UNIVERSITY OF AGDER

Modeling and Simulation of Stabilizer for Remote Controlled Helicopter

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This Master's Thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

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Abstract

When working with a complex system it will always be useful to try to control the systems behavior to make it follow a given command. To make this possible different approaches can be taken and solutions purposed adjusted for the given system.

The main focus of this thesis has been to look at different methods and ways to control the movement and behaviour of a radio controlled(RC) helicopter. A comprehensive litterature study is performed to present the possiblilities and the different approaches to such a difficult and complex problem.
A method on how to obtain a linearized helicopter model is presented. Different control methods for multiple-input multiple-output systems is explained and tested on a linearized model. A controller for postition reference is modeled with Linear quadratic regulator (LQR) with integral action in addition to a kalman filter.

This project shows how to model helicopter behavior and how this can be represented with different linearized models. A Linear quadratic gaussionan controller with integral action is simulated and is shown to have satisfactory results. How to implement and test susch a soultion on a physical helicopter is discussed.

Keywords: Small scaled helicopter, Linearization, Integral action, Matlab/Simulink.

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Contents

1. Introduction

Ever since the first helicopter was invented it has never stopped to contribute to the everyday life around the world. As time has passed technological innovation have made it possible for every ordinary man to own a small helicopter and maneuver it by the help of a remote controller. Where the pilot in a full scale helicopter would make use of a onboard computer to control the movement a person with a RC helicopter would need to rely on his finger skills and vision to make sure the helicopter maintain the desired path via remote control from the ground. This has often shown to bee quite difficult for first time users and may in many cases lead to expensive crashes throughout a long period of training. It would save both money and time if it in some way would be possible to reduce the time spent on training by connecting something to the helicopter and thereby control and stabilize the helicopter for the user.

The idea is to investigate the possibility of constructing a stabilizer which reduces this complexity. A desired solution is to let the pilot control one movement in one direction or rotation around one axis while a microcontroller attached to the helicopters body controls the remaining degrees of freedom.

An important question is how this should be done and what kind of model this would be based on. It would be useful if it was possible to implement this control system on a onboard microcontroller in assosciation with sensors and measuring devices.
This thesis is ment to give a novice reader a good overview over the complexity of controlling a helicopter. In chapter ?? the concept helicopter movement is explained in detail and the differntial equations of motion is presented. In chapter ?? the differnt linearized models is explained and suitable contol methods is presented.. A purposed simulation model is constructed. At last some simulaton results is presented in ?? alnong with a chapter about instrumentation and how one could implement this on a physical helicopter.

2. Concept of Helicopter movement

2.1 Introduction

To fully understand the concept of how to stabilize a helicopter it is important to obtain some knowledge related to helicopter movement. As any other moving object a helicopter will have a certain amount of degrees of freedom. A degree of freedom relates to a direction in a coordinate system which a object can move frealy around or along. If an object is located in a three dimensional coordinate system x,y,z movement along x axis is oone degree of freedom and angular movement around the same axis is another.

If a local coordinate system is placed in the middle of a RC helicopter (figure 2.1) it is common to have the z axis pointing downwards, the x axis to the right and y to the left. The helicopter will have a total amount of six degrees of freedom. These are the translative movements along x,y and z axis as well as angular movemnt around each of them. The helicopter consists of different key parts which is connected to a remote conroller which makes the movements possible.

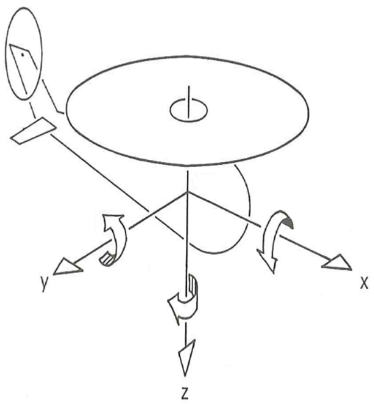


Figure 2.1: Helicopter with local coordinate system

2.2 Movement related to the Z axis

The helicopter has two main parts which contributes to change of the movement. A main rotor is located at the top of the fuselage and is the main source to change the postion along the z-axis. To lift of the ground or gain altitude the main rotor must generate more lift force then the force of gravity pullling it down.

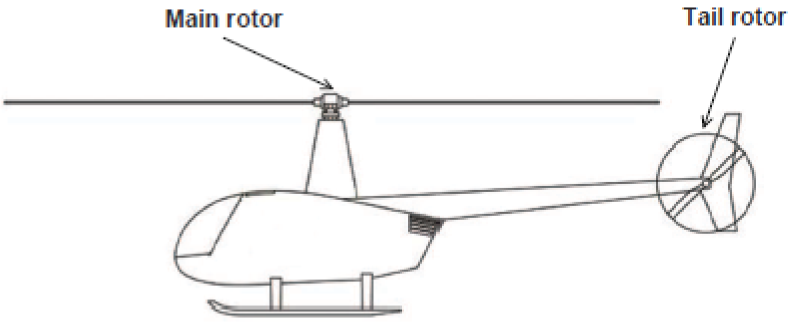


Figure 2.2: Main parts of Helicopter

At a distance from the main fuselage there is a tail rotor whichs main task is to control the angular movemnt around z.

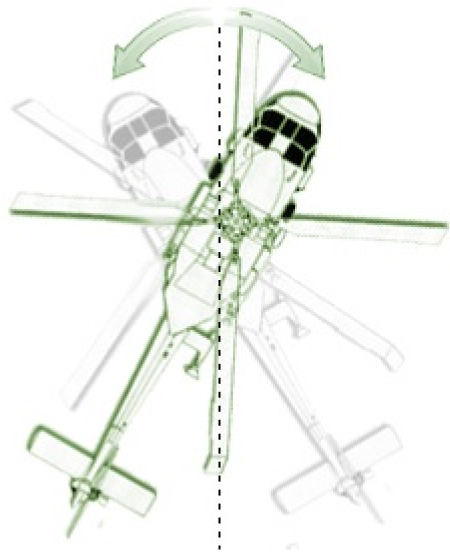


Figure 2.3: Angular movemnt around z

The main and tail rotor consists of blades. These blades are shaped to be as aerodynamical as possible. The angle between the blade and the reference plane is called the angle of attack or pitch angle.

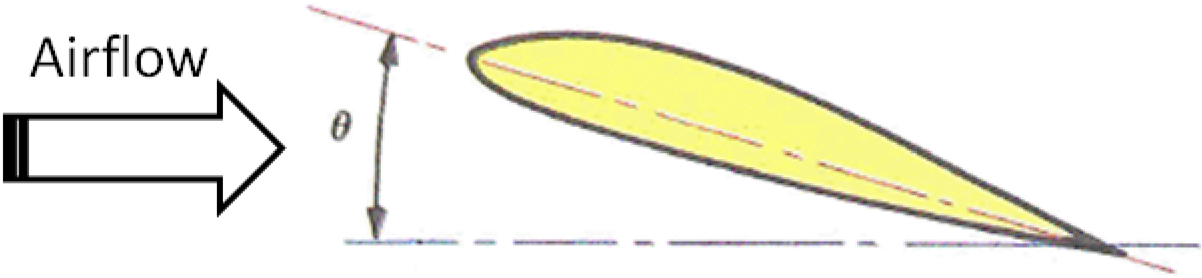
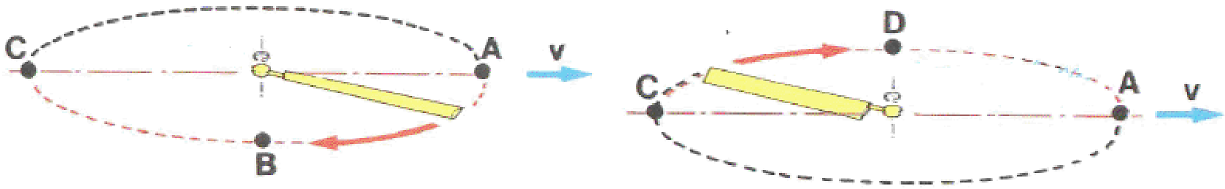


Figure 2.4: Pitch angle

When the angle is increased or decresed for all blades on the main rotor alonge the whole rotation, the helicopter will change position along the z-axis more rapidly while usually maintaing the same thrust as its changes posistion. This feature is called collective pitch. The translative motion along z is denoted as w and the agular r.

2.2.1 Cyclic Pitch

Where the collective pitch changes the pitch angle for all blades around the whole rotation the cyclic pitch changes the angle of the blades only for a certain part of the rotation. This leads to translative movement along the x and y axis.



(a) Cyclic pitch in x direction

(b) Cyclic pitch in y direction

Figure 2.5: Cyclic pitch

To make cyclic pitch possible the helicopters need a mechanism to transfer lateral and longitudinal stick input to change of cyclic pitch. This mecahnism is located under the mainrotor and is called a swashplate.

2.3 Movement realted to X axis

When the helicopter fly forward the rotor disc tilitd forward trhough the application of cyclic pitch - pitching the blades down on the advancing side and pitching up on the retreating side. This maneuver directs the thrust vector forward and apllies a pitching moment around y axis (2.6) to the helicopter fuselage and accelerating the hekicopter into forward flight. The translative motion along x is denoted as u and the agular p

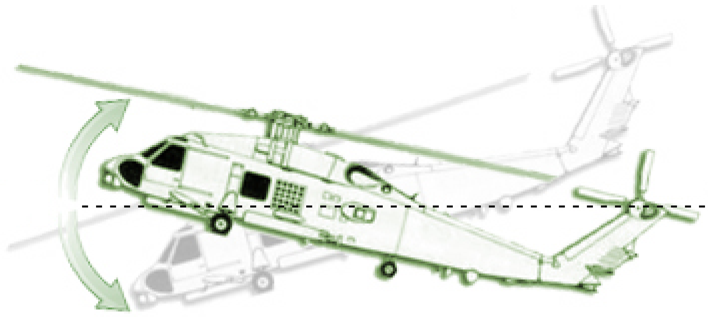


Figure 2.6: Angular movemnt around Y

2.4 Movement realted to Y axis

The same concept of angular and tranlsative movement around X also happens when the helicopter is moving from side to side (2.7) The translative motion along y is denoted as v and the agular q

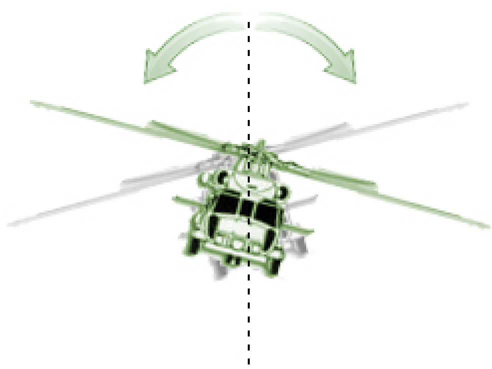


Figure 2.7: Angular movement around Y

2.4.1 Summary

u and $\dot{v}$ describes the lateral and longitudinal motion of the fuselage.
p and $\dot{q}$ describes the pitch and roll motion of the fuselage.
w describes the verical motion of the fuselage.
r describes the fuselage yaw motion.

