant) can be assumed to satisfy (2) without loss of generality [28], [29].

Remark 3: It is noted that the system structure in [4] considers a production network with constant delays in local capacity adjustments and in compare to our case do not center on time-varying delays, i.e., the results in [4] cannot be directly applied to the systems with time-varying delays.

Equations (1a)–(1g) can be combined to obtain a discrete-time singular model for the system

$$\begin{aligned} EX\left((n+1)T\right) &= AX(nT) + BHX\left((n-d(n))\,T\right) \\ &\quad + CW(nT) \qquad (3a) \\ Z(nT) &= L_1X(nT) + L_2HX\left((n-d(n))\,T\right) \\ &\quad + L_3W(nT) \qquad (3b) \end{aligned}$$

where $X(nT) := [w_i(nT)^T,\ w_o(nT)^T,\ c_m(nT)^T]^T$, $W(nT) := [i(nT)^T,\ wip_p(nT)^T,\ c_p(nT)^T,\ w_d(nT)^T,\ c_d(nT)^T]^T$, $A = A_1 + k_c\,A_2$, and $C = C_1 + k_c\,C_2$ with

$$E = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & -I \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ I & -I & 0 \end{bmatrix}, B = \begin{bmatrix} R^T \\ I \\ 0 \end{bmatrix},$$

$$H = \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}^T, C_1 = \begin{bmatrix} I & 0 & R^T & 0 & -R^T \\ 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -I & 0 & I & 0 \end{bmatrix},$$

where $Z(nT)$ denotes controlled outputs of the network and the constant matrices L_1, L_2, L_3 are defined in Section IV.

Then, the local capacity H_∞ control problem to be addressed in this paper can be formulated as finding the controller gain k_c in (1g), the same throughout the network for each work system, such that

i) The system (3a) is asymptotically stable when $W(nT) = 0$.
ii) Under the zero-initial condition and for any nonzero $W(nT) \in L_2$, the controlled output $Z(nT)$ satisfies the H_∞ performance measure

$$\|Z(nT)\|_2 < \gamma\,\|W(nT)\|_2 \qquad (4)$$

where γ is a prescribed scalar.

Remark 4: The order-flow structure R and the choice of k_c, T and $d(n)$ will affect the fundamental dynamic properties of the network (possibility of free oscillation when disturbed, time required to respond to changes in plans, time required to recover from disturbances, etc.) and the dynamic properties of the network may change with time if the order-flow structure is not constant. Furthermore, significant deviation of WIP from plan can be easily illustrated if the work system capacity required to satisfy order flows entering from outside network and from other work systems deviates significantly from planned capacity by the case of constant inputs.

III. Main Results

In this section, sufficient conditions for the solvability of the local capacity H_∞ control problem are proposed using the Lyapunov method and an LMI approach is developed.

A. Stability Analysis

In this section, assuming that the control gain k_c is known, new delay-range-dependent sufficient conditions for the local capacity H_∞ control problem formulated in the previous section are presented.

Theorem 1: For given scalars $d_1, d_2 > 0$, the system (3) is asymptotically stable and satisfies the H_∞ performance bound γ by the control gain k_c, if there exist some matrices N_1, N_2, N_3 and positive-definite matrices P and Q such that the following matrix inequality is feasible:

$$\begin{bmatrix} \Pi_{11} & T_1^TN_2^T & N_1T_2 + T_1^TN_3^T & A^TP & L_1^T \\ * & -Q & N_2T_2 & B^TP & L_2^T \\ * & * & sym\{N_3T_2\} - \gamma^2 I & C^TP & L_3^T \\ * & * & * & -P & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \qquad (5)$$

with $d_{12} = d_2 - d_1$

$$\begin{aligned} \Pi_{11} = &-E^TPE + (d_{12}+1)H^TQH + (d_{12}+1)H^TQH \\ &+ sym\{N_1T_1\}, \\ T_1 := &[k_cI, -k_cI, -I] \end{aligned}$$