Helicopter input	**Movement**
Left/right cyclic	Roll
Forward/backwards cyclic	Forward/backward
Left/right tail	Yaw
Collective pitch/Throttle	Climb/dive

3. Modeling of the helicopter

3.1 Introduction

To fully understand the problem of how to model a helicopter in a simulation inviroment it is necesarry to investigate all possibile methods and the different approaches which can be made. In the book 'Helicopter Flight Dynamics' by Padfield [2] the construction of a helicopter dynamic model is explained in detail along with a linearized model. This modeling prochedure is based on a full scale helicopter and it consits of quite complex terms. This model is used in 'Small-Size Unmanned Helicopter Guidance and Control' by Kurusu [13] and in 'Constructing and Simulating a Mathematical Model of Longitudinal Helicopter Flight Dynamics' by Fahad A Al Mahmood [14] to control the behavior of the helicopter. The problem with such a approach is the prochedure which needs to be gone through to obtain all parameters needeed. In [7] and [17] a detailed description of all forces and equations is given. [7] uses the nonlinear helicopter model presented by Padfield in addition to lead compensators and inner and outer control loops. The linearized model presented by Padfield is further adopted to small scaled helicopter by Mettler [3]. Mettlers approach is often used in projectr related to control and simulation of RC helicopters. Versions of this modeling method is shown to be valid in various reports and thesis. ([16, 9, 8, 6] and [11]).

In this chapter the path from the nonlinear model by Padfielfd through to Mettlers version is explained to give a good description on how a suitable simulation model can be obtained. This chapter will present the helicopter model which later will be controlled. First the spesifications of the helicopter will be defined. Second the nonlinear model, the linearized model and the simultion model used will be presented.

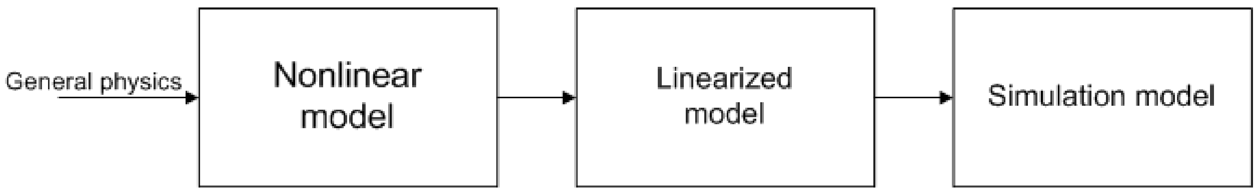


Figure 3.1: Modeling process

To simulate and try to control the behavior of the helicopter we need to construct a dynamical model which gives a better representation of the differrent aspects of helicopter movement.

3.2 Nonlinear model

3.2.1 Reference geometry

When constructing a nonlinear dynamical model of the helicopter it is important to denote the forces and moments in the correct way. Recalling that the translative velocities in relation to a local cooordinate system is denoted as u,v,w and the angular movemnts p,q,r. This local coordinate system is also known as the body frame. In relation to the navigational frame the angular velocities is θ ,ϕ, ψ also known as pitch, roll and yaw.

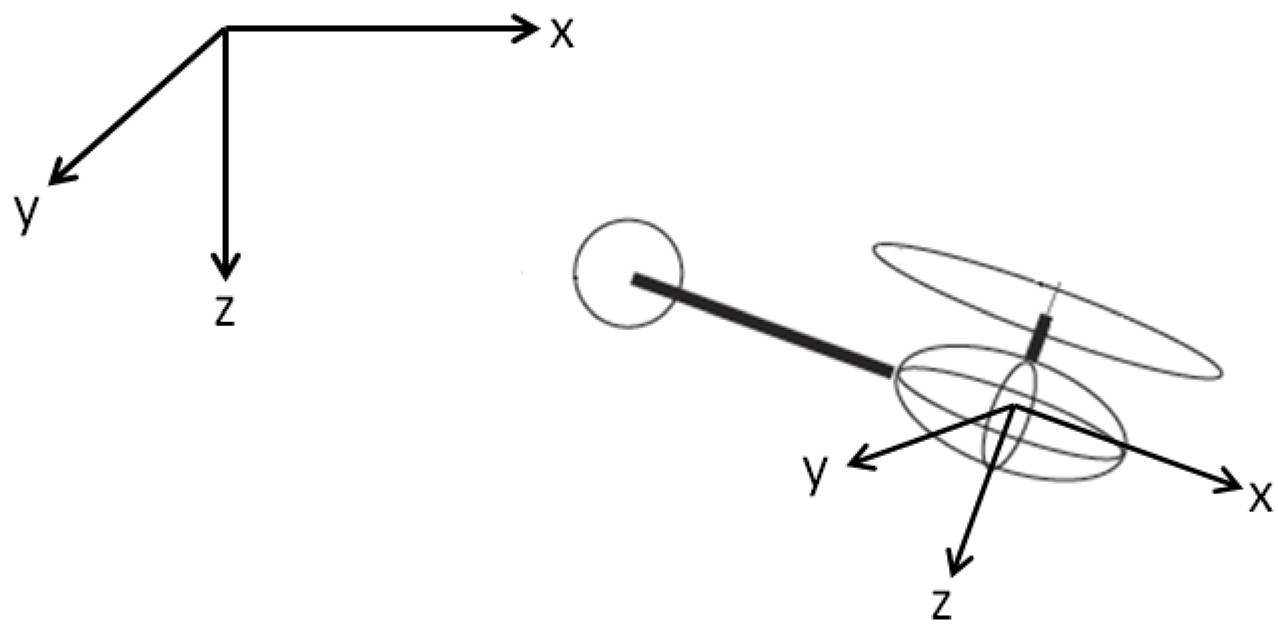


Figure 3.2: Reference axis

To rotate a vector about a single axis it has to be multiplied by a transformation matrix. Rotation abot each of the three axis is given by:

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} R_z(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The total conversion from navigation frame to bodyframe is given by multiplying the three transformation matrices together:

$$C_b^n = R_x(\phi)R_y(\theta)R_z(\psi) \tag{3.1}$$

$$C_b^n = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\cos\theta \end{bmatrix}$$

3.2.2 Translational movement

Newton second law states that:

$$F = \frac{d}{dt}(mV_T) \tag{3.2}$$

F=total external force aplplied to the body.
V_T=Total velocity of the body.
This can be further explained as:

$$\frac{d}{dt}(V_T)_E = (\frac{d}{dt}V_T)_B + \omega \times V_T s \tag{3.3}$$

where $\omega \times V_T$ is denoted as the Coriolis effect which is known as the motion resulting from the relative angular veliocity of the moving frame with respect to the moving frame ([13])
In the body frame the vectors is expressed as

$$\vec{V_T} = \vec{i}u + \vec{j}v + \vec{k}w \tag{3.4}$$
$$\vec{\omega} = \vec{i}p + \vec{j}q + \vec{k}r \tag{3.5}$$

The total force $\vec{F}$ acting on the helicopter fuselage is then

$$\vec{F} = m\left\{(\dot{u} + qw - vr)\vec{i} + (\dot{v} + ur - pw)\vec{j} + (\dot{w} + pv - uq)\vec{h}\right\} \tag{3.6}$$

$\vec{F}$has components acting in each direction and can thereby be expressed as

$$\vec{F} = \vec{i}F_x + \vec{j}F_y + \vec{k}F_z \tag{3.7}$$

The gravity component will always point downwards in the navigation frame and this is threrfore needed to be multplied by the transformation matrix.

$$F_{gb} = m \cdot C_b^n \cdot \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix}$$

$$F_{gb} = m \cdot g \cdot \begin{bmatrix} \sin(\theta) \\ -\sin\phi\cos\theta \\ \cos\phi\cos\theta \end{bmatrix}$$

Combining the to previos equations and adding the components of gravity gives us the following differntial equations for the translational movement.

$$\dot{u} = -wq + vr - g \cdot sin(\theta) + \frac{F_x}{m} \tag{3.8}$$

$$\dot{v} = wp - ur + g \cdot sin(\phi)cos(\theta) + \frac{F_y}{m} \tag{3.9}$$

$$\dot{w} = uq - vp + g \cdot cos(\phi)cos(\theta) + \frac{F_z}{m} \tag{3.10}$$

Where:

u, v and w	Translative velocity in x,y and z direction respectivly
p, q and r	Angular velocity around x,y and z axis respectivly
θ, ϕ and ψ :	Angular velocity navigational frame (yaw, pitch, roll)
m	Mass of helicopter
F_x, F_y, F_Z :	Forces acting in x, y and z direction respectivly.

The forces $\frac{F_x}{m}$ etc. will be presented as X,Y,Z since they consits of multiple forces acting on the helicopter.

3.2.3 Angular movement

Newtons second law on rotational form states:

$$\vec{M} = \frac{d}{dt}(\vec{H}) \tag{3.11}$$

M=total torque applied to the body. H=Total angular momentum.
This can be further explained as:

$$\vec{H} = I\vec{\omega} \tag{3.12}$$

$$\frac{d}{dt}(\vec{H})_E = \frac{d}{dt}(\vec{H})_B + \vec{\omega} \times \vec{H} \tag{3.13}$$

The total moment $\vec{M}$ in the body frame is expressed as:

$$\vec{M} = (\dot{p}I_{xx} + qr(I_{zz} - I_{yy}))\vec{i} + (\dot{q}I_{yy} + pr(I_{xx} - I_{zz}))\vec{j} + (\dot{r}I_{zz} + pq(I_{yy} - I_{xx}))\vec{k} \tag{3.14}$$

The Moments $\vec{M}$ has components acting in each direction and can thereby be expressed as:

$$\vec{M} = \vec{i}M_x + \vec{j}M_y + \vec{k}M_z \tag{3.15}$$

Rearrangeing the terms with repect to acceerations gives us:

$$\dot{p} = \frac{1}{I_{xx}}(-qr(I_{yy} - I_{zz}) + M) \tag{3.16}$$

$$\dot{q} = \frac{1}{I_{yy}}(-pr(I_{zz} - I_{xx}) + N) \tag{3.17}$$

$$\dot{r} = \frac{1}{I_{zz}}(-pq(I_{xx} - I_{yy}) + L) \tag{3.18}$$

Where:

p, q and r	Angular velocity in x,y and z direction respectivly
I_{xx}, I_{yy} and I_{zz}	Moments of inertia in x,y and z direction respectivly
L, M, N :	Moments acting in x, y and z direction respectivly.

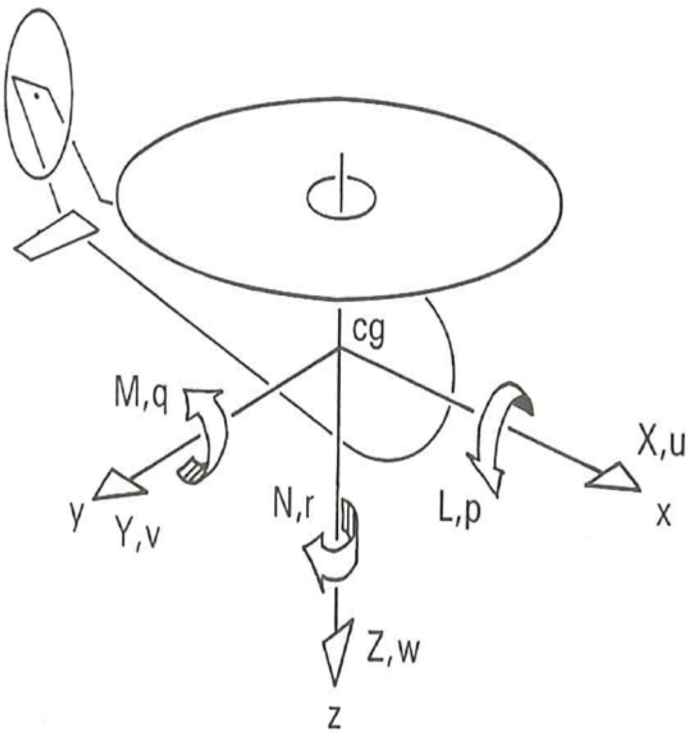


Figure 3.3: Helicopter with moments and forces

The equations of motion for the helicopter related to the local coordinate system is now been explained in detail and can be summarized by figure??. The moments in L, M and N can be further explained in the following way

$$L = L_R + L_{TR} + L_f + L_{tp} + L_{fn} \tag{3.19}$$
$$M = M_R + M_{tp} + M_f \tag{3.20}$$
$$N = -Q_e + N_{vf} + N_{tf} \tag{3.21}$$

$$X = X_{mr} + X_{fus} \tag{3.22}$$
$$Y = Y_{mr} + Y_{fus} + Y_{tR} + Y_{vf} \tag{3.23}$$
$$Z = Z_{mr} + Z_{fus} + Z_{ht} \tag{3.24}$$

This shows that the forces and moments have different components based on the given specifications of the helicopter.

The angular movement which now is expressed is the with respect to the body of the helicopter. We need to transform to earth related angular movement, better known as roll, pitch and yaw. These are represented as:

$$\dot{\phi} = p + \tan(\theta)(q\sin(\phi) + r\cos(\phi)) \tag{3.25}$$

$$\dot{\theta} = q\cos(\phi) - r\sin(\phi) \tag{3.26}$$

$$\dot{\Psi} = \sec(\theta)((q\sin(\phi)) + r\cos(\phi)) \tag{3.27}$$

Nine sets of differntial equations have now been presented and concludes the explanation of the nonlnear model of the helicopter.

3.3 State space

As any other sytem the helicopter will give a output based on a given input. The helicopter is a multiple-input multiple-output (MIMO) system. A common way of represent a MIMO system is in the form of state space. This makes us able to represent our system in a compact form and makes the simulation proceduere more visuable. The shape of the system is given as:

$$\dot{x} = Ax + Bu \tag{3.28}$$

$$y = Cx + Du \tag{3.29}$$

Where x is denoted as the internal variables of the system or states. Y equals the outputs of the sytem. Using the state space form makes it easier to modern control methods on bigger systems.

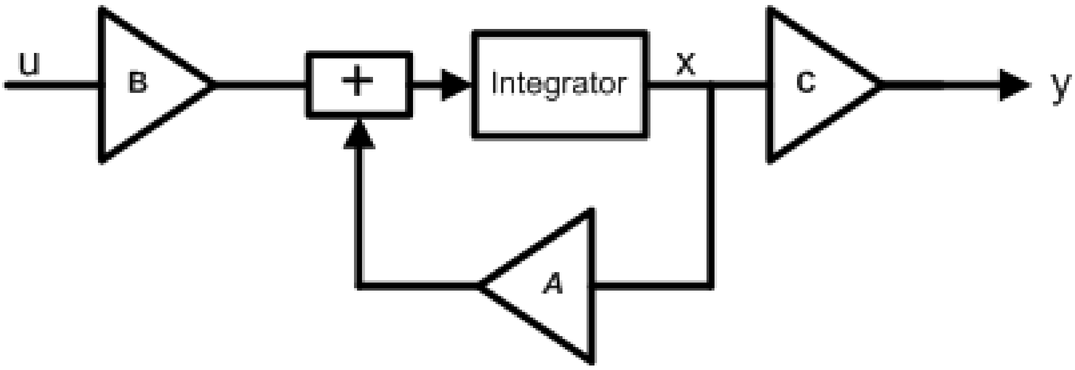


Figure 3.4: State space block diagram

3.4 Linearization

When we linearize a nonlinear differential equation, we linearize it for small signal inputs about the steady state solution when the small-signal input is equal to zero.[1] The steady state equals the equilibrium of the sytem. Its commom to say that we linearize the nonlinear equations with a linear equation for small excursions about the equlibrium point.

$$f(x) - f(x_0) \approx \left.\frac{df}{dx}\right|_{(x=x0)} (x - x0) \tag{3.30}$$

To make the linear system reflect the behavior of the nonlinear system we have to linearize around a trim point to be able to implement methods known from modern control theory.
By a trim point we mean a operating point of the helicopter such as a specified torque input on the two rotors. It can be shown that linear control methods used with a linerized model only will function when applied with a constant refrence value and not with a continous function such as a sine or cosine function.

When a nonlinear sytem such as this is linearized it is assumed that the external X,Y,Z and moments L,M and N can be represnted as analytic functions of the diturbed motion and their derivatives. As explained in section?? the forces and moments consists of different components. Using Taylors theorem for analytic functions gives as an example the terms for the force X.

$$X = X_e + \frac{\delta X}{\delta u}\delta u + \frac{\delta X}{\delta v}\delta v + .. \tag{3.31}$$

$$\frac{\delta X}{\delta u} = X_u \tag{3.32}$$

The states of the helicopter are defined as

$$x = [u, v, p, q, \phi, \theta, a, b, w, r] \tag{3.33}$$

To linearize our eight sets of equations we need to compute the steady state equations for each sate. As an example we solve the lineariztion problem for the transliational movement in x direction u. As explained the equation for u is:

$$\dot{u} = -wq + vr - g \cdot sin(\theta) + \frac{F_x}{m} \tag{3.34}$$

Rewriting terms which is depended on time gives:

$$\frac{du}{dt} = -w(t) \cdot q(t) + v(t) \cdot r(t) + X - g \cdot \sin(\theta(t)) \tag{3.35}$$

This equation csn be further linearized using the method in...

$$u \;=\; \frac{d}{du}|_{ss}(X) \cdot \overline{u} + \frac{d}{dv}|_{ss}(X + r_{ss}) \cdot \overline{v} + \frac{d}{dp}|_{ss}(X) \cdot \overline{p} + \frac{d}{dq}|_{ss}(X_q + w_{ss}) \cdot \overline{q} \tag{3.36}$$

$$- \;\frac{d}{d\phi}|_{ss}(g \cos\theta_{ss}) \cdot \overline{\phi} + \frac{d}{dw}|_{ss}(X_w - q_{ss}) \cdot \overline{w} + \frac{d}{dr}|_{ss}(X + v_{ss}) \cdot \overline{r} \tag{3.37}$$

Doing this for all differential equations for all states gives an A matrix for the system. To see the full terms for each state see([2])

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{p} \\ \dot{q} \\ \dot{\phi} \\ \dot{\theta} \\ \dot{w} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} X_u & X_v + R_e & X_p & X_q + R_e & -g\cos\theta_e & 0 & X_w - Q_e & X_r + V_e \\ Y_u - R_e & Y_v & Y_p + W_e & Y_q & g\cos\phi_e\cos\theta_e & -g\sin\phi_e\sin\theta_e & Y_w + P_e & Y_r - U_e \\ L_u & L_v & L_p.. & L_q.. & 0 & 0 & L_w & L_r - .. \\ M_u & M_v & M_p - .. & M_q & 0 & 0 & M_w & M_r.. \\ 0 & 0 & 1 & \sin\phi_e\tan\theta_e & 0 & \Omega_a\sec\theta_e & 0 & \cos\phi_e\tan\theta_e \\ 0 & 0 & 0 & \cos\phi_e & \Omega_a\cos\theta_e & 0 & 0 & -\sin\phi_e \\ Z_u + Q_e & Z_v - P_e & Z_p - V_e & Z_q + U_e & -g\sin\phi_e\cos\theta_e & -g\cos\phi_e\sin\theta_e & Z_w & Z_r \\ N_u & N_v & N_p.. & N_q.. & 0 & 0 & N_w & N_r.. \end{bmatrix} \begin{bmatrix} u \\ v \\ p \\ q \\ \phi \\ \theta \\ w \\ r \end{bmatrix}$$

3.4.1 Linearized for RC helicopter

The linearization method described in previosus section can be shown to be simplified when we are working with a small scaled rotorcraft such as a RC helicopter.
It is assumed that the the linarization is done around a equlibrium point in hover where $[u, v, w] = [0, 0, 0]$.
Mettler purposes a method with more simplifications then Padfield. This because there are forces and moments which can be neglected and simplified when dealing with a small scaled rortorcraft in comparison to a full scale helicopter.
Notice how the terms for the tranlative and angular acceleration hav been reduced.

$$\dot{u} = X_u u - g\phi + X_a \tag{3.38}$$
$$\dot{v} = Y_v v + g\theta + Y_b b \tag{3.39}$$
$$\dot{\phi} = p \tag{3.40}$$
$$\dot{\theta} = q \tag{3.41}$$