and

$$T_2 := [0, -k_cI, 0, k_cI, 0].$$

Proof: Consider the Lyapunov function candidate in the following form:

$$V(nT) = X(nT)^T E^T PEX(nT) + V_1(nT) + V_2(nT) \quad (6)$$

where

$$V_1(nT) = \sum_{i=n-d(n)}^{n-1} X(iT)^T H^T QHX(iT) \quad (7)$$

and

$$V_2(nT) = \sum_{j=-d_2+2}^{-d_1+1} \sum_{l=n+j-1}^{n-1} X(lT)^T H^T QHX(lT). \quad (8)$$

Then, one obtains

$$\begin{aligned}
&V_1((n+1)T) - V_1(nT) \\
&\quad = \sum_{i=n+1-d(n+1)}^{n} X(iT)^T H^T QHX(iT) \\
&\qquad - \sum_{i=n-d(n)}^{n-1} X(iT)^T H^T QHX(iT) \\
&\quad = \sum_{i=n+1-d(n+1)}^{n-d_1} X(iT)^T H^T QHX(iT) \\
&\qquad + X(nT)^T H^T QHX(nT) \\
&\qquad - X((n-d(n))T)^T H^T QHX((n-d(n))T) \\
&\qquad + \sum_{i=n+1-d_1}^{n-1} X(iT)^T H^T QHX(iT) \\
&\qquad - \sum_{i=n+1-d(n)}^{n-1} X(iT)^T H^T QHX(iT) \\
&\quad \le \sum_{i=n+1-d(n+1)}^{n-d_1} X(iT)^T H^T QHX(iT) \\
&\qquad + X(nT)^T H^T QHX(nT) \\
&\qquad - X((n-d(n))T)^T H^T QHX((n-d(n))T). \quad (9)
\end{aligned}$$

Now, by some calculation, we derive

$$\begin{aligned}
&V_2((n+1)T) - V_2(nT) \\
&\quad = \sum_{j=-d_2+2}^{-d_1+1} \Big[X(iT)^T H^T QHX(iT) \\
&\qquad\qquad + \sum_{l=n+j}^{n-1} X(iT)^T H^T QHX(iT) \\
&\qquad\qquad - \sum_{l=n+j-1}^{n-1} X(iT)^T H^T QHX(iT)\Big] \\
&\quad = d_{12} X(nT)^T H^T QHX(nT) \\
&\qquad - \sum_{j=n+1-d_2}^{n-d_1} X(iT)^T H^T QHX(iT). \quad (10)
\end{aligned}$$

From (2) and (7)–(10), it is easy to see that

$$\begin{aligned}
&V_1((n+1)T) + V_2((n+1)T) - V_1(nT) - V_2(nT) \\
&\quad \le (d_{12}+1) X(nT)^T H^T QHX(nT) \\
&\qquad - X((n-d(n))T)^T H^T QHX((n-d(n))T) \\
&\qquad + \sum_{i=n+1-d(n+1)}^{n-d_1} X(iT)^T H^T QHX(iT) \\
&\qquad - \sum_{i=n+1-d_2}^{n-d_1} X(iT)^T H^T QHX(iT) \\
&\quad \le (d_{12}+1) X(nT)^T H^T QHX(nT) \\
&\qquad - X((n-d(n))T)^T H^T QHX((n-d(n))T). \quad (11)
\end{aligned}$$

Using the obtained inequalities (10) and (11), the following result is obtained:

$$\begin{aligned}
&V((n+1)T) - V(nT) \\
&\quad \le (AX(nT) + BHX((n-d(n))T) + CW(nT))^T \\
&\qquad \times P(AX(nT) + BHX((n-d(n))T) + CW(nT)) \\
&\qquad - X(nT)^T E^T PEX(nT) \\
&\qquad + (d_{12}+1) X(nT)^T H^T QHX(nT) \\
&\qquad - X((n-d(n))T)^T H^T QHX((n-d(n))T). \quad (12)
\end{aligned}$$