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{p} \\ \dot{q} \\ \dot{\phi} \\ \dot{\theta} \\ \tau_f \dot{a} \\ \tau_f \dot{b} \\ \dot{w} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} X_u & 0 & 0 & 0 & 0 & -g & X_a & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & g & 0 & 0 & Y_b & 0 & 0 \\ L_u & L_v & 0 & 0 & 0 & 0 & 0 & L_b & L_w & 0 \\ M_u & M_v & 0 & 0 & 0 & 0 & M_a & 0 & L_w & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\tau_f & 0 & 0 & -1 & A_b & 0 & 0 \\ 0 & 0 & -\tau_f & 0 & 0 & 0 & B_a & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_a & Z_b & Z_w & Z_r \\ 0 & N_v & N_p & 0 & 0 & 0 & 0 & 0 & N_w & N_r \end{bmatrix} \begin{bmatrix} u \\ v \\ p \\ q \\ \phi \\ \theta \\ a \\ b \\ w \\ r \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & Y_{ped} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{col} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_{lat} & A_{lon} & 0 & 0 \\ B_{lat} & B_{lon} & 0 & 0 \\ 0 & 0 & 0 & Z_{col} \\ 0 & 0 & N_{ped} & N_{col} \end{bmatrix} \begin{bmatrix} \delta_{lat} \\ \delta_{lon} \\ \delta_{ped} \\ \delta_{col} \end{bmatrix}$$

By insection of the matrix we see a clear difference between this and the first more general method described in 3.4

3.5 Simulation model

The method by Mettler however is designed for an advanced helicopter and it is necessary to simplify it further to make it suitable for testing. The yaw motion will be neglected since this doesnt play a part in the complex dynamics and the yaw motion is often controlled by a gyro anyway.

Since the helicopter is in linearized in hover mode the helicopter is assumed to be whitout collective pitch. This to make the simulation process easier. Then the possible inputs and the complexity of the model have been reduced to only two inputs.

4. Helicopter Control

4.1 Introduction

To make the helicopter follow a desired path or maintain a certain heading or altitude it is necesarry to implement some sort of control method. The control systems main task is to stabilize the plant and make sure it performs according to the given spesification. It is also interesting to look at the complexity of the solution. If it is ment to be implentet on a microcontroller it would be useful if the solution dont take up to much space or is to complex to execute.

To grasp the concept of this control problem it is imprtant to get a good overview over the different approaches and the key differences between them. First we think of the helicopter as a full system with a given inputs and outputs. Recalling section?? that the helicopter has two inputs and a given number of outputs. One would think that the easiest way of controlling such a plant is by implementation of control methods such as PID control. The problem with such an approach is that PID is a single-input single-output(SISO) control method which makes the it hard to implemnt caused by the fact that the helicopter consits of many different variables which nis dependent on each other.

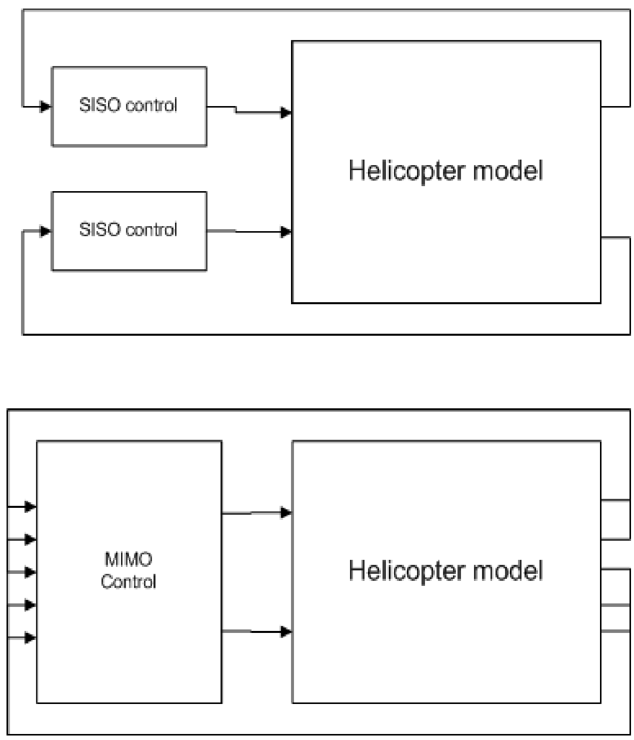


Figure 4.1: Ways of controlling a plant

Extensive resarch have been done around the world to try to control te behavior with different approaches and results. In [5] Linear quadratic regulator (LQR) in combination with PID control is used along with an Extended Kalman filter. A LQR controller is also used in [14]. In [4]a controller is designed for attitude,heave and yaw. Linear Qudratic Gaussian is used in [12] with setpoint tracking. This design is based on the prediction error method. Tracking of a reference point is implemented also in [16] and [11]. LQR and LQG along with tracking tends to be the most commonly used approaches, but ther are people which uses Lyapunov as in [15]. There are also aprroaches which results in bigger solutions such as [7] which controls a nonlinear helicopter model using lead compensators.

4.2 Linear Quadratic Regulator

The LQR controller is founded on thre assumptions
1) All states are available for feedback
2)All of the unsttable modes are acontrollable
3) All unstable modes are observable
The LQR seeks to minimize the cost function:

$$J = \int_0^{\infty} \left[x^T Q x + u^T R u \right] dt \tag{4.1}$$

$$\tag{4.2}$$

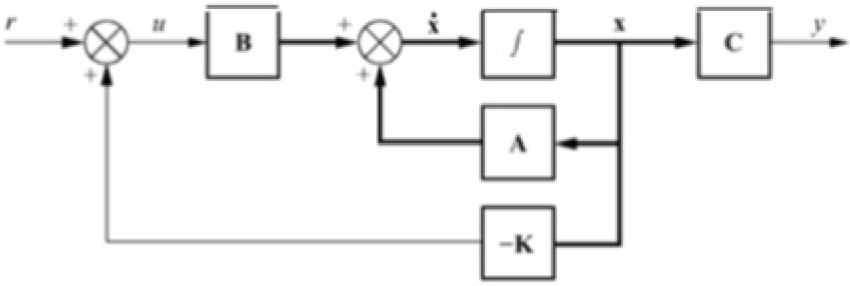


Figure 4.2: LQR blocks

where Q and R are used to tune the perfermance of the controller.

4.2.1 Controllability

The lqr regulator will try tostabilize the plant or in fact place the poles of the system. If an input to a system can be found that takes every state variable from a desired initial state to a desired final state , the sytem is said to be controllable; otherwise the sytem is uncontrollable.[1]
The controllability matrix is given by:

$$C_M = [B\, AB\, A^2 B ... A^{(n-1)} B] \tag{4.3}$$

If the matrix C_M is of rank n the system is said to be completly controllable.[1] The rank equals the number of linearly independent rows or columns in the controllabilty matrix.

4.3 State estimation

4.3.1 Kalman filter

Kalman filter is used to estimate startes in a given plant baced on a mathematiacal model. The filter is a modelbased algorithm used to estimate stateswhich is under tghe influence of random noise both in the plant and related to the process mesurements.

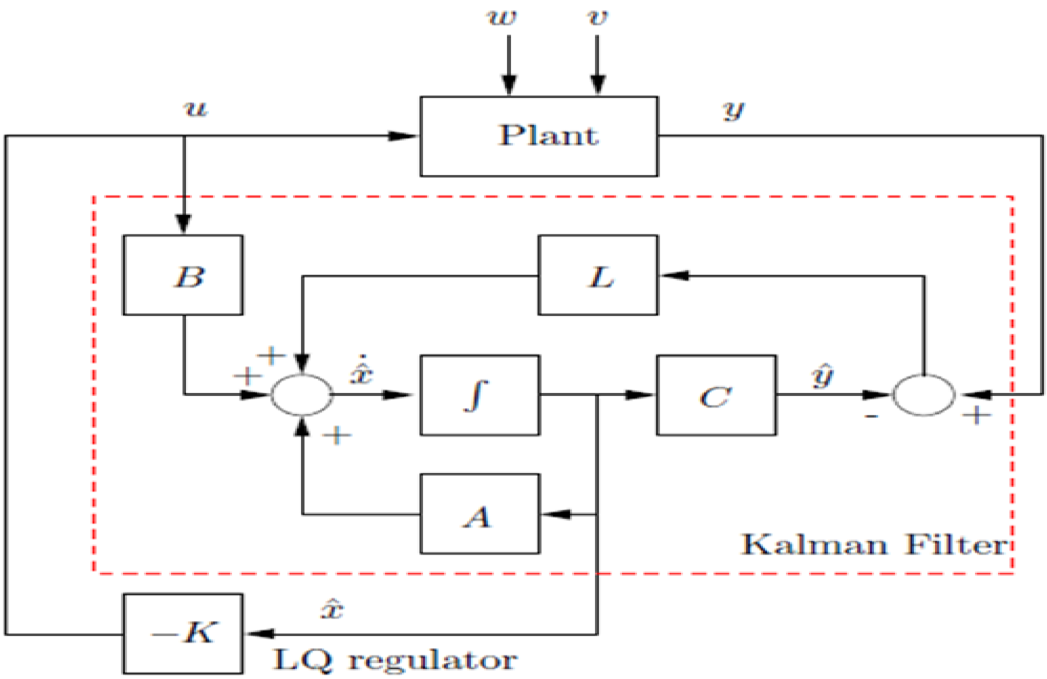


Figure 4.3: Block diagram of regulator with filter

4.4 Linear Quadratic Gaussian

$$\dot{x} = Ax + Bu + w_d \tag{4.4}$$

$$y = Cx + Du + w_n \tag{4.5}$$

The Linear Quadratic Gaussian (LQG) control method is a optimal controller for a linear system with white Gaussian noise. This type of controller is commonly used when the linear system is uncertain. Each state can be weighted by the quasratic matrix Q. The LQG method is a combination of a linear quadratic regulator(LQR) and a linear quadratic estimator(LQE). The objective of the the LQG is to minimize the cost function:

$$J = E\left\{lim\frac{1}{T}\int_0^T \left[x^TQx + u^TRu\right] dt\right\} \tag{4.6}$$

$$Q = Q^T \geq 0 \tag{4.7}$$

$$R = R^T \geq 0 \tag{4.8}$$

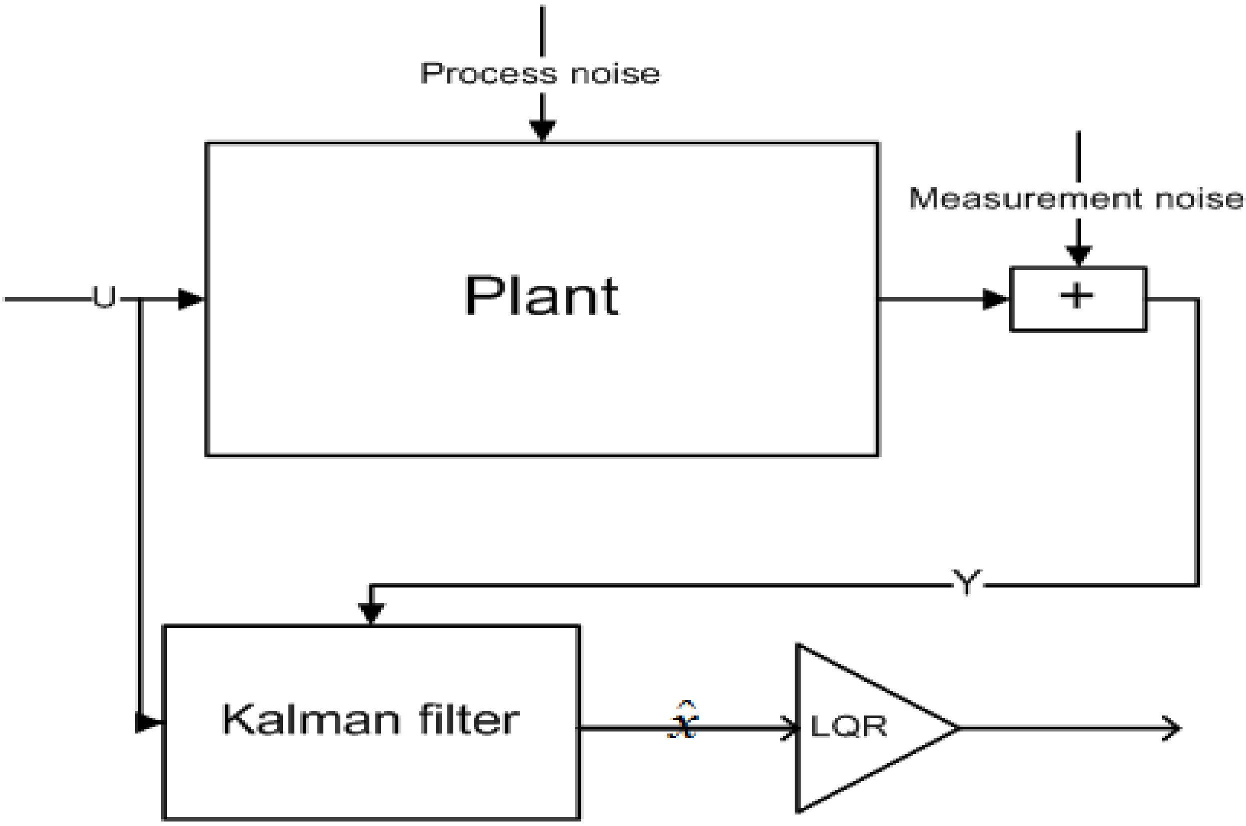


Figure 4.4: LQG concept

with the control $U(t) = -Kx(t)$ requires the avalibility of all states through process measurement. When the state variables are not accesible we can use $U(t) = -K\hat{x}$ where $x\hat{(t)}$ is an estimate of x(t)based on the output y.

The way of doing this is by repecating the process dynamics. We construct a copy of the system on the form:

$$\dot{\hat{X}} = A\hat{X} + Bu \tag{4.9}$$

We define the state estimation error as $\dot{e} = Ax - A\hat{x} = Ae$. If A is stable, the ereor will go to zero asymtotically. If A is unstable, e is unbonded and $\hat{x}$ will grow further apart from x. To avoid this problem we considere a correction term where the output y is fed back to the estimator:

$$\dot{\hat{X}} = A\hat{X} + Bu + L(y - \hat{y}) \tag{4.10}$$

$$\hat{x} = 0 \tag{4.11}$$

where L is thr observer gain matrix. The state estimationerror is now

$$\dot{e} = Ax - A\hat{X} - L(Cx - C\hat{x} = (A - LC)e \tag{4.12}$$

$$e(0) = 0 \tag{4.13}$$

The observer error will go to zero if L is chosen such that A-LC is stable

4.5 Tracking with integral action

While the LQR stabilizes all states it does not guarantee that the given plant go to a desired reference signal. To do this it is nescsarry to introduce the use of integral action. This means a intgerator and a gain is added in combination with the lqr controller. In addition to the states x it the sates z. The plant consitaing of both the x and z is called the augmented plant. Matlab has a buildt in function LQI which produces a big controller which consists of both the lqr and the gain integral acton. The introduction of integral action to track a refernce signals gives a state space model of

$$\dot{x} = Ax + Bu \tag{4.14}$$

$$\dot{z} = y - y_d \tag{4.15}$$

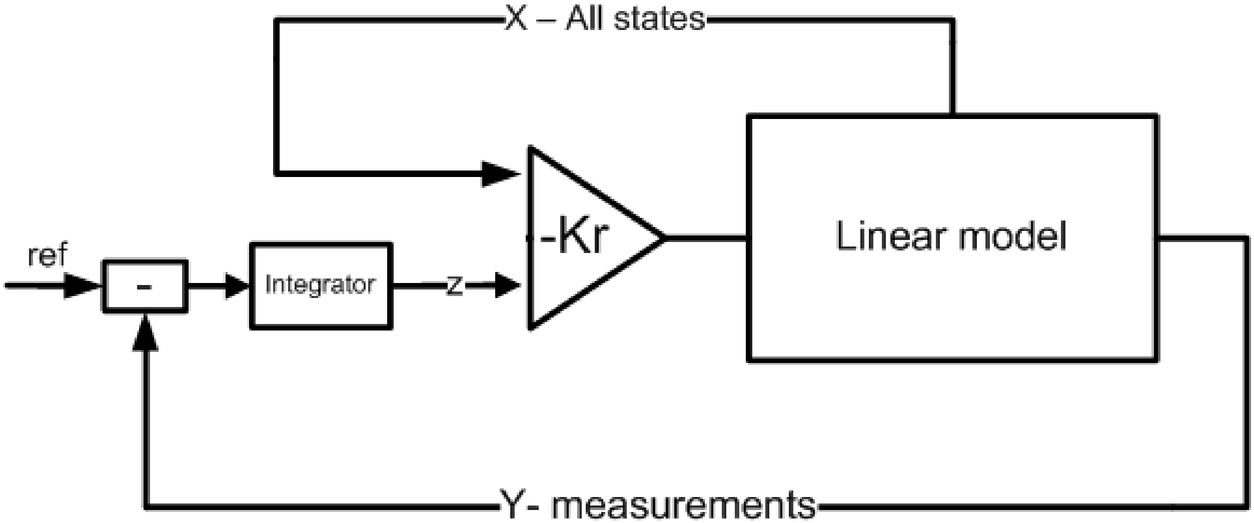


Figure 4.5: Linear qudratic regulator with integral action

$$\begin{bmatrix} \dot{X} \\ \dot{Z} \end{bmatrix} = \dot{X}_{aug} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} X_{aug} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u - \begin{bmatrix} 0 \\ 1 \end{bmatrix} y_d$$

This means that in order to get construct a controller for the new augmented plant we need to use A_new and B_new as system matrices. We wil then end up with a large controller and then we need to seperate the new big controller into the lqr controller and the gain matrix used in the integral action.

4.6 Control system layout

The RC helicopter has many states and variables. In order to develop a controller for a certain amount of degrees of freedom
the focus of this thesis hav been to used a method which is easy to understand and is proven to work whitin the given specifications. A study of the littaruture mentioned results in a conlusion that the approach in [11] is a good way of controlling the helicopter. Some modifications has though been done due to the parameters avaialble and the desired resutlts. The first thing which would me normal to assume is to adopt control algorthm directly on the model described in [16]. However this is shown to be difficult caused be interaction of roll and pitch movement as shown in [16, 11].

4.6.1 Attitude controller

First a Attitude controller is designed based on the model described in 3.4.1. Since it is decided that the system only will have input from the longitudinal and lateral input the terms relateds to these input is estracted into the B matrix.

$$A_1 = \begin{bmatrix} X_u & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & g & 0 \\ \hline L_u & L_v & 0 & 0 & 0 & 0 \\ M_u & M_v & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} B_1 = \begin{bmatrix} Alon & Alat \\ Blo & Blat \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The C matrix is designed such that the controller have meaurements from the roll and pitch angles, θ and ϕ.

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The attitude controller is designed with LQR and tracking with integral action.

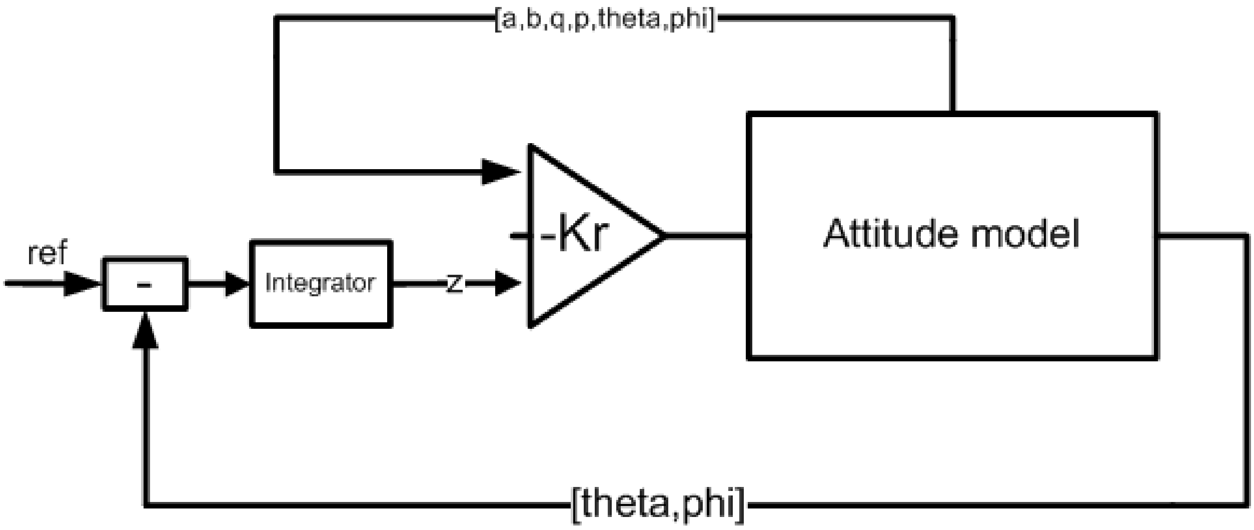


Figure 4.6: Attiude model

4.6.2 Full model

The atttiude controller is then implemented in the full model along with the lateral and longitudinal motion.

$$A = \left[\begin{array}{cccc|cc} \multicolumn{4}{c|}{(A_1 - B_1 * K_r)^{12x12}} & \multicolumn{2}{c}{0^{8x4}} \\ G & 0^{2x2} & G & 0^{2x2} & V & 0^{2x2} \\ \hline \multicolumn{4}{c|}{0^{2x6}} & I & 0^{2x4} \end{array}\right] \; B = \begin{bmatrix} 0^{6x2} \\ I \\ 0^{4x2} \end{bmatrix}$$

Where the G and V matrix is

$$G = \begin{bmatrix} -g & 0 \\ 0 & g \end{bmatrix} V = \begin{bmatrix} X_u & 0 \\ 0 & Y_v \end{bmatrix}$$

The lateral and longitudinal motion then becomes:

$$\dot{u} = X_u u - g(\theta + a) \tag{4.16}$$

$$\dot{v} = Y_v v + g(\phi + b) \tag{4.17}$$

The C matrix makes sure the X and Y position of the helicopter is measured.