Moreover, from (1a)–(1g), the following equation holds for any matrices N_1, N_2, and N_3 with appropriate dimensions:

$$\left(X(nT)^T N_1 + c_m((n-d(n))T)^T N_2 + W(nT)^T N_3\right) \times (T_1 X(nT) + T_2 W(nT)) = 0. \quad (13)$$

Furthermore, in the case of $W(nT) = 0$, it follows from (12) that:

$$\begin{aligned}
V((n+1)T) - V(nT) &\le \hat{\xi}(nT)^T \hat{\Pi} \hat{\xi}(nT) \\
&\le -\lambda_{\min}(-\hat{\Pi}) \left|\hat{\xi}(nT)\right|^2 \quad (14)
\end{aligned}$$

where

$$\hat{\xi}(nT) = \left[X(nT)^T, c_m((n-d(n))T)^T\right]^T$$

and

$$\hat{\Pi} = \begin{bmatrix} \Pi_{11} + A^T PA & A^T PB \\ * & B^T PB - Q \end{bmatrix}. \quad (15)$$

On the other hand, considering the Lyapunov function (6), one gets

$$\begin{aligned}
\lambda_{\min}(E^T PE) |X(nT)|^2 &\le V(nT) \\
&\le \alpha_1 |X(nT)|^2 + \alpha_1 (d_{12}+1) \\
&\quad \times \sum_{l=n-d_2}^{n-1} |X(lT)|^2 \quad (16)
\end{aligned}$$

where

$$\alpha_1 = \max\left\{\lambda_{\max}(E^T PE), \lambda_{\max}(H^T QH)\right\}.$$

Define

$$J_m = \sum_{n=0}^{M} \left[Z(nT)^T Z(nT) - \gamma^2 W(nT)^T W(nT) \right] \quad (17)$$

where M is a positive integer scalar.

Now, noting the zero initial condition and (12) and adding the left-hand side of the (13) to the right-hand side of the inequality (12), one has

$$\begin{aligned} J_m = & \sum_{n=0}^{M} \left[Z(nT)^T Z(nT) - \gamma^2 W(nT)^T W(nT) \right. \\ & \qquad \left. + V((n+1)T) - V(nT) \right] - V((N+1)T) \\ \leq & \sum_{n=0}^{\infty} \left[Z(nT)^T Z(nT) - \gamma^2 W(nT)^T W(nT) \right. \\ & \quad + (AX(nT) + BHX((n-d(n))T) \\ & \qquad + CW(nT))^T \\ & \quad \times P(AX(nT) + BHX((n-d(n))T) \\ & \qquad + CW(nT)) - X(nT)^T E^T PEX(nT) \\ & \quad + (d_{12}+1) X(nT)^T H^T QHX(nT) \\ & \quad - X((n-d(n))T)^T H^T QHX((n-d(n))T) \\ & \quad + \Big(X(nT)^T N_1 + c_m((n-d(n))T)^T N_2 \\ & \qquad \left. + W(nT)^T N_3 \Big) (T_1 X(nT) + T_2 W(nT)) \right] \\ \leq & \sum_{n=0}^{\infty} \xi(nT)^T \Pi \xi(nT) \end{aligned} \quad (18)$$

where $\xi(nT) = \left[\hat{\xi}(nT)^T, W(nT)^T\right]^T$ and where (19) is shown at the bottom of the page. Now, by the Schur complement formula, it follows from (5) that $\Pi < 0$, which together with (18) ensure that (4) holds under the zero-initial condition.

Moreover, the condition $\Pi < 0$ implies $\hat{\Pi} < 0$. Therefore, from (14) and (16) it is easily concluded that the system (3) is asymptotically stable. ■

Remark 5: The reduced conservatism of Theorem 1 benefits from the construction of the new Lyapunov function in (6), using a free weighting matrix technique, and no bounding technique is needed to estimate the inner product of the involved crossing terms. It can be easily seen that the derived sufficient conditions are discrete-delay-range-dependent. Therefore, it makes the treatment in the present paper more general with less conservative in compare to most existing results in the literature which are delay-range-independent, see for instance [18] and [20].