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

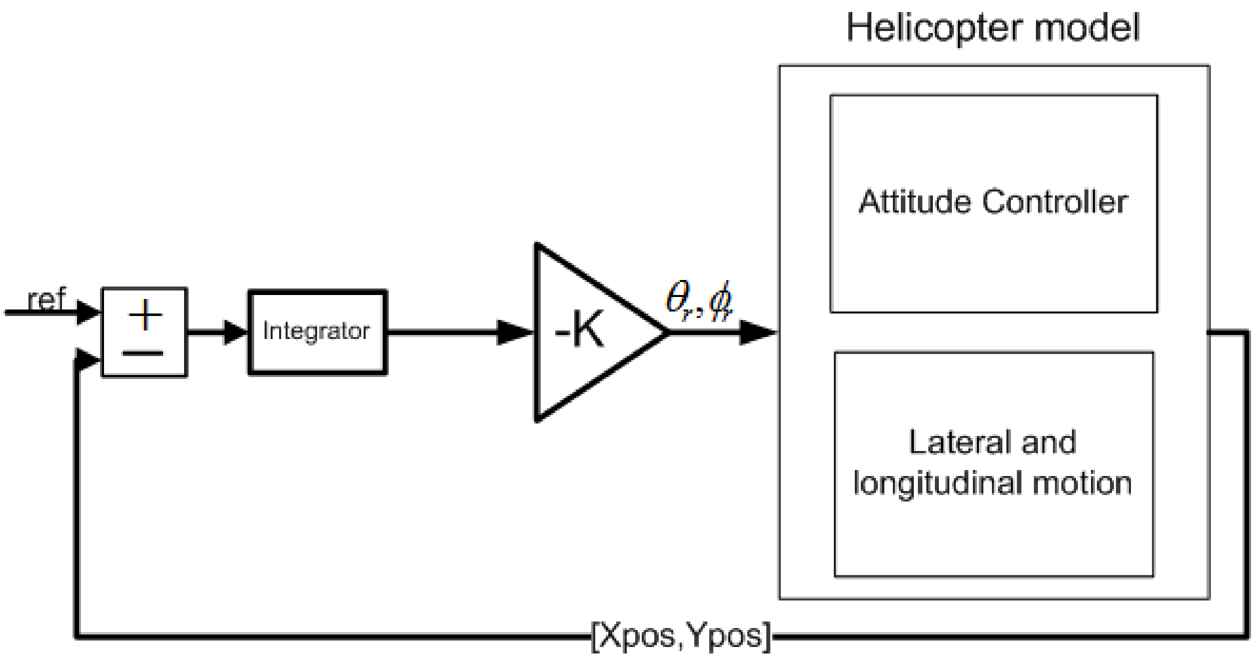


Figure 4.7: Control of helicopter model

5. Simulation and implementation

5.1 Simulation Objectives

The objective in this thesis is to model and simulate a stabilizer for a RC helicopter. Recall that a model which tracks the lateral and longitudinal position of the helicopter is descided to be constructed with a control system consisting of a LQR regulator with integral action and disturbance filter. This control system have to be tested on a linear system of a helicopter before it can be implemented on a physical helicopter.
The model which is simulated is obtained in the described in section?? and the parameters obtained is located in [16].
The operating operating point for which the model was linearized have the following parameters:

Helicopter model: Raptor 90 SE.
Main rotor angular speed: 1250 Rpm
Tail rotor angular speed: 5000 Rpm.

The other parameters used can be found in the appendix.

5.2 Attitude controller

The augmented plant for the attitude controller is:

$$a = \begin{bmatrix} A1 & 0 \\ -C1 & 0 \end{bmatrix} b = \begin{bmatrix} B1 \\ 0 \end{bmatrix}$$

This small part of the system is simulated to show how the pitch and roll moveemnt can be implemented.
A controller for lqr and a gain for the tracking is construucted using the *lqr* command in *Matlab* on the augmented plant. The total controller K_r becomes:

$$K_r = \begin{bmatrix} 16.1570 & 0.1264 & 3.2695 & -0.0041 & 15.2435 & -0.0140 & -31.6228 & 0.0299 \\ 0.1518 & 35.3960 & 0.0027 & 3.0921 & 0.0145 & 14.8676 & -0.0299 & -31.6228 \end{bmatrix}$$

The attitude controller is modelled in Matlab/simulink with the constructed controller. The integral action is implemented and tested for a given reference of 10 degrees for pitch angle and -10 degrees for roll which equals +-0.175 radians.
This reults in a measurement of the tracked states which behaves satisfactory5.1.

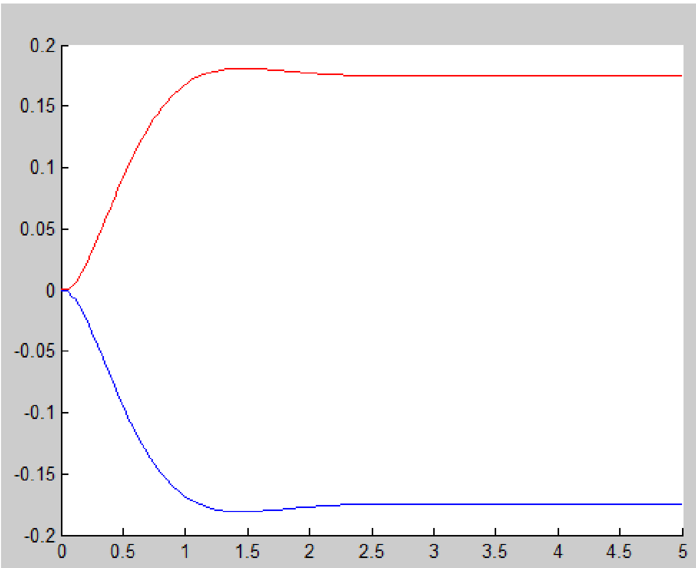


Figure 5.1: Results from attitude model

5.3 Simulation of full model

The full scale model is simulated with initial values for speed in x and y direvtion of 0.001. This is done to not make the simulation crash and to create a more life like enviroment. First it useful to check the respons of the system witout any control and with just measurements of the positions.

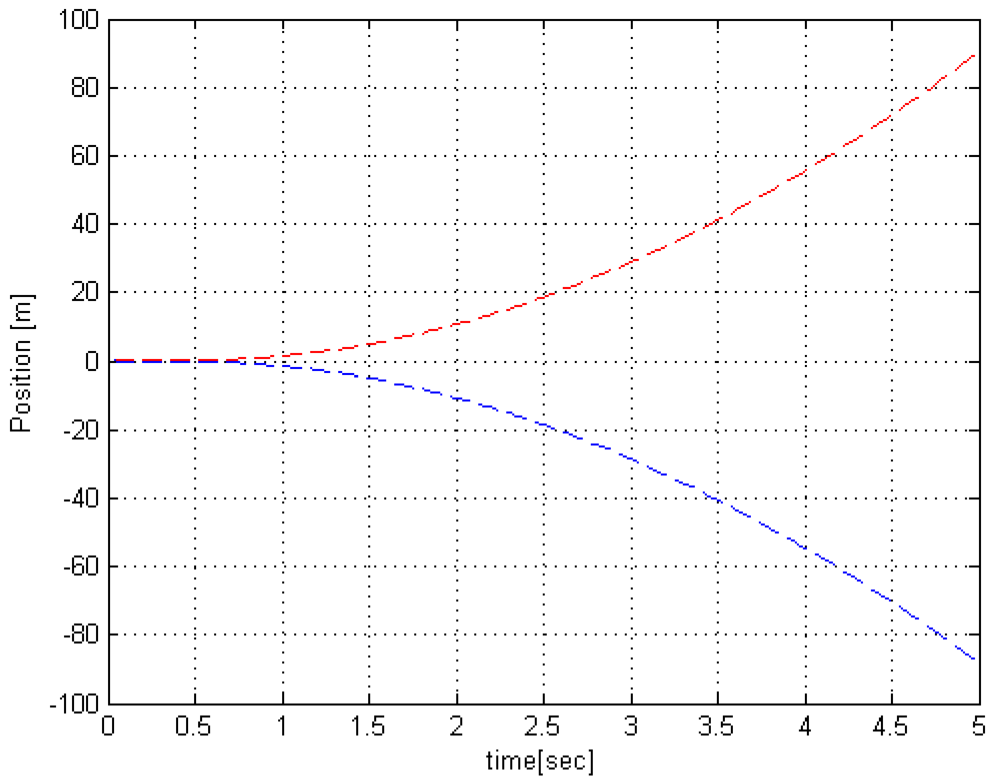


Figure 5.2: Position with no control

Figure 5.2 shows an uncontrolled respons which behaves as expected. The helicopter just moves far away in both directions.

5.3.1 Simulation with LQR controller

A LQR controller is constructed along with a gain for integral action using the *lqi* command in *Matlab*. First a test with only LQR controller and a random reference is tested.
The LQR gain F_1 is found to be

$$F_1 = \begin{bmatrix} 0.4039 & 0.0019 & 0.0323 & -0.0001 & 1.1473 & -0.0002 & 10.2313 & 0.0007 & 1.6191 & 0 \\ 1.1532 & 0 & & & & & & & & \\ 0.0009 & 0.4267 & -0 & 0.0248 & -0.0004 & 0.9909 & 0.0007 & 10.5470 & 0 & -1.6 \\ 0 & -1.11597 & & & & & & & & \end{bmatrix}$$

The respons in the position output(5.3) along with the control signal(5.4) has satisfactory results. The output becomes stable at a given reference.

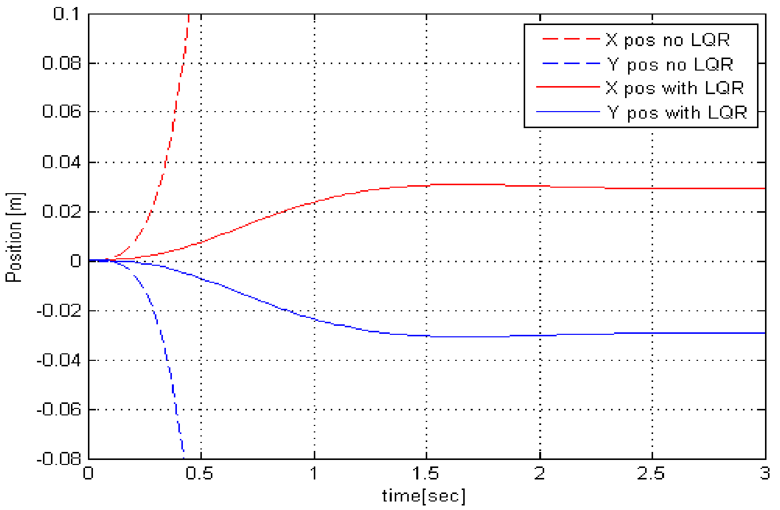


Figure 5.3: Position

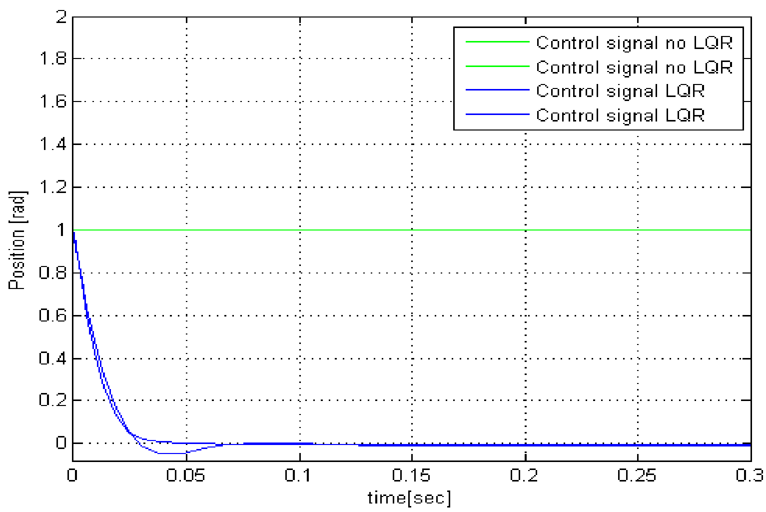


Figure 5.4: Control signal lqr

Stability

An important concept of lqr control is to make the system become stable. A bode plot is constructed to show the frequency response with respect to stability. This also show the gain and phase margins of the system.

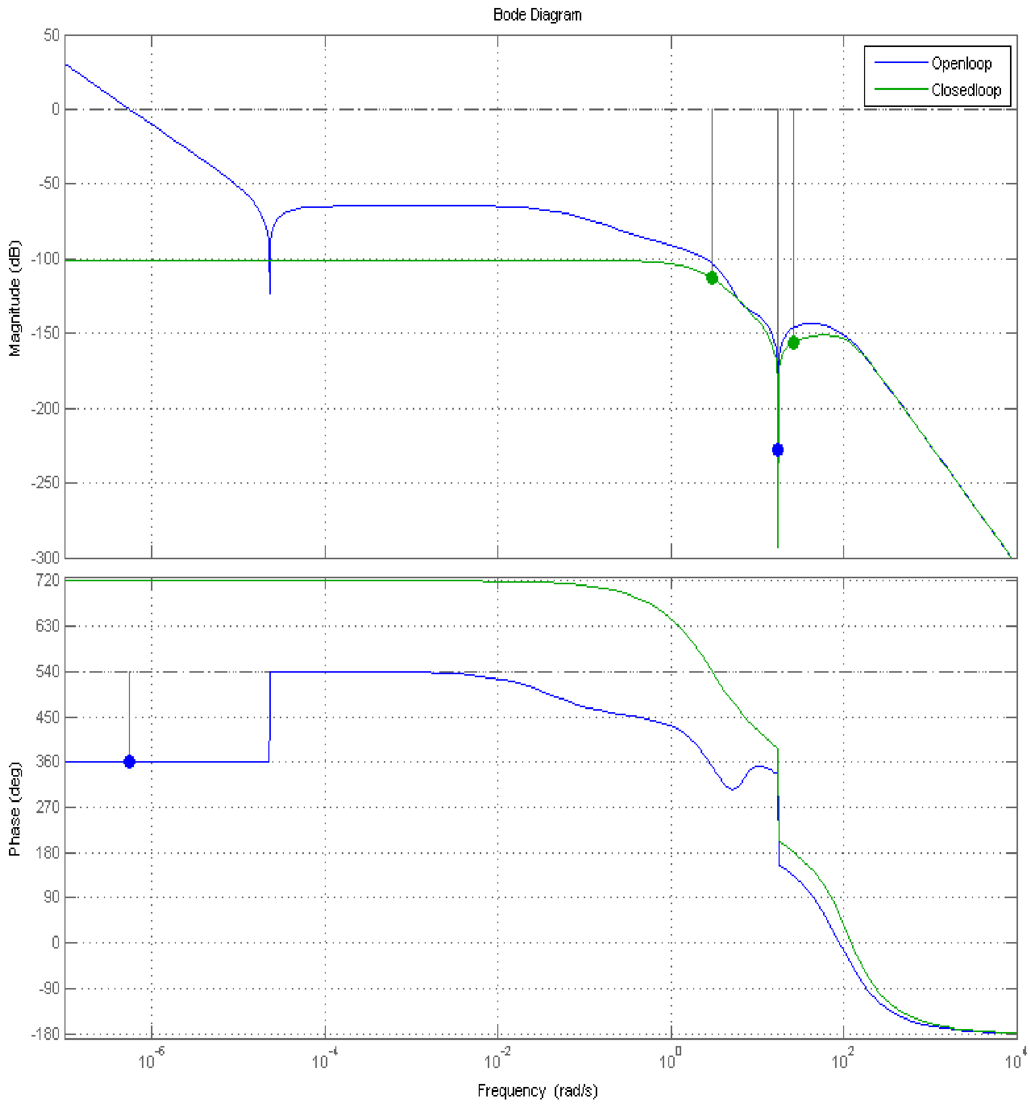


Figure 5.5: Bode plot of closed vs open loop

5.3.2 Simulation of tracking and kalman filter

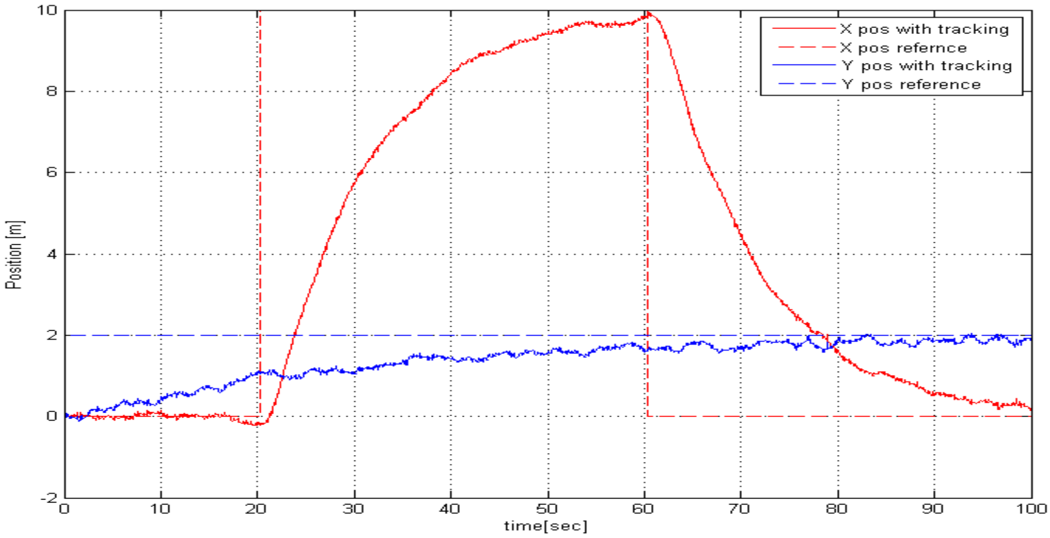


Figure 5.6: Respons of fulll model

The LQR gain is combined with a klamn filter and tracking with integral action. (5.6) A reference signal which goes to 10 meters after 20 seconds is given to the X positon reference while a reference of 2 meters is given to the y positon. It is clear that the respons of the system is quite slow. This has to do with the fact that the angle of the swashplate (longitudinal and lateral angel) must not exceed +-10 degrees [11]. To make this happend it was necesarry to adjust the Q matrix accoringly and have penalize the states for X and Y position with a value of 0.01. The R matrix has the form of a two by to identity matrix with 0.01 as a multiplied gain.