B. Control Design

This subsection is devoted to the design of the local capacity H_∞ control gain k_c by using the results in Theorem 1. Obviously, the matrix inequality (5) includes multiplication of the matrices P, N_i and the control gain k_c. In the literature, more attention has been paid to the problems having this nature (see for instance [18]). In the sequel, it is shown that, based on the Finsler's Lemma a convex programming algorithm in terms of LMIs is developed to solve the bilinear matrix inequality (5).

Lemma 1: (Finsler's Lemma [30]) Consider a vector $x \in \Re^n$, a symmetric positive definite matrix $\Theta \in \Re^{n \times n}$ and a matrix $\vartheta \in \Re^{m \times n}$, such that $rank(\vartheta) < n$. The following statements are equivalent:

i) $x^T \Theta x < 0, \forall x \text{ such that } \vartheta x = 0, x \neq 0;$
ii) $\vartheta^{T\perp} \Theta \vartheta^{\perp} < 0;$
iii) $\exists \tau \in \Re : \Theta - \tau \vartheta^T \vartheta < 0;$
iv) $\exists R \in \Re^{n \times m} : \Theta + sym\{R\vartheta\} < 0;$

The following theorem gives a sufficient condition for the existence of a local capacity H_∞ controller for the work systems (1a)–(1g).

Theorem 2: For prescribed $\gamma > 0$, $d_i > 0$, there exist a local capacity H_∞ controller in the form of (1g) such that the system (3) is asymptotically stable and with an H_∞ performance γ for any delay satisfying (2), if there exist matrices N_1, N_2, N_3 and positive-definite matrices P and Q such that the following LMI is feasible:

$$\vartheta^{\perp T} \hat{\Pi} \vartheta^{\perp} < 0 \quad (20)$$

where

$$\hat{\Pi} := \begin{bmatrix} \hat{\Pi}_{11} & \hat{T}_1^T N_2^T & N_1 \hat{T}_2 + \hat{T}_1^T N_3^T & A_1^T P & L_1^T \\ * & -Q & N_2 \hat{T}_2 & B^T P & L_2^T \\ * & * & sym\{N_3 \hat{T}_2\} - \gamma^2 I & C_1^T P & L_3^T \\ * & * & * & -P & 0 \\ * & * & * & * & -I \end{bmatrix}$$

with

$$\begin{aligned} \hat{\Pi}_{11} = & -E^T PE + (d_{12}+1) H^T QH + sym\{N_1 \hat{T}_1\}, \\ \widetilde{T}_1 = & [I, -I, 0], \\ \hat{T}_1 = & [0, 0, -I] \end{aligned}$$

and

$$\widetilde{T}_2 := [0, -I, 0, I, 0].$$

In this case, a desired control gain k_c can be obtained from the following inequality:

$$\hat{\Pi} + k_c \, sym\{\xi \, \vartheta\} < 0 \quad (21)$$

$$\Pi = \begin{bmatrix} \Pi_{11} + A^T PA + L_1^T L_1 & L_1^T L_2 + A^T PB + T_1^T N_2^T & L_1^T L_3 + A^T PC + N_1 T_2 + T_1^T N_3^T \\ * & L_2^T L_2 + B^T PB - Q & L_2^T L_3 + B^T PC + N_2 T_2 \\ * & * & L_3^T L_3 + C^T PC + sym\{N_3 T_2\} - \gamma^2 I \end{bmatrix} \quad (19)$$

with

$$\xi := \begin{bmatrix} N_1 & 0 \\ N_2 & 0 \\ N_3 & 0 \\ 0 & P \\ 0 & 0 \end{bmatrix},$$

$$\vartheta := \begin{bmatrix} \widetilde{T}_1^T & A_1^T \\ 0 & 0 \\ \widetilde{T}_2^T & C_1^T \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T,$$