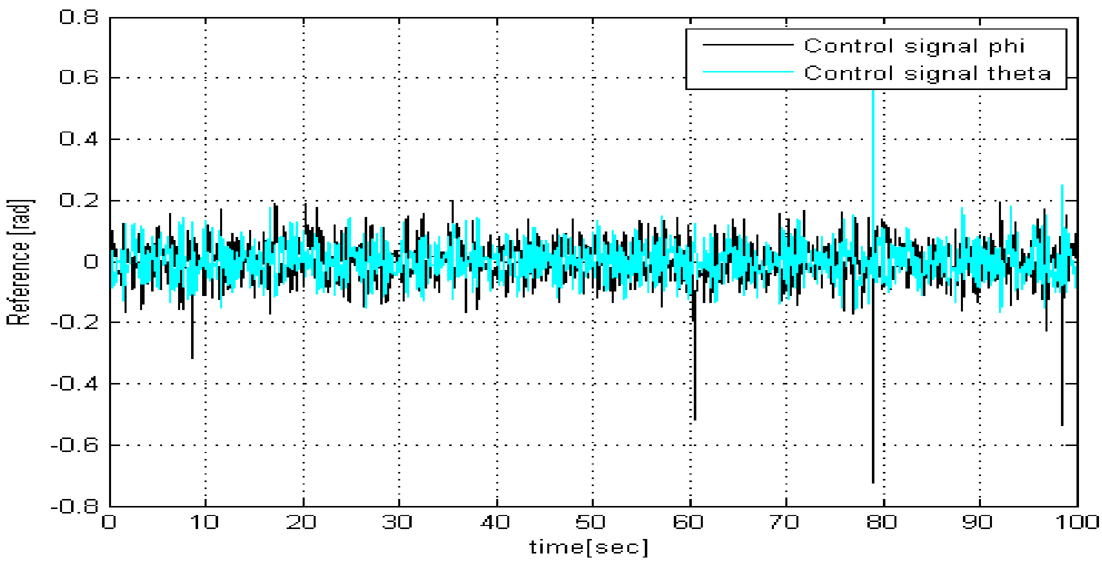


Figure 5.7: Control signal

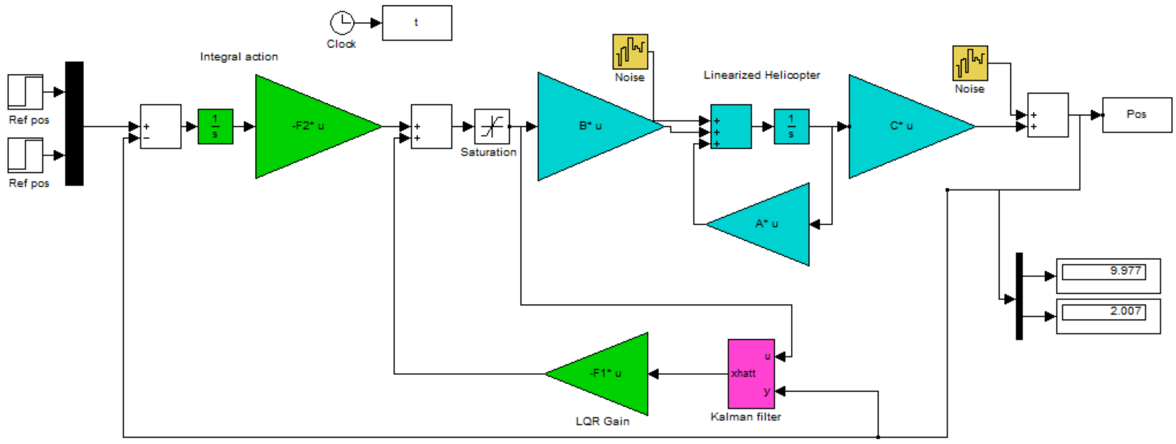


Figure 5.8: Full scale tracking model with kalman filter

5.4 Evaluation

The control sytem which is tested in based on a model presented in [11] and the parameters used is from [16]. This may create some deveation from the results which has been achieved in [11]. The controller can not exceed +-10 degrees and thid makes the system follow a refernece slowly.

5.5 Implementation

The regultaion and simulation of the linearized model may seem easy to control on the computer. Another thing is to actually immplement it and test it on a physical helicopter. Observer design have to be taken into consideration. The solution presented in this thesis assumes that all states are measurable.

If sensors were to be imlemnted there may occure problems in relations to electromagnetic interference from motor and speed controller if such is inplemnted [5]. This interference may cause the microcontroller to perisically reset.

5.5.1 Generation of C code

To nake use of the regulation algortihn it is nec nesesarry to generat C code from Matlab. This can be done in different ways and depends on the way the code is made in matlab. Both methods is additonal programming tools avaliable with Matlab. One way is to use Matlab Coder whch is makes it possible to generate C code directly from Matlab algorithms. This however demands that your matlab program only consits of code in a script. Another way which may be more useful is use of the Simulink Coder. This add-on generates C code from Simulink blocksets and makes it ready for a real-time target.

6. Conclusion and further work

The purpose of this thesis has been to invesigate the possibilities of constructing a stabilizer for a RC helicopter by implemenation of various control methods. A litterature study have been performed in order to understand the control problem of helicopter behaviour. A solution has been developed for parameters given for a Raptor 90 in hover. A solution for tracking of position refernce in x and y direction have been implemented and proven to have satisfactory results. How to implement such a solution on a physical helicopter have been discussed

This thesis presents good oppurtunities for further work and implentation possiblilities for future projects. One approach is to start with the linearized model presented by Mettler3.4.1 in [3]. The paramters in this model can be determined through experiments. This can however be quite difficult and require a high degree of technical insight. [3] and [16] presents paramteres for a range of RC helicopter types. Control of these models can be tested in Matlab/simulink and implentesd on a microcontroller with methods described in this thesis. It might be useful to construct a testbench such as the one desbribed in [10] which reduces the degrees of freedom and makes sure that the helicopter stays whitin a controlled inviroment.

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List of Figures

7. Appendix

```
clc
clear all
close all


N=2.982;
Ab=0.773;
Ba=0.618;
Lb=1172.4817;
Lu=-0.0244;
Lv=-0.1173;
Ma=307.571;
Mu=0.2542;
Mv=-0.06013;
Np=0;
Nr=-10.71;
N=2.982;
Nw=-0.7076;
taufi=30.71;

Xa=9.389;
Xu=-0.03996;
Yb=-9.389;
Yv=-0.05989;
Za=0;
Zb=0;
Zr=0;
Zw=-2.055;
g=-9.389;

Alo=4.059;
Alat=-0.01610;
Blo=-0.01017;
Blat=4.085;
Zcol=-13.11;
Ncol=3.749;
Nped=26.90;



A1=[(-taufi) Ab -1 0 0 0;...
    Ba (-taufi) 0 -1 0 0;...
    Ma 0 0 0 0 0;...
    0 Lb 0 0 0 0;...
    0 0 1 0 0 0;...
    0 0 0 1 0 0];
B1=[Alo Alat;Blo Blat;0 0;0 0;...
    0 0;0 0];


C1=[0 0 0 0 1 0;...
   0 0 0 0 0 1];

D1=zeros(2,2);
systest=ss(A1,B1,C1,D1)
```

```
%--------Creating Augmented plant--
a=[A1 zeros(6,2);-C1 zeros(2,2)];
b=[B1;zeros(2,2)];
c=eye(size(a));
d=zeros(size(b));
sys=ss(a,b,c,d);

[n,m]=size(B1);

Qnew=eye(size(a));
Qnew(7,7)=100;
Qnew(8,8)=100;
Rnew=eye(m,m)*.1;

Kr=lqr(a,b,Qnew,Rnew);
Krp=Kr(1:m,1:n);
Kri=Kr(1:m,n+1:n+2);


%----------------Creating big augmented plant--

G=[-g 0; 0 g];
I=eye(2,2);
V=[Xu 0;0 Yv];
 A=[a-b*Kr  zeros(8,4);...
       G zeros(2,2) G zeros(2,2) V zeros(2,2);...
       zeros(2,8) I zeros(2,2)];
B=[zeros(2,6) eye(2,2) zeros(2,4)]';

C=[0 0 0 0 0 0 0 0 0 0 1 0;...
   0 0 0 0 0 0 0 0 0 0 0 1];
Dtop=zeros(2,2) ;
Qnew_top=eye(14,14);
Qnew_top(14,14)=0.001;
Qnew_top(13,13)=0.01;
Qnew_top(10,10)=1;
Qnew_top(9,9)=1;
Rnew_top=eye(2,2)*0.01;


SYStop=ss(A,B,C,Dtop);
[Krtest,Stest,etest]=lqi(SYStop,Qnew_top,Rnew_top);
F2=Krtest(1:2,13:14);
F1=Krtest(1:2,1:12);
[Kfull,Lfull,Pful] = kalman(SYStop,eye(2,2)*0.001,eye(2,2)*0.001);

[NUM,DEN] =ss2tf(A,B,C,Dtop,2);
[NUM1,DEN1] =ss2tf((A-B*F1),B,C,Dtop,2);

Openloop=tf(NUM(1,:),DEN);
Closedloop=tf(NUM1(1,:),DEN1);
```

```
%-------------------Plots-------------------
figure
sim('helicopter_main_lqr')
plot(t.signals.values,Pos.signals.values(:,1),('r--'));
hold on
plot(t.signals.values,Pos.signals.values(:,2),'b--');
hold on
sim('helicopter_main_lqr')
grid on
plot(t.signals.values,Pos1.signals.values(:,1),('r'));
hold on
plot(t.signals.values,Pos1.signals.values(:,2)),('b');
legend('X pos no LQR','Y pos no LQR','X pos with LQR','Y pos with LQR')
axis([0,3,-0.08,0.1])
xlabel('time[sec]')
ylabel('Position [m]')
grid on
hold off

figure
sim('helicopter_main_lqr')
plot(t.signals.values,Pos.signals.values(:,1),('r--'));
hold on
plot(t.signals.values,Pos.signals.values(:,2),'b--');
xlabel('time[sec]')
ylabel('Position [m]')
grid on


figure
grid on
bode(Openloop)
hold on
bode(Closedloop)



figure
sim('helicopter_main_lqr')
plot(t.signals.values,Control.signals.values(:,:),('g'));
hold on
plot(t.signals.values,Control1.signals.values(:,:),'b');
legend('Control signal no LQR','Control signal no LQR','Control signal
LQR','Control signal LQR')
axis([0,0.3,-0.08,2])
xlabel('time[sec]')
ylabel('Position [rad]')
grid on



figure
sim('WORKING_helitrack')
plot(t.signals.values,Pos.signals.values(:,1),('r'));
```

```
hold on
plot(t.signals.values,Ref.signals.values(:,1),'r--');
hold on
plot(t.signals.values,Pos.signals.values(:,2),'b');
hold on
plot(t.signals.values,Ref.signals.values(:,2),'b--');
legend('X pos with tracking','X pos refernce','Y pos with tracking','Y pos
reference')
xlabel('time[sec]')
ylabel('Position [m]')
grid on

figure
sim('WORKING_helitrack')
plot(t.signals.values,Control.signals.values(:,1),'k');
hold on
plot(t.signals.values,Control.signals.values(:,2),'c');
legend('Control signal phi','Control signal theta')
xlabel('time[sec]')
ylabel('Reference [rad]')
grid on
```