$$\vartheta^\perp := \begin{bmatrix} S_1 & S_2 & 0 \\ I & 0 & 0 \\ S_3 & S_4 & 0 \\ 0 & 0 & I \\ 0 & I & 0 \end{bmatrix}$$

such that

$$\begin{bmatrix} \widetilde{T}_1 & \widetilde{T}_2 \\ A_1 & C_1 \end{bmatrix}^\perp = \begin{bmatrix} S_1 & S_2 \\ S_3 & S_4 \end{bmatrix}.$$

Proof: Let $T_1 := k_c\widetilde{T}_1 + \hat{T}_1$ and $T_2 := k_c\tilde{T}_2$. Then, the matrix inequality (5) can be rewritten as the inequality (21). Based on the Finsler's Lemma, it follows that (21) has a solution if the LMI (20) holds. ■

Remark 6: Theorem 2 provides sufficient conditions for the solvability of the local capacity H_∞ control problem for the work systems (1a)–(1g). It is shown that a desired controller can be constructed by solving the condition (21) in terms of the gain k_c when the LMI condition (20) is satisfied.

Remark 7: It is worth noting that the number of the variables to be determined in the LMI (20) is $2N(7N+1)$, where N is the number of interacting work systems. It is also observed that the LMI (20) is linear in the set of matrices N_1, N_2, N_3, P_2, Q, and the scalar γ. This implies that the suboptimal solution to the problem of delay-dependent local capacity H_∞ control can be found by solving the following convex optimization problem

$$Min \quad \lambda$$
$$subject\ to\ LMI\ (20)\ with\ \lambda := \gamma^2.$$

IV. Numerical Results

Consider the case of a supplier of components to the automotive industry and for which production data documents orders flowing between five work systems over a 162-day period. These work systems and the order-flow structure over this period is illustrated in Fig. 2. In this network, all order flows are unidirectional; therefore, the fundamental dynamic properties of capacity adjustment in the individual work systems are independent. Then, the internal flow of orders is approximated using the following matrix [4]:

$$R = \begin{bmatrix} 0 & 106/341 & 235/341 & 0 & 0 \\ 0 & 0 & 0 & 188/401 & 204/401 \\ 0 & 0 & 0 & 100/236 & 129/236 \\ 0 & 0 & 0 & 0 & 268/295 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

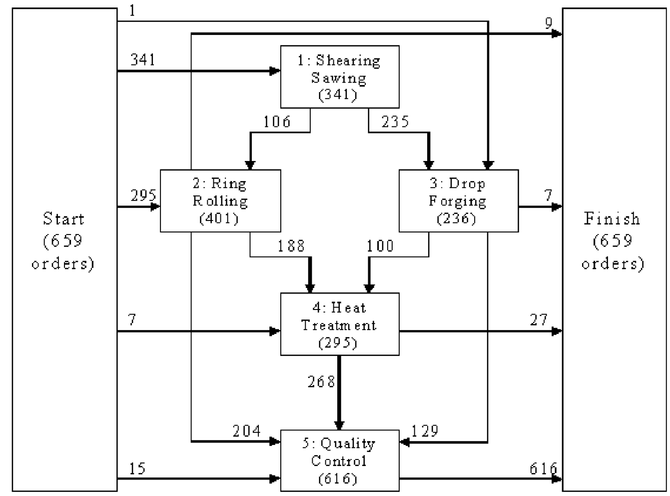

Fig. 2. A production network consisting of five work systems.

in which element R_{ij} is the total number of orders that went from work system i to work system j divided by the total number of orders that left work system i. Generally, in the special case of a unidirectional order-flow structure, upstream work systems do not receive work from downstream work systems, and the work systems can be numbered such that $R_{ij} = 0$ for $j \le i$.

Consider $L_1 = [I - R^T\ 0]$ and $L_2 = L_3 = 0$ in (1a)–(1g) with the sampling time $T = 1$ scd. It is required to find a controller gain k_c in (1g) such that the system (3a) is asymptotically stable and the H_∞ performance measure is satisfied as well. To this end, in light of Theorems 1 and 2, the LMIs (20), (21) using Matlab LMI Control Toolbox for different values of parameter d_2 with $d_1 = 0$, and different values of the H_∞ performance bound γ, are solved and the values of the parameter k_c are obtained and shown in Table I.

For simulation purposes, changes in the local capacity $(c_m(nT))$ of the work systems are considered under the controller gain $k_c = 0.1scd^{-1}$ and the H_∞ performance bound $\gamma = 0.15$. In this case, in response to a one-order step planned levels $(wip_p(nT))$ at the shearing-sawing work, time behavior of the local capacity changes at the shearing-sawing work system is depicted in Fig. 3 for four different values of the upper bound of the time-varying delay $d(n)$, i.e. $d_2 = \{1, 2, 3, 4\}$. It is also noting that a lower value of control parameter k_c tends to produce a more slow-acting dynamic system and, within limits, a higher value of k_c tends to produce a more fast-acting system.

V. Conclusion

The problem of local capacity H_∞ control for a class of production networks of autonomous work systems with time-varying delays in the capacity changes was investigated in this paper. The system under consideration was modelled as a discrete-time singular form. Attention was focused on the design of a controller gain for the local capacity adjustments which maintains the WIP in each work system in the vicinity of planned

TABLE I
CONTROLLER GAIN k_c W.R.T. d_2 AND γ

	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
$d_2 = 1$	0.6290	0.6705	0.7110
$d_2 = 2$	0.4185	0.4550	0.4955
$d_2 = 3$	0.2010	0.2725	0.3015

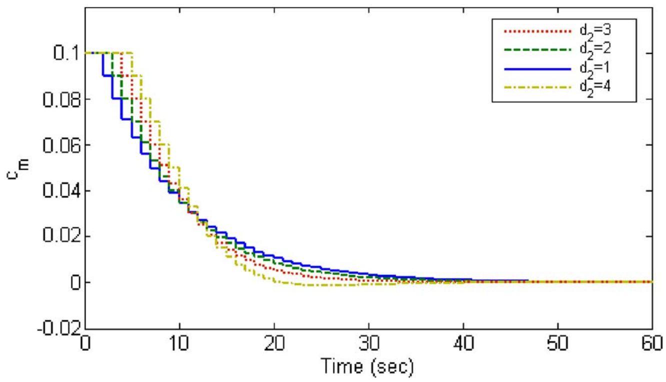

Fig. 3. Time behavior of the local capacity changes at the shearing-sawing work system.

levels and guarantees the asymptotic stability of the system and reduces the effect of the disturbance input on the controlled output to a prescribed level. In terms of a matrix inequality, a sufficient condition for the solvability of this problem was presented using an appropriate Lyapunov function, which is dependent on the size of the delay and is solved by existing convex optimization techniques. When this matrix inequality is feasible, the controller gain can be found by using LMI Toolbox Matlab. Finally, numerical results were provided to demonstrate the proposed approach.

In the proposed method, no information is shared between work systems and it has been shown that a good capacity plan is required if steady-state WIP errors are to be avoided. The provision of such a plan by sources external to the network may be a challenge because work systems require capacity to process both external order flows from outside the network and order flows within the network. Therefore, this method may be more appropriate when the order-flow structure is unidirectional. Delay in capacity adjustment has been included to represent the inability to make instantaneous adjustments, but variations in cost, delay and feasibility with adjustment magnitude, as well as capacity limits, have not been modeled. It is noted that for the special case of unidirectional order-flow structures, the local dynamic behavior of the work systems is not affected by the order-flow structure, and that the network dynamic behavior is simply characterized by series combinations of work system dynamics. However, order-flow information sharing still is beneficial in this case because it curtails propagation of disturbances to downstream work systems.

